National Income:
Statics and Dynamics

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To Anne

Contents

Preface

This work was developed as the result of a need for better textual materials in courses taught by the author at the University of Alabama in national income economics and business cycles. In the area of national income a source explaining fully the basic building blocks for analysis was needed. In business cycles, material establishing a deeper and broader relation to national income was required. My attempt to meet these needs resulted in a text stating national income theory in a way which leads directly to current ideas on business cycles, inflation, and economic growth. The work is specifically designed as a text in national income economics and, in addition, as the unifying material for a course in business cycle theory or macro-dynamics to be supplemented by readings from original sources. Problems are included in later chapters to reinforce the student's understanding of the subject and to sketch out less important theoretical topics.

For a course in national income, Chapters 1 through 7 should be assigned. If desired, Chapter 6 on Employment Theory can be omitted. If the instructor wants a deeper treatment of economic policy, he can assign Chapter 8, Economic Policy—More Advanced Techniques. Together with assigned outside readings in original sources, this material is adequate for a rather leisurely course in national income theory. It omits the topic of national income accounting which may be introduced via outside readings. In its static section the text presupposes no training other than a reasonably strong course in principles of economics. However, it is aimed at the undergraduate with maturity gained through taking some other courses in economics or related areas. The plan suggested omits dynamics, but the instructor can add as much of this as he feels appropriate, in particular, Chapter 9 on the Simple Economic Dynamics of National Income.

For a course in business cycle theory or macro-dynamics it should be assumed that Chapters 1–8 have been covered. A brief

class review of these chapters with a complete reading by the student should serve as an introduction. Part II, including Chapters 9–15, covers topics in the basic aspects of business cycles, inflation, and growth theory. These chapters contain the central contribution of national income theory to economic dynamics. As such, they form a proper foundation for readings in original sources. No list is included, because the emphasis in courses of the type under discussion is subject to wide variation. A fair amount of mathematics has crept into the discussion, but its use is limited to algebra covered in the first college course. Chapter 12, entitled Notes on the Value of Capital and the Marginal Efficiency of Capital, is designed to strengthen understanding of the concept of the marginal efficiency of capital, but its omission will not hamper understanding of the other topics. Chapter 13, on Investment Growth and Its Effect on Income, presents a development of multiplier theory. While relatively important in itself, this chapter may be omitted without loss to the understanding of the material in other chapters. The philosophy of the dynamics section of the text is that it should serve as a point of departure for readings in this rapidly changing field.

The basic framework of analysis was provided by John Maynard Keynes's *General Theory of Employment, Interest, and Money* and Hicks' convenient diagram summarizing that theory. Other basic parts of the apparatus were provided by Wicksell's concepts of the natural and money rates of interest, Gurley's graphical picture of fiscal policy, Samuelson's model of the interaction of the accelerator and multiplier, Kaldor's model of the cycle, Harrod's (Domar's) theory of growth, and the cost-push, demand-pull concepts of inflation. My main contribution lies in an attempt to synthesize the various elements into a unified and workable theory. A contribution to this unification was made by the publisher in a suggestion for the inclusion of additional policy material. This led to the development in Chapters 7, 8, and 15 of policy concepts related to the purely theoretical parts of the text.

My interest in this area of study was first aroused by the lectures of Dr. E. M. Bernstein, now associated with the International Monetary Fund. Later work with Dr. H. L. McCracken of Louisiana State University led me to further investigations.

My colleagues, especially Drs. R. M. Havens and Paul Paustian, have been helpful in ways both intellectual and practical. My main debt is to those faithful guinea pigs, my students in the classes already mentioned. The various ideas were successively presented, discussed, and modified in a sort of Hegelian process. Considerable help was provided by one of my assistants, David Woodham, who contributed materially to the presentation at a number of points, especially the chapter on Employment Theory. Mrs. Thomas Dobbins was most helpful in the preparation of the manuscript. Finally, my wife persisted in encouraging my work through all the vicissitudes incident to its completion.

A grant from the University Research Committee of the University of Alabama for secretarial assistance and supplies provided help in the preparation of the manuscript without which its appearance would have been long delayed.

Any errors in the text are, of course, the sole responsibility of the author.

JOHN S. HENDERSON

PART 1

NATIONAL INCOME FROM THE STATIC VIEWPOINT

In Part I the basic concepts of national income economics are outlined. These concepts, together with the notion of an equilibrium, serve to explain the levels which national income, employment, and other variables assume at a given ime. Since attention centers on the *levels* of national income and other variables, the approach tends to be static in character. The principal issue discussed is whether an economic system moves automatically to a full employment level, or whether it may fall short of or exceed this level.

CHAPTER 1

Introduction to national income economics

The inception of national income theory

Compared with the study of other branches of economics, the study of national income is comparatively new. From a practical viewpoint it dates from the publication in 1936 of John Maynard Keynes's book, *The General Theory of Employment, Interest, and Money.*[1] Prior to this time various writers had discussed national income, but none produced any basic change in economic thinking. In fact, Say's Law, the only principle of income determination then known to economists, asserted that full employment income was always attainable. In Keynes's work the contrary view is advanced —that income may be determined at a level which falls short of or exceeds the full employment level. If Keynes was right, earlier writers had failed to provide usable tools of income analysis.

ECONOMICS PRIOR TO PUBLICATION OF THE GENERAL THEORY

As a formal discipline, economics is generally believed to have had its beginning in 1776 with the publication of the *Wealth of Nations* by Adam Smith. Smith and certain of his successors in England established what is known as the classical school. Of his successors, the most influential was David Ricardo, who wrote a book which was accepted by most economists as the final word in economics for over half a century. It appeared in 1817 under the title, *Principles of Political Economy and Taxation.*

Both Smith and Ricardo took note of the problem of national income or aggregate demand. In particular, Ricardo adapted from a French economist, J. B. Say, a very definite view on the subject, known as Say's Law.

[1] John Maynard Keynes, *The General Theory of Employment, Interest and Money*, New York, Harcourt, Brace, 1936.

Stated in national income terminology, Say's Law asserts that "production creates its own demand." Without detailing the argument, it is evident that this is a very reassuring doctrine. If true, it guarantees that an economy can always operate at capacity. If production falls short of capacity, new workers can be hired who will be paid incomes. As they turn out additional goods, the extra workers receive income which will be used to buy up the additional flow of goods. Continuing in this way, the economy could always move to capacity production and full employment of resources.

Although a famous contemporary of Ricardo, T. R. Malthus, strongly disagreed with this notion, Say's Law held the field as an economic principle for many years. From a pragmatic viewpoint, a principle is "true" if it works. More precisely, if the available data support a contention, then that contention may be accepted as a working hypothesis. Since the first half of the nineteenth century witnessed the expansion of the political and economic system of the West, depressed economic conditions were exceptional. For this reason conditions were favorable to the acceptance of Say's Law. Unimpressed by Ricardo's argument, Malthus pointed to repeated periods of depressed economic activity during which Say's Law did not seem to operate. Since his theoretical argument was somewhat weak, Malthus was unable to capitalize on his empirical study, and his viewpoint did not prevail. In work subsequent to that of Ricardo and Malthus, interest was maintained in the possibility of variations in employment, production, and national income. However, with Say's Law as a central doctrine, the possibility of any prolonged lapse from capacity production appeared to be out of the question. Since this phase of economics appeared to be in perfect order, students of the subject turned to other matters.

For about a century economists busied themselves with the study of individual firms, households, and industries. Over the course of time their theories were developed to a rather high pitch. Meanwhile, Say's Law found continued acceptance. Theories of economic cycles were interpreted to explain only temporary lapses from a full employment situation.

In 1929, however, history finally supervened. In that year began the biggest, deepest, and most prolonged depression hitherto

recorded. So severe was this depression that by 1933, 25 percent of the labor force of the United States was unemployed, the banking system was in a state of collapse, and the system of private farm ownership was crumbling. Gross per capita production in real terms stood at about two-thirds of its 1929 value. Against such a background it is small wonder that faith in Say's Law began to weaken. At this juncture a prophet was needed to declare the falsity of Say's Law and to proclaim new truth. Keynes was this prophet.

During the early 1930's economists rationalized severe unemployment in two main ways. First, they argued, wage rigidities such as those imposed by labor unions prevented wages from falling. As the prices of products fell under the influence of reduced demand, while costs remained rigid, profits fell. In turn, this drop in profits discouraged production and employment. If wages could only be cut sufficiently to bear their former proportion to prices, it was argued, unemployment would tend to be eliminated. This view in a sophisticated form was adopted by a distinguished English economist, A. C. Pigou. In the face of massive unemployment, however, this view could scarcely bear the weight of the facts. And proponents of wage cuts did not command a very enthusiastic audience in the 1930's—even among academic economists. Pigou's view is theoretically important, however, and receives examination later.

A second view of unemployment laid stress on monopoly conditions. Under rather general conditions it can be shown that an industry which is monopolistically organized tends to employ fewer workers than one organized competitively. Perhaps, it was argued at the time, increased unemployment is traceable to the unquestioned increase in large-scale monopolistic enterprise. Although economists readily concede that monopoly may lead to secular unemployment, they are not prone to admit that short-run fluctuations in employment are traceable to this cause. In consequence, few economists accepted this as a complete cause of the collapse of employment following 1929.

The foregoing reputable, orthodox theories were supplemented by notions of economic oscillation. However, these views tended to indicate that the economic system passed straight through a

period of depression into a period of recovery and then boom. In the United States between 1929 and 1940 serious unemployment existed at all times. Indeed, 15 percent of the work force was still unemployed in 1940. To all appearances we were brought out of this period of prolonged unemployment only by defense preparations and then war. Such prolonged depression amounted to economic stagnation and could not be brushed off as a mere cyclical departure from recurrent periods of good times. Accordingly, the theories of economic cycles then accepted did not appear to hold the answer to the problems of the time.

To summarize, economic collapse produced intellectual collapse. The so-called Great Depression was a condition that Say's Law, sticky wages, monopoly, and existing theories of cycles failed to explain to the satisfaction of economists. As the existing tools were applied to the situation, they broke in the hands of those using them. Evidently, a new set of tools was called for, and such a set was provided by Keynes's book.

The General Theory of Employment, Interest, and Money was produced in the crucible of an intense need in the world for a practical remedy to unemployment. At the same time it was called forth by the intense intellectual activity of economists working on a problem that had defied previous attempts at solution. In a very real sense the *General Theory* was addressed to explaining the problem of the hour—unemployment. Reactions to Keynes's work were varied. Since the book itself urged the economist to abandon his old intellectual armory, it was bound to arouse some strong negative responses. On the other hand, those who had tried the old tools and found them wanting were awaiting a call to a new intellectual cause. As a result, few readers had a neutral response to the new theory. Also, the tone of the book encouraged strong reactions. By turns brilliant, obscure, iconoclastic, dogmatic, filled with illuminating analogies, and plagued by worthless mathematics or paradoxical identities, the *General Theory* was an almost indigestible dose. Yet, somehow, it filled a need.

Even though basic notions such as "underemployment equilibrium" were accepted readily, extended arguments arose over proposed definitions. Certain concepts, such as "user cost," were found to be worthless. An almost unlimited amount of comment

poured forth on every aspect of the material. Ultimately, the body of the material contained in the *General Theory* was sifted down to a set of consistent, usable doctrines. As one distinguished economist, A. C. Pigou, has put it, we can now have "Keynes without pain."

Without doubt, the most important idea in the *General Theory* was the notion that an economic system generates a defined level of income and employment. Moreover, this level of income and employment corresponds of necessity neither to capacity production nor to full employment. In fact, this doctrine leads to a different economic philosophy than does Say's Law. Whereas acceptance of Say's Law implies belief in an economy which regulates itself, the new doctrine indicates the possibility of a continued economic problem even in "equilibrium." If one believes the new doctrine, one is led to the conclusion that full employment can only be ensured by some kind of planning to operate on the system's basic determinants. Although this does not imply a belief in detailed governmental planning, it does lead away from a purely private economic system.

On the side of economic method, Keynes made one important contribution. He introduced in an effective manner the concept of economic aggregates. Although other economists had discussed the economic system in terms of aggregates, their systems lacked a critical element: the concept of a functional relation between two or more economic aggregates. In particular, Keynes advanced the notion that aggregate consumption is functionally dependent on aggregate income. With the aid of this powerful new concept, Keynes effected an important scientific breakthrough. In fact, this concept is a heroic, but justifiable, simplification which cuts through countless problems in analysis.

A preliminary summary of basic doctrines

After completing the reading of a book we often find that the basic ideas have quite escaped us. In the paragraphs that follow the basic ideas to be developed in the text are stated without explanation. With these ideas in hand the reader will have some notion of what to look for in the text. Since no short explanation of

these principles is possible, none is given here. In fact, such an explanation is the task of the entire text.

Doctrine I. There are two convenient concepts of income. (1) Income can be thought of as the *cost of producing the aggregate output* of society. "Income at factor cost" is a measure of output at the cost of producing the output. This cost includes the wages of labor and such returns to the other factors as will cause them to be forthcoming. Hence it measures output conceived of as "supply." (2) Income may also be regarded as the *value of output*. In this sense, income measures the expenditure of society on output. Consequently, income as the value of output may be thought of as "demand."

Doctrine II. Income and employment are so determined that aggregate supply and aggregate demand are equal When this occurs, all available goods are cleared from the market, and producers are properly remunerated for their efforts. In this situation, income, at factor cost, is made exactly equal to total expenditure, and this equal value is called "equilibrium national income."

Doctrine III. The value of income determined by this process is not necessarily one which exactly employs all the factors of production. This doctrine is at variance with Say's Law.

Doctrine IV. Aggregate consumption is functionally dependent on national income. This principle announces the existence of the so-called "consumption function" or "propensity to consume."

Doctrine V. National income is functionally dependent on investment. According to this doctrine, income varies by a multiple of any change in investment. The relation between income and investment in numerical form is known as "the multiplier."

Doctrine VI. The rate of interest is determined by the supply of and the demand for money. Conditions in the money market are such as to preclude a fall in the interest rate below a certain minimum. This limits the power of the monetary authorities to stimulate investment through lower interest rates and, thereby, to bring about recovery from depression.

Doctrine VII. Employment is determined by the aggregate supply of and demand for output, not by the supply of and demand

for labor. The supply of and demand for labor define conditions of under- and overemployment and the wage.

Doctrine VIII. Income, consumption, saving, investment, interest, and the quantity of money are determined simultaneously. This doctrine of simultaneous determination of variables corresponds to the theory of simultaneous determination of prices of goods and services.

Beyond these eight principles are certain post-Keynesian doctrines. Some of the ideas which have been worked out since the *General Theory* are set forth in the text. Although they develop naturally out of Keynesian analysis, they are not specifically Keynesian. Only rather well-accepted ideas are listed here.

Doctrine IX. If this period's expenditure generates next period's income, an increase in investment generates a rise in income. Income follows a defined path in time, approaching an upper limit which is not exceeded with the passage of time.

Doctrine X. If this period's expenditure generates next period's income, and if investment depends on the increase in consumption, income tends to fluctuate in a periodic manner.

Doctrine XI. If investment increases by a constant amount per year, income rises following a specific path, until it increases by a (different) constant amount per year.

If investment increases by a constant percent per year, income rises following a specific path until it rises by the same percentage as investment.

Doctrine XII. If investment depends on the increase in consumption, and expenditure generates corresponding income in the same period, an increase in income at a constant percentage rate can be sustained.

CHAPTER 2

National income–statistical concepts

It is a basic principle of modern science that theoretical concepts should be susceptible to measurement and testing. National income theory constitutes a rather compact, simple set of concepts, which should satisfy the principles of science. National income may be used in several senses, and in each such sense should be measurable. Furthermore, as a matter of statistical calculation, each should be capable of being derived from the other.

Real and money income

By long tradition economists distinguish between economic quantities expressed in money and those expressed in goods or labor. On the one hand, then, economists speak of the sum of the money incomes received by the members of society as national money income. On the other hand, they refer to the flow of goods and services which the members of society may enjoy as national real income. To illustrate the distinction, suppose an individual receives a monthly income of $400. His only purchases consist in food packages at $1 a package. His money income is $400, while his real income is 400 food packages. If money income remains the same, while the price of food packages rises to $2, then real income drops to 200 food packages. On the other hand, if the individual's money income rises to $800, and the price of food packages rises to $2, then real income remains the same—400 food packages. In short, real and money income do not necessarily move together.

The circular flow and its components

As a background to the study of national income it is pertinent to consider the flow of money and goods in the economic system. This is described by the well-known circular flow chart, pictured in Figure 1.

A. The firm hires the factors of production from the households or individuals who own them. These factors are land, labor, capital, and entrepreneurship. Although the land may be hired from individuals, that is, rented, it is more commonly purchased outright with the aid of capital. Labor is provided by the working force of the firm, capital by stockholders or bondholders in a corporation or by the owner in an individual proprietorship, and the entrepreneurship by top management and associated groups.

Circular-Flow Diagram

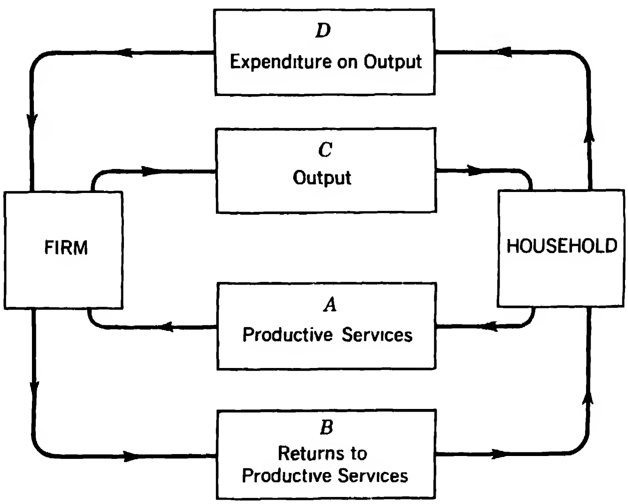

Figure 1. The Circular Flow of Economic Life.

B. In return for services rendered to it the firm pays out income. The owner of the land receives rent; the laborer receives wages; the capitalist receives interest; and the profits are distributed to the owners or top managers. The sum of all these factor payments is referred to as the national income.

C. With the income received from factor payments, the household purchases output. The flow of output to the household constitutes real income because it represents a flow of goods which may be enjoyed by the consumer. In this connection an awkward

problem presents itself. Since the quantities of different commodities are incommensurable, it is impossible to sum the quantities of the various outputs directly. Thus, 2 dozen apples + 3 bottles of ink + 1 shot of penicillin = What? Evidently, the measurement of real income is not an elementary task.

D. In exchange for this flow of output the household spends money and thus contributes to a flow of money to the firm. This expenditure on output may be regarded as a money measure of the output. When each good is valued at the price prevailing in the market, a common unit is established in terms of which each commodity may be measured. This unit, of course, is the dollar or monetary unit. By adding up the total amounts spent on the several goods measured in dollars, we arrive at the total expenditure on output. In fact, however, this procedure serves to measure expenditure and not output. This value of output can be taken as a measure of income.

Ways of calculating national income

INCOME AS THE SUM OF FACTOR PAYMENTS—FLOW B IN FIGURE 1

Under this approach income is regarded as the sum of factor costs. To arrive at income you simply find the income shares received by the various factors and add them. It is difficult to find statistical information which corresponds precisely to the traditional division into wages, interest, rent, and profit. Table 1 comes as close as possible to this design. Such an approach to statistical calculation of income is not currently in favor.

INCOME AS EXPENDITURE OR THE VALUE OF OUTPUT—FLOW D IN FIGURE 1

In calculating the value of output we may follow two methods. According to the first, we find the several totals of the goods produced, multiply by their respective average prices during the year to find values, total the values, and thus arrive at the total value of output. In effecting such a calculation it is necessary to keep sight of two problems. First, it is necessary to count only those products which are sold for the last time, the final product. If an automobile

company buys certain raw materials which it embodies in its product, the raw material should not be counted in addition to the automobile. Second, it is necessary to understand the meaning of "final product." Let us define a final product as any item which is

TABLE 1. National Income, 1959—the Sum of Factor Payments

Wages (compensation of employees)			
Wages and salaries		257.8	
Supplements to wages and salaries		19.6	
Total compensation of employees		277.4	277.4
Interest (plus some pure profits, rent, and wages)			
Undistributed profits	11.3		
Cash dividends	13.2		
Corporate profits after tax	24.5	24.5	
Corporate profits tax liability		23.3	
Inventory valuation adjustment		−.6	
Corporate profits and inventory valuation adjustment		47.2	
Net interest		15.6	
Total interest income		62.8	62.8
Rent on land (plus some interest on capital, including real estate improvements and capital rented with land)			
Rental income of persons			12.0
Mixed returns (wages, interest, rent, and profit)			
Unincorporated business and professional income		34.5	
Farmers' income		11.8	
Total mixed returns		46.3	46.3
National Income			398.5

SOURCE: *Federal Reserve Bulletin*, May 1960, p. 560. The arrangement of the material follows Whittaker, *Economic Analysis*, New York, Wiley, 1956, p. 394, Table 35.

the object of current productive activity and which cannot be resold during the current period. For example, a newly produced automobile just added to stock by a manufacturer which is not sold during the period in question, is counted as a final product. If the automobile is sold to the consumer for $2200 during the current period, this sum contributes a corresponding amount to the national income. However, we may *not* count the raw materials entering into the production of the good, or the selling activities involved in

transferring it to the consumer. To count these would be to duplicate some of the elements entering into the final product.

A second procedure for finding the value of output starts by determining the amount each stage of production adds to the value of the final product, and concludes by finding the sum of the several amounts. Such an approach is known as the "value-added

TABLE 2. Income by Value-Added Method, Extended Hypothetical Auto Example

Value Added in Industry			
A. Steel[a]			$ 200
B. Rubber[a]			100
C. Glass[a]			100
D. Auto manufacturing			
Sales to retailers		$1600	
Less purchases:			
Steel	200		
Rubber	100		
Glass	100		
	400	400	
Value Added in Auto Manufacturing		1200	1200
E. Auto selling			
Sales to consumers		2200	
Less purchases from manufacturer		1600	
Value Added in Auto Selling		600	600
Value of Production or National Income			2200

[a] Industry makes no purchases from other firms (by assumption).

method." Returning to our example, we find the value of automobile sales to dealers and subtract the purchases of products, such as steel, glass, and rubber, from other producers. The difference is the value added (to the materials with which the firm begins) by the activities of the firm. Suppose steel, rubber, and glass are industries which are completely self-contained as to raw materials. We can then add the values of steel, glass, and rubber to value *added* in autos to get the total value of the product at the manufacturer's level. Finally, we may add the selling contribution of the dealer. All this is displayed in Table 2.

To recapitulate, we may think of national income in two ways. First, national income may be regarded as the sum of those payments made to the factors of production to reward them for their services. In this sense national income is the sum of wages, interest, rent, and profit. Clearly, national income thus understood measures the factor cost of the output currently being produced. To calculate national income in this sense it is necessary to add the income shares earned by the several factors.

Second, national income may be thought of as a flow of goods which society will enjoy and use. In order to measure the flows of disparate goods, each good is valued at the price for which it is sold in the market. By adding the values of the several outputs sold in the market during the year we arrive at the total value of output for society. This is a measure of the flow of real income expressed in money and is also called national income. Clearly, it expresses the expenditure of society on output. Income in this sense can be measured either by adding the values of goods sold for the last time, neglecting those not sold for the last time, or by summing the values added by every firm in the productive process.

Government and the calculation of national income

The bulk of government's contribution to the national income takes the form of services rendered directly to the people. Almost all local governmental expenditures take this form, consisting of fire and police protection, the administration of the laws, the upkeep of highways, and the maintenance of schools. The Federal Government devotes the bulk of its outlays to the provision of defense. Services of these kinds are rendered directly to the public without the mediation of any exchange transaction. Since the value of these services cannot be measured by the expenditure of the public, the value of output method cannot be used. Instead, economists resort to the factor costs of rendering these services. In this area the outlays required to render these services are used to measure the contribution to the national income. At the same time the costs of rendering these services are defrayed by various taxes levied on the people. Finally, the national income is measured by the value of final output privately produced plus government

services at cost. Since most government activity is not inspired by the profit motive, the factor costs of government services do not include profit. This sets the treatment of government-generated income on a different plane from that of private business.

Gross and net production

In using the value of output method of computing income, two major problems arise. Both revolve about the fact that gross and net production are not the same. National income is a measure of that flow of real income which can be used in any desired way without encroaching on the stock of capital. When production is measured in terms of the money paid for it in the market, two extraneous elements appear.

DEPRECIATION

In the first place, part of the flow of production is required to replace capital that is being wasted through time and use. The most important element here is depreciation, or the wastage of capital as a result of use through time. However, capital may also be wasted as a result of obsolescence, or "acts of God" such as fire or flood. Thus, new and more modern machinery renders old production methods and equipment obsolete. As a result, this equipment loses its value and must be replaced. In order to find the flow of goods which society may enjoy, we must deduct depreciation from the gross value of output.

Suppose that the capital equipment of a country consists of ten machines. Each machine has a life of ten years and costs $100. Assume that the acquisition of the machines was staggered and that, in consequence, one machine wears out each year. With the aid of this capital society produces a gross output of $1100. If this $1100 can be used in any desired way without encroaching on the supply of capital, it measures national income.

Suppose that in the first year under consideration the entire gross output is devoted to the production of goods which are consumed during that year. Evidently, one machine will wear out by the end of the first year. Since no production is allocated to replacement, the capital stock will have declined to nine machines, or $900. If this course is pursued for ten successive years, the capital

stock will be worn out at the end of a decade. By this time capitalistic production will have ceased.

From the consequences of consuming the entire $1100 it should be clear that society cannot dispose of its entire gross output. If society sets aside $100 to replace the annual wastage of capital, $1000 is left. Any desired use of this sum can be made without encroaching on the stock of capital. In this case production can be indefinitely sustained. Consequently, it is *gross output less capital wastage* which measures the flow of income available for free use by society. In schematic form this may be represented as follows:

Gross production	$1100.00
Less depletion of capital	100.00
Net production (National Income)	$1000.00

INDIRECT TAXES

Our second problem may be posed in the form of a question. Does the entire value of output correspond to a flow of goods? From price theory we know that a sales tax imposed on the sale of a product tends to raise the price of the article. Next, we know that the proceeds of such taxes are used to finance governmental services such as defense or fire protection. In calculating the national income the services rendered by government are entered (at cost). In short, the tax corresponds to an element included in value of output under government services rendered. If the value of private output contains an element of tax in the form of higher prices of goods, the element has been counted twice. To recapitulate, the tax was included once in the price of private output, higher by the amount of the tax, and included also in the guise of government services which the taxes finance.

To illustrate, suppose a suitcase is sold for $36, of which $6 or 20 percent of the net price of $30, constitutes tax. The government uses this $6 to hire a clerk to type up some forms. We then have an income calculation which goes as follows:

Suitcase	$36.00
Less tax	6.00
Private output	30.00
Government output (clerk)	6.00
National income	$36.00

By deducting the $6 we avoid the double counting discussed.

A second way of regarding the same matter consists in looking at the flow of factor incomes. The $6 paid to the seller in tax is not available to be passed on to privately hired factors of production. In fact, the government intercepts this money before such a thing can happen. Obviously, this tax money cannot become private income. It is of no avail to say that the money is used to finance government output. Payments for factor services rendered to the government are calculated independently.

From the preceding discussion it is clear that two deductions must be made from total private output at sale prices to get national income. These are depreciation and indirect taxes.[1] The Department of Commerce of the United States Government refers to the value of output at sale prices as Gross National Product or GNP. The result obtained by subtracting capital wastage from the value of output is entitled Net National Product or NNP. The result obtained by subtracting indirect taxes from NNP and making some minor adjustments is the National Income. We shall capitalize the names of all income concepts of the Department of Commerce.

There are many ways of considering National Income. We might consider the contribution to National Income made by the several industries. Likewise geographical breakdowns might be made. Although such breakdowns would be interesting and enlightening, we shall confine our attention to the broadest aggregates.

TABLE 3. National Income in 1959
(In Billions of Dollars)

Gross National Product	$479.5
−Capital Consumption Allowances	40.2
Net National Product	439.3
−Indirect Business Taxes (and other minor items[a])	40.7
National Income[b]	$398.5

[a] Other minor items included in the total amount to $1.3, combined.
[b] Difference between value shown and result of subtraction is the outcome of a rounding error.
SOURCE: *Survey of Current Business*, February 1960, p. 13.

[1] To avoid double counting it is judged necessary to deduct indirect (e.g., sales) taxes from gross output in securing national income. Why not deduct direct taxes also? By definition a direct tax is one which is intended to be borne by the persons on whom it

From personal income to national income

National income measures the amount earned by the factors as a result of the productive contributions of the current period. Since all factors are owned by someone, it is clear that all income should make its way into someone's hands. However, there are several ways in which money may be added to or withdrawn from the income stream before it reaches the consumer or household. Such additions and withdrawals are associated with corporations and governments. Let us consider the various adjustments which must be made to "take-home pay" in order to get factor incomes.

Let us start with the consumption and savings of individuals. The sum of the consumption and savings of individuals equals Disposable Personal Income. This is a title used by the Department of Commerce to designate the income actually received by households. If we add personal taxes, we get what is called Personal Income. Quite obviously, the household cannot "dispose of" the entire amount of Personal Income owing to the tax liability attached. Personal taxes consist mainly of direct taxes such as personal income taxes, inheritance taxes, gift taxes, and taxes on land owned by persons. In general, economists hold to the opinion that direct taxes are paid ultimately by the persons on whom they are levied. On the other hand, they believe that at least part of indirect business taxes, such as sales taxes, are passed on from the businessman to the consumer. For this reason these items are handled differently in the accounts.

From Personal Income we must subtract an item known as "transfer payments." A transfer payment is a current inflow of funds to a person which is not received in virtue of a current productive service rendered by the recipient. As a typical example, benefits received by a retired individual under the social security

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is levied, whereas an indirect tax is levied on one set of persons in the expectation that it will be passed on to others. According to most tax theorists, personal and corporate income taxes cannot be passed on to others in the form of higher prices (at least in the short run). Consequently, Gross National Product is *not* believed to contain a significant element representing an attempt by corporations or individuals to recover income tax. In turn, this eliminates the necessity for a deduction to avoid double counting. Consequently, sales taxes which are believed to be roughly indirect require an adjustment, whereas income taxes do not.

system are classed as government transfer payments. Such outlays as business gifts to nonprofit organizations are classified as business transfer payments, because the organizations in question are assumed to distribute the proceeds directly to individuals in the form of services. Evidently, the recipients of these benefits, e.g., disaster victims securing food from the Red Cross, are granted goods and services which they do not earn. Since Personal Income includes such an unearned element, it must be subtracted in moving to earned income, that is, National Income.

Next, we subtract dividends paid to stockholders. This eliminates from Personal Income the only part of corporate profits which enters into the former total. In short, our subtotal at this point excludes corporation profits completely.

Next come the additions. In the first place, all corporation profits (plus inventory valuation adjustment) belong in National Income, since they have been earned as a result of the firms' productive activities. The only portion of such profits entering Personal Income has been deducted by the subtraction of dividends. Consequently, it is now appropriate to add corporation profits (plus inventory valuation adjustment).

Finally, we add contributions for social insurance. Although these contributions do not enter into anyone's take-home pay, they have been earned during the year and must be included in National Income. We finally add any excess of wages earned over wages paid, the excess of wage accruals over disbursements.

After all the above adjustments have been made to Disposable Personal Income we arrive at National Income. The adjustment process is recorded in systematic form in Table 4.

The preceding discussion indicates the difference between National Income and Disposable Personal Income. In any practical discussion of the determination of income it is impossible to ignore this difference. Since consumption is based on take-home pay or Disposable Personal Income, rather than National Income, a knowledge of DPI is necessary to determine consumption. In turn, consumption is one of the basic elements of society's expenditure on output. Finally, expenditure on output is a way of measuring National Income.

TABLE 4. Disposable Personal Income and National Income, 1959 (in Billions)

Consumption	$311.6	
+Personal savings	23.1	
=Disposable Personal Income	334.6[e]	
+Personal tax and related payments	45.5	
=Personal Income	380.2[e]	380.2
−Government transfer payments	25.1	
−Net interest paid by government[a]	6.8	
−Business transfer payments[b]	1.7	
Total transfer payments	33.6	−33.6
−Dividends		−13.2
+Corporate profits and inventory valuation adjustment[c]		47.2
Contributions for social insurance		17.9
Excess of wage accruals over disbursements[d]		.0
=National Income		398.5

[a] Interest on private debt is included in the National Income, whereas net interest on government debt is not. To account for this, consider that the main component of government debt is Federal debt which consists principally of money raised during wars. Most of the money was used for purposes such as the pay of soldiers and the purchase of equipment which soon lost its value. In short, little or no capital was created with the aid of the money. Consequently, no interest is earned by the government. Such sums as are paid in interest are included in Personal Income, and must be deducted in moving to National Income.

[b] Business Transfer Payments consist of such items as business givings to charitable institutions. In turn, these institutions presumably make these sums available to individuals in money or in kind. To go from Personal to National Income, it is necessary to subtract these unearned amounts from Personal Income.

[c] Inventory valuation adjustment is equal to the excess of the accounting cost attributed to the inventory over the replacement cost. Insofar as accounting cost diverges from replacement cost, the relevent economic concept, an error in the computation of profit and, therefore, of income is introduced and must be eliminated.

[d] Less than $.05 billion.

[e] Failure of items to add to totals is the result of rounding errors.

Special problems in calculating income

In the calculation of income by the counting of final products, the method used by the Department of Commerce, several problems present themselves. First, is it proper to include services in the totals? To find an answer, let us recall from elementary economics the commonly accepted definition which states that "production is the creation of utilities." On the one hand the "utilities" may consist of material goods, such as food and clothing, which yield satisfaction to the consumer. On the other hand, the "utilities" may consist of personal services such as those rendered by a physician. Evidently, the outputs measured may be both tangible and intangible in form, consisting of goods and services. Clearly, this gives an affirmative answer to the question posed.

As a second problem in handling services, the method of counting final products requires a precise interpretation of "final products." From an earlier discussion emerged the notion that it is improper to add the cost of materials used in production (such as glass) to the price of the final product (the automobile). By the same token it is inappropriate to add inputs of services to outputs. Consider the following cases: clearly, it is proper to include in output the services of a physician to a patient, since the latter experiences satisfaction as a result of the service. By way of contrast, the worker painting a car coming down an assembly line renders a service or input which helps to create an output, the automobile. In computing the value of output, it is appropriate to count the physician's services and the value of the automobile, but *not*, in addition, the wage of the painter. Since the painter's wage enters into the price of the automobile, its inclusion would involve doubling counting. From another viewpoint, the painter's activity constitutes input in contrast to the physician's services which constitute output. Evidently, the addition of input (painter's wages) to output (automobiles and physician's services) can lead to nothing sensible. By definition the inputs, painter's wages, are excluded in calculating the value of output.

To summarize these points: (1) output consists of both goods and services; (2) only goods and services which can be classified as output are to be included in the value of the product.

Consider now a problem of quite a different sort. In calculating aggregate output the economist encounters certain goods and services which are quite hard to value and, therefore, to combine with the remainder of the output. Taking a rather practical viewpoint, the Department of Commerce includes such output only when a regular exchange process values them. For example, the output produced by farmers, but retained for their own use, is included. Such output is sold under competitive conditions on the open market, is comparatively homogeneous in quality, and can be accurately valued. On the other hand, the services of a housewife, though valuable, cannot be calculated quantitatively. On this ground they are excluded.

As a third problem consider the treatment of sums received by households, and which constitute personal income to them, but

which involve the rendering of no corresponding productive service. To be brief, such transfers are excluded from National Income by the Department of Commerce. Income is thought of as a flow of money in return for productive services currently rendered. If a thug hits you on the head in a dark alley and takes twenty dollars from your billfold, National Income remains unchanged. The thug merely has transferred the money from your possession to his. When the government taxes A, and gives an unearned, though perhaps justifiable, handout to B, National Income is not changed by this act. Income is transferred, not created.

Breakdown of national income by expenditure classes

National Income considered as the value of output serves to represent total expenditure. Considerable light can be shed on National Income by classifying these expenditures. One difficulty which appears immediately in connection with investment expenditure is that it is quite difficult to obtain truly accurate and consistent measures of depreciation. Rather than put on the record uncertain reports on net investment, gross investment less depreciation, government statisticians prefer to deal with the gross concept. This figure includes that part of output designed to replace the depletion of capital in the current period. Since statistical considerations require the use of gross investment, consistency demands the use of gross income or GNP as the object of study. In Table 5 Gross National Product is broken down into four parts: personal

TABLE 5. Gross National Product by Expenditure Classes, 1959 and 1929 (in Billions of Dollars and Percentages)

	1959		1929	
Personal consumption expenditure	$311.6	65.0%	$79.0	75.7%
Gross private domestic investment	71.1	14.8	16.2	15.5
Net foreign investment[a]	−.8	−.2	.8	.7
Government purchases of output	97.6	20.4	8.5	8.1
Gross national Product	$479.5	100.0%	$104.4[b]	100.0

[a] Net foreign investment means roughly the excess of goods and services rendered to foreigners by the United States.
[b] Failure of items to add to total is the result of a rounding error.
SOURCE: *Federal Reserve Bulletin*, May 1960, p. 328.

consumption expenditure, gross private domestic investment, net foreign investment, and government purchases of goods and services.

Undoubtedly, the outstanding feature of this table is its revelation of the increased importance of government. In 1929 government's share in total production was about 8 per cent. By 1959 its share had risen to about 20.4 percent, or roughly one-fifth of total production.

A word may be needed on the item "net foreign investment." This item gives the excess of sales abroad (and other activities causing foreigners to owe us money) over purchases from abroad (or other activities by foreigners causing us to owe them money). The difference defines the net amount that foreigners owe us as a result of the year's activities. Since they owe us the money, this sum may be regarded as a "loan" or as "net foreign investment." The item is small for two reasons. First, purchases of goods tend to equal sales, and so the two elements tend to be equal, and the difference, zero. Second, international transactions constitute only a small part of our National Income.

If we owe foreigners more than they owe us, net foreign investment will be negative. In fact, the United States is generally in the position of selling more than it buys, because its advanced technology and abundant resources make it a source of goods which cannot easily be obtained elsewhere. A second point is that for every dollar of net foreign investment the United States has, there is a corresponding negative item for some country trading with us. In short, this item tends to be zero for all countries taken together.

Final run-down on national income concepts

In the treatment given in this chapter the theoretical concepts of income as the sum of factor payments and as value of output have been coordinated with the specific statistical concepts of the United States Department of Commerce. This is a practical necessity, because this department is the main source of information concerning national income. As a final step in creating familiarity with the income concepts used by the Department of Commerce, we will present and comment briefly on a typical National Income table.

This table presents the relations between all the income concepts which have been mentioned. In this presentation we start with Gross National Product and end up with Disposable Personal Income.

In Table 6, Tables 3 and 4 have been combined. The material of Table 3 has been expanded to fill out the detail actually given in

TABLE 6. Relation of Gross National Product, National Income, Personal Income, and Saving, 1959 (in Billions of Dollars)

Gross National Product			$479.5
−Capital Consumption Allowances			−40.2
=Net National Product			439.3
−Indirect business taxes and other adjustments:			
Indirect Bus. tax and related liabilities		42.0	
Business transfer payments[a]		1.7	
Statistical discrepancy[b]		−2.3	
−Subsidies less current surplus of government-operated enterprise[c]		−.7	
−Total of above four items			−40.7
=National Income			398.5
−Corporate profits and inventory valuation adjustment			−47.2
−Contributions for social insurance			−17.9
−Excess of wage accruals over disbursements			.0
+Dividends			13.2
+Government transfer payments	25.1		
Net interest paid by government	6.8		
Business transfer payments	1.7		
Total transfer payments			33.6
=Personal Income			380.2
−Personal Tax and related payments			45.5
=Disposable Personal Income			334.6
−Personal consumption expenditure			311.6
=Personal Savings			23.1

[a] Like indirect business taxes, business transfer payments are included in GNP, the total quantity of goods produced at the prices for which these goods are sold. Since the sums are transferred to individuals or nonprofit organizations, the money is not available to be paid out to the factors as income. In short, this constitutes a part of GNP which does not flow into factor income.

[b] An entry made to reconcile divergent statistical estimates.

[c] Subsidies consist of elements such as soil conservation payments to farmers. Whereas no immediate improvement in the sale value of the farmers' output is registered, the subsidy corresponds to an improvement in the capital value of the farm. In short, this is treated as an investment in capital goods. Surplus on government-operated enterprise implies that government output—say, electricity—is being sold at a profit. Since government output is consistently valued at cost, it is logical to subtract this item to put all government output on a consistent basis.

Survey of Current Business, Febuary, 1960, p. 13; *Federal Reserve Bulletin*, May 1960 p. 561

the usual presentation. Notes explain the new entries. The material in Table 4 is unchanged, but the direction of movement is reversed. Such a table is sometimes referred to as a "reconciliation" of the various income concepts. This means that Table 6 reveals the way in which the various concepts can be derived from each other. Note, however, that a statistical discrepancy appears in the statement.

If we now appear to have a plethora of income concepts, the reader can take comfort from the fact that National Income will be used consistently as the basis of further theoretical argument. For practical purposes it is essential to have other measures, but we cannot make theoret cal use of them all. With the above introduction the reader should be able to read National Income statistics with some idea of their meaning.

CHAPTER 3

Consumption

By far the largest of the major components of expenditure on output is consumption. Consumption may be defined as the amount of money spent on things which yield direct satisfaction. Under this definition an automobile is consumed when the customer signs the purchase agreement. At the same time, the purchase of a bond which yields an income stream in the future does not yield direct satisfaction. Accordingly, its purchase is not classified as consumption. Saving is defined as that part of income which is not consumed; it is what is left over after the consumption decision has been made.

Consumption function

From the viewpoint of National Income economics certain relationships between aggregates are assumed to exist. Of these the most prominent is a relationship between aggregate consumption and National Income. For every level of National Income there is assumed to be a corresponding level of consumption. From the viewpoint of National Income economics it is perfectly natural to start with such an assumption and appeal to the facts for justification. On the other hand, we may also refer back to individual behavior as our starting point. Pursuant to the first approach our argument commences with society as a whole.

The level of consumption at a given level of National Income depends on a number of considerations. Among these are: (1) factors relating to the disposition of income, (2) the rate of interest, (3) the desire to hold cash, (4) the price level. Let us note briefly the nature of these influences.

FACTORS RELATING TO THE DISPOSITION OF INCOME

The distribution of National Income is, after the level of income, the most important influence on consumption. With a very uneven

distribution, income is largely concentrated in the hands of a small fraction of the population. Since individuals with large incomes are proportionately large savers, aggregate saving is increased at the expense of consumption. On the contrary, a very even distribution of income leads to a concentration of the bulk of all income in the hands of individuals with moderate incomes. Such individuals do not save a great deal. In consequence, savings tend to be held to a low level and consumption increased. To summarize, aggregate consumption varies directly with equality in the distribution of income.

Next, consumer tastes constitute an important determinant of the level of saving and consumption. Insofar as the social attitude toward saving is favorable, saving will be encouraged. So, according to Benjamin Franklin, "a penny saved is a penny earned." In the nineteenth century a Calvinistic attitude prevailed among the upper income groups. According to the pattern of thinking which then prevailed, a saver benefited society by enabling it to equip itself with capital goods. In turn, the additional capital permitted the creation of a larger national income. By his act of saving the individual conferred a benefit upon society. Since the saving was mostly accomplished by persons with large incomes, it was the well-to-do who managed to achieve virtue in this manner.

In the present century an attitude less favorable to saving has developed. Especially in America, a very complex machinery has developed for making high-level consumption desirable, painless, and even a social necessity. By means of the continuous introduction of new products, more conventional goods are made to seem undesirable. Even when the product consists in some form of durable equipment, obsolescence brought on by an altered product induces the consumer to replace the old product while it is still serviceable. Concurrently, a financial apparatus has been set up by which the consumer can pay for goods a little at a time. By book credit and easy payments the consumer can make his consumption average out closer to his income than ever before. To complete the picture, psychological conditioning convinces the consumer that he should buy the article he does not "need" but "wants." For such reasons consumption generally amounts to about 92 percent of Disposable Personal Income in the United States.

Consider now the role of the corporation in the disposition of income. Since corporations dispose of the net income which they earn, they exercise an influence on consumption. In theory, corporations are owned by the stockholders. If all earnings were paid to stockholders, or if stockholders made decisions about the disposition of this income, corporation earnings would create no new analytical problem. However, corporations are run by hired executives who claim but a minor ownership in the business. Their decisions determine the fraction of corporate earnings which are paid out in dividends.

In recent years corporations have saved annually about half as much as individuals. They do this by retaining a fraction of corporate earnings after taxes. In the act of retaining earnings or saving, corporations reduce consumption correspondingly. Evidently, the fraction of corporate earnings to be retained is not unimportant in determining the level of saving and consumption. For this reason the factors determining corporation decisions concerning retained earnings are important determinants of consumption.

Finally, consider the influence of government. Government affects consumption in two main ways: by taxing corporations and by transfer payments enlarging the personal incomes of certain persons or groups. About half of current corporation earnings are drained off by corporation income taxes. Since corporations do one-third of all the saving, high corporation taxes reduce saving sharply and increase the capability for consumption. In particular, distribution of the tax receipts in direct payments to individuals would permit consumption to increase. At the same time high taxes on individuals with large incomes and high saving rates would reduce their saving sharply and consumption less sharply. If these receipts are distributed to the poor, the consumption level would be more than recovered, but the level of saving would suffer. For these reasons any change in tax policy or in the distribution of tax moneys may affect consumption.

RATE OF INTEREST AND CONSUMPTION

In the past economists inclined to the belief that the quantity of money saved increases as the rate of interest rises. In recent years growing acceptance has been accorded the view that saving

increases up to a given level as the rate of interest rises, but that at still higher rates, less is saved.

By way of comment, let us note that among the motives for saving may be listed: (1) the desire to increase future income, and thereby future satisfaction; (2) the urge to achieve greater security through the accumulation of a given sum of money. Taking the first case, suppose the objective of the individual is the attainment of a *given* income from the purchase of income-earning assets. As the rate of interest rises, the desired total income can be earned by a smaller amount saved and spent on the higher-yield assets.

Considering the second case, if the individual wishes to accumulate a *given* sum of money in a certain number of years, he does so by acquiring assets whose value grows at compound interest to the desired amount. A higher rate of interest will permit this to be done with a smaller contribution of savings. In either case the volume of savings may decline with a rise in the rate of interest. In the absence of some empirical test, we cannot say anything definite about the influence of interest on saving. Perhaps the safest statement is that the influence is somewhat weak. The same applies to the influence of interest on consumption.

THE DESIRE TO HOLD CASH

For reasons to be discussed later the individual and the firm find it desirable to hold certain reserves in the form of cash. If changes take place in the attitude of the public toward the holding of cash, this fact may affect the attitude of income recipients toward consumption. For one thing an increased desire for cash could manifest itself directly in reduced consumption and increased saving designed to satisfy liquidity needs. Also such a desire for cash could induce the public to sell securities, driving down the price of securities and raising the earnings or interest rate. With a higher interest rate consumers might reduce consumption with a view to increased saving. Finally, any outside factor altering the cash balance of the consumer or the purchasing power of that cash may affect consumption. An undesigned increase in the effective cash balance may lead to increased consumption, since the consumption may afford more satisfaction than the extra balance. This matter is mentioned further below.

THE PRICE LEVEL

Price level considerations fall under two main headings: static and dynamic. From a static viewpoint the main notion is the so-called "real balance effect." A fall in the price level raises the real value or purchasing power of the money balances held by individuals. After the change they may hold more purchasing power in liquid form than they need; that is, the surplus money may be used to better advantage to acquire bonds or to purchase consumer goods. Let us concentrate on the latter possibility. According to the real balance approach advocated by Patinkin,[1] a fraction of the increment of surplus real balances will be spent on consumption. Thus a decline in the price level leads to a rise in the level of consumption.

If the surplus real balances are devoted to the expenditure on bonds, the outcome is approximately the same. In this case the price of bonds is bid up, the percentage return declines, and the rate of interest falls. In turn, the lower rate of interest may (but this is not certain) prompt individuals to save less and, by the same token, to consume more. As a matter of fact, the influence of the rate of interest on investment is likely to be more striking.

Consider now the dynamic effects of the price changes. These cannot be predicted with any confidence. However, there is one point on which a good many authorities agree. A decline in prices, sustained for some time, tends to lead to an expectation of a continuation of the price change. If expected future prices are lower than present prices, people will tend to postpone present consumption in favor of future consumption.

A decline in prices has repercussions on the need for liquid balances. If the declining price trend is sustained, it is clear that it is desirable to "get out of goods and into money," because money is gaining in purchasing power relative to goods. Such a tendency would lead to accumulation of additional balances out of income, thus reducing the level of consumption.

It seems reasonably clear that it makes a good deal of difference whether the change in prices is a unique event event or part of a continuing trend. In the first case the effect of a decline in prices is

[1] *Money, Interest, and Prices*, Evanston, Row, Peterson, 1956, chap. 2.

probably to increase consumption. In the second case the effect is probably to reduce consumption. What is needed is some empirical evidence which will give us more to go on in reaching an evaluation of this effect.

The rest of this chapter will be concerned with the relation between consumption and income. The discussion proceeds generally on the assumption that the other influences on consumption remain the same. For accurate results, consideration must be given to the element of time. We proceed now to analyze the relation between consumption and income with attention to the element of time.

Let us assume that some definite distribution of income exists at every level of income. Under such a condition a definite consumption function will exist. In order to proceed further with the analysis it is necessary to distinguish between time periods. Empirical study has shown that it is convenient to distinguish between the short run (a period of time shorter than that of a complete business cycle) and the long run (a period of time in excess of the period of the cycle). More precisely, the short-run period describes a set of conditions under which national income does not attain a full employment level. Thus income may vary within any range short of the full employment level in the short-run period. Presumably such variation occurs between the peak of one business cycle and the subsequent peak, each peak roughly representing full employment income.

The long-run period describes a period in which the consumption of society has ample time to adjust itself to a given level of income. It is to be presumed that this level of income is one in an expanding sequence, each one of which corresponds to full employment. Evidently, more is involved in the situation than the passage of time. Certain assumptions about the behavior of the economic system are involved, and have been briefly stated in this text (pp. 40–42).

The short-run consumption function

Let us consider a consumption function which an economist regards as properly representing short-run tendencies. Its properties may be illustrated graphically. In Figure 2 consumption is

plotted against income, Y, to give the consumption line C. For this construction C is plotted along the vertical axis and Y along the horizontal axis. Since some of the properties of the consumption

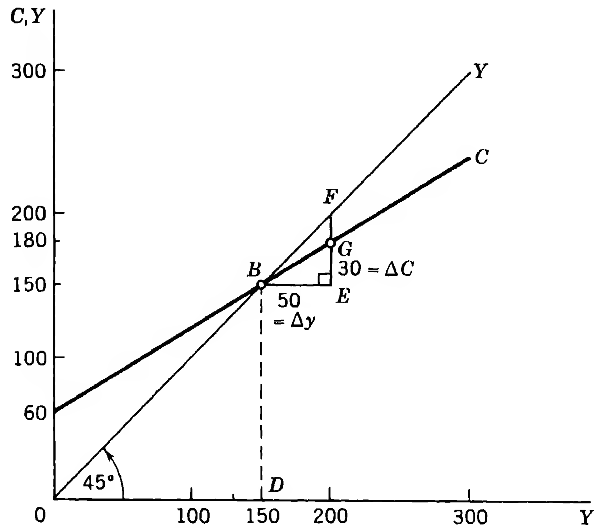


Figure 2. Short-Run Consumption Line and Its Properties.

function are shown by comparing C with Y, it would be distinctly convenient to be able to refer them to the same axis. To effect direct comparisons between C and Y it is necessary to plot Y values on the vertical axis against the same Y values along the

TABLE 7. Hypothetical Short-Run Consumption Schedule

Income	Consumption	ΔY	ΔC	$\frac{\Delta C}{\Delta Y}$	$\frac{C}{Y}$
$ 0	60	—	—	—	—
50	90	50	30	.6	1.80
100	120	50	30	.6	1.20
150	150	50	30	.6	1.00
200	180	50	30	.6	.90
250	210	50	30	.6	.84
300	240	50	30	.6	.80

horizontal axis. So an income value of \$50 is plotted along the vertical axis against a corresponding value of \$50 on the horizontal axis; \$100 is plotted against \$100 and so forth. The line connecting these points is called the income, or *Y*, line, since it gives the value of income on the vertical axis at the same value on the horizontal axis.

The *Y* line makes a 45-degree angle with the horizontal axis. Although the statement has no economic content, let us prove it. In any triangle the sides opposite equal angles are equal. In triangle *OBD*, sides *BD* and *OD* have a common value of 150 by construction. For this reason angles *BOD* and *OBD* are equal. The sum of the angles in a triangle equals 180 degrees and angle *ODB* is a right angle (90 degrees) by construction. Since the other angles have the remaining 90 degrees and are equal, each is 45 degrees. Accordingly, angle *BOD* is 45 degrees.

THE MARGINAL PROPENSITY TO CONSUME (MPC)

By definition MPC is the ratio of the increment in the consumption of society to the increment of income that caused it. Suppose income increases from 150 to 200. Consumption increases from the level indicated at *B* to that indicated at *G*. Extend line *B* horizontally to the right, and extend a line downward from *G*, intersecting the former line in *E*; further, extend the line upward from *G* intersecting the *Y* line in *F*. Line *BE* is parallel to the horizontal axis. Since parallel lines make equal angles on any line passing through them, angle *FBE* is a 45-degree angle, being equal to angle *BOD*. Moreover, angle *BEF* is a right angle by construction, making it 90 degrees. Consequently, angle *BFE* is also 45 degrees. Since sides opposite equal angles are equal, *BE* equals *EF*. *BE* represents the increase in income, amounting to 50; we have just shown that the increase in income can also be measured by the vertical distance *EF*.

As income increases from 150 to 200, consumption rises by *EG* or 30. MPC can be expressed by the ratio,

$$\frac{\Delta C}{\Delta Y} = \frac{30}{50} = \frac{EG}{BE} = \frac{EG}{EF}$$

Since *EG* is smaller than *EF*, MPC is less than 1. Obviously, this is

verified by the numerical data. In general, the line C passes through the point B, the break-even point where all income is consumed. If EG is less than EF, the C line must extend below the Y line to the right of B and extend above it to the left. In short, an MPC less than 1 is a characteristic of the short-run consumption function. This property is geometrically evidenced by the fact that the C line is flatter than the Y line.

THE AVERAGE PROPENSITY TO CONSUME (APC)

As the words suggest, the average propensity to consume is the fraction of all income spent on consumption. In Figure 2 the average propensity to consume, C/Y, exceeds 1 whenever the C line lies above the Y line. At incomes lower than 150 or to the left of B, APC exceeds 1. At B with an income of 150, APC is exactly equal to 1, and all income is exhausted in consumption. At incomes in excess of 150 or to the right of B, income exceeds consumption. At G the level of consumption is 180, while income is 200, giving an APC equal to .9.

ECONOMIC INTERPRETATION OF CONDITIONS

Up to this point our argument has been confined merely to exhibiting the geometrical properties of a consumption function arbitrarily constructed in Table 7 and Figure 2. Nothing has been said which would tend to justify the properties which such a line has been shown to possess.

First, the MPC was chosen to be less than 1—in this case, .6. The latter is not a magic number. Economically speaking, this condition simply implies that society wishes to enjoy part of *additional* income in the form of consumption but to reserve a part for saving also. This is not to say that society wants to set aside a part of total income for future use at any level of income. Rather, it means that an improvement in the income level occasions an effort on the part of society to increase that part of income set aside for future use or to reduce the consumption of existing capital occurring when consumption exceeds income. A rich person who receives additional income will doubtless wish to add to the savings he is making at his current income level. A very poor person who is obliged to consume more than income will wish to devote part of

any additional income to narrowing the gap between consumption and income. This amounts to an increment in savings or to having the increment in consumption less than the increment in income.

Two reasons are advanced for believing in the condition, MPC less than 1, apart from the above simple reasoning from individual behavior. First, the available statistical evidence indicates that the condition is true. Second, under standard conditions used in national income analysis an MPC greater than 1 leads to runaway inflation or deflation. An MPC equal to 1 under the same conditions leads to complete indeterminacy in the income level. Only an MPC less than 1 is consistent with a stable and determinate level of income. Since the real world does not exhibit runaway inflation, or deflation on the one hand or indeterminate income on the other, it seems plausible to conclude that the value of MPC must be consistent with stable income levels. In short, MPC should be less than 1. To put it bluntly, basic national income analysis can only explain observed behavior on the basis of this assumption.

In the representation we have given, the consumption line's MPC is .6 throughout. Every time income increases by 50, consumption increases by 30. The ratio, MPC $= \Delta C/\Delta Y = 30/50 = .6$ between every two levels of income. This property of the line is registered in Table 7, but we will defer discussion of the possible economic basis of this property until later.

If this consumption line gives a true picture of the situation, APC exceeds 1 at low levels of income, meaning that consumption exceeds income. At this point it is necessary to know what income concept we have in mind. Suppose the concept used is Disposable Personal Income. How can a person consume more than his income even in the short run? In the first place, the consumer may draw down his bank balance in order to consume. Suppose the consumer has an income of $400 a month, consumes $500 a month, and has a balance of $300 at the beginning of the month before deposit of his monthly paycheck. At the beginning of the month, his monetary assets amount to $700. After he has spent $500 on consumption during the month, the balance dwindles to $200 at the end of the month. Clearly, this process can only continue for two more months before the consumer will have exhausted his balance.

Can the consumer continue to spend more than his income after his balance is exhausted? Obviously he can do so only by borrowing. If another household or firm with excess balances lends him money, the consumer can continue to overspend his income. In effect, this is the same case, since it represents utilization of bank balances to consume in excess of income.

If the quantity of money in society is held constant, consumption can exceed income only by the process of utilizing excess bank balances. However, suppose that the individual purchases from a business firm on credit, thus permitting consumption to exceed income. Since the firm needs to maintain its working capital, it cannot continue to deplete it by extending credit to consumers. Remember that consumers are not repaying loans, because consumption consistently exceeds income. In such circumstances the firm itself may look around for a source of funds. Since all other firms are in the same boat, the firm cannot borrow from them. Neither can it borrow from consumers, because they are borrowing from the firm. In these circumstances the only possible source of funds is a bank.

In books on elementary economics, money, and banking, it is explained how a bank can create loans and deposits. These loans and deposits can be used to acquire goods and services, and so qualify as money. If the firm borrows money from a bank, it acquires a new quantity of funds which it can dispense to consumers. In turn, this permits households to consume more than income. In some cases where the household can secure the bank loans directly from the bank, it can avoid going through the firm for the extra money. To sum up, consumption can exceed income by two methods: drawing down bank balances or creating new money through the medium of the banks.

Suppose the income concept used is National Income. As a general rule Disposable Personal Income is considerably less than National Income. In order to get consumption to exceed National Income when this condition prevailed would be quite a trick. However, in deep depression and at a low income level Disposable Personal Income can exceed National Income. In fact, this event occurred in the deep depression year, 1932. In such years corporate and individual taxes are low, thus cutting the withdrawals

from National Income. On the other hand, corporations pay out more in dividends than they earn, reversing the normal tendency to retain earnings. Further, the government makes substantial transfer payments to the unemployed, creating in this way Disposable Personal Income that does not enter into National Income, all of which is earned from the employment of a factor. In this way Disposable Personal Income is made to exceed National Income. At such times consumption can more easily exceed National Income than Disposable Personal Income. In 1932 consumption exceeded Disposable Personal Income. Since Disposable Personal Income exceeded National Income, consumption exceeded the latter by an even larger margin.

In real terms the meaning of consumption in excess of income is that the flow of consumer goods and services is in excess of the quantity currently created. How can this occur? Clearly, this can only happen by drawing on some source of supply other than current production. Such a source consists in the stock of accumulated capital, both circulating and fixed. It is possible to consume the stock of circulating capital directly. During the seven fat years in Egypt Joseph saw to it that wheat was accumulated. During the seven lean years the populace drew on the accumulated stock of wheat.

In the second place, society may consume its stock of fixed capital. In order to maintain this capital a certain part of the current productive effort is normally devoted to turning out a stream of replacements for pieces of capital which are wearing out. If society defers replacement by retaining overage equipment, the productive effort normally devoted to producing replacements can be turned to the production of consumer goods. As this part of the stream of goods and services is consumed, the capital stock will run down to the same extent. During World Wars I and II this country, by not maintaining the capital of the nation's railroads, released a quantity of steel and other scarce goods for military uses. Obviously, this procedure represented a form of consumption of capital.

Evidently, consumption can exceed income for a short period of time. Since MPC is less than 1, consumption increases less rapidly than income. Ultimately a time will be reached when consumption will fall short of income. This is the normal state of affairs and requires no comment.

Long-run consumption function

In the long run it is quite evident that APC cannot exceed 1. If consumption exceeds Disposable Personal Income, all consumers are drawing down bank balances and running up debt. Ultimately, consumers will tend to exhaust their credit. We recall, however, that Disposable Personal Income may be supplemented by transfer payments from government, payment of unearned dividends or unemployment compensation. In this way consumption might be raised to exceed National Income but not the larger value of Disposable Personal Income. Such a condition could not continue indefinitely. In real terms it would imply that the flow of consumer goods exceeded the flow of output currently produced. In short, capital would be steadily encroached upon. Ultimately, the stock of capital would be exhausted by such a process and, obviously, that process would then have to cease.

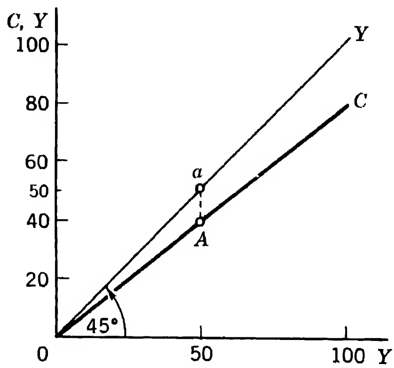


Figure 3. Long-Run Consumption Line.

Following this argument we draw the consumption line in Figure 3 in such a way that it lies below the income line at every point. In this case the C line is drawn straight outward from the origin, making a smaller angle with the horizontal axis than does the Y line. Such a straight line through the origin has the property that MPC equals APC throughout, and both are constant. In this case MPC = APC = .8 throughout. At zero income consumption is zero, while at an income of 50, consumption is 40. Over this range ΔC is 40, whereas ΔY is 50, so that $\Delta C/\Delta Y = 40/50 = .8$. At the same time $C/Y = 40/50 = .8$, indicating the equality of these two magnitudes. All these data can be verified from Table 8. Compare this information with the results shown in Figure 2 and

Table 7 relating the values of MPC and APC for the short-run function.

The long-run consumption function is drawn up on the premise that any given level of income will be maintained for a long time. By the same token households are permitted time to adjust their consumption levels to the given level of income. From this assumption it is perhaps clear why the MPC in the long run is assumed to

TABLE 8. Hypothetical Long-Run Consumption Schedule

Income	Consumption	MPC	APC
10	8	.8	.8
20	16	.8	.8
30	24	.8	.8
40	32	.8	.8
50	40	.8	.8

be larger than in the short run. It was .6 in our short-run function and .8 in our long-run function. When income increases in the short run, households are reluctant to change their consumption levels for two reasons. In the first place, they have established certain habit patterns which are not quickly broken down. A man in the low income class with a modest house and car does not abandon his spending habits immediately on receiving a substantial promotion. Time is required for the household to adapt to the new circumstances arising when income increases substantially. In the second place, income changes which occur in the short run tend to be regarded as temporary. In such cases consumers are reluctant to go ahead with spending on the assumption that the increase is to continue. A better car and a larger house and grounds tend to lead to a permanently higher level of spending. Such changes in spending are only undertaken when the income change is expected to be long lasting.

Short and long run compared

Let us see what can be done to coordinate the short- and long-run concepts of consumption. In Figure 4 the long-run consumption

schedule of Table 8 appears as consumption line C_L. Suppose the level of income 50 is experienced for some considerable period of time. Let us suppose also that the economy is operating at a level close to full employment. Under these conditions consumption will be 40, a figure found at R on the long-run consumption line C_L. Now suppose income expands gradually from 50 to 100

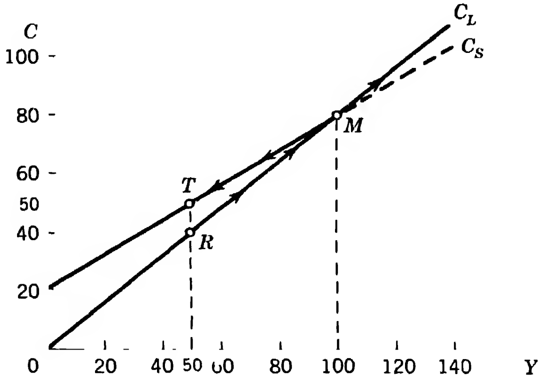


Figure 4. Short- and Long-Run Consumption Lines.

over a succession of time periods, say in a sequence 50, 60, 70, and so forth. As this happens consumers are put in a frame of mind to expand their consumption steadily along the long-run consumption line C_L. By the time an income of 100 is reached (point M on the long-run function) consumption is 80 which is in the same .8 ratio to income as the consumption of 40 was at an income of 50.

At this juncture suppose there is a recession in business activity which carries national income from 100 to 50 in a relatively short period of time. Consumption contracts along the short-run consumption line from M to T, or from a consumption of 80 to 50. Let us note that consumption at an income level 50 is now 50, whereas before the attainment of the income level 100 it had been 40. What accounts for this increase? When the recession occurs, the consumer is likely to feel that it will not last forever. Sooner or later, he reasons, the income level of 100 will be reattained, and he can resume his former spending level. In the meantime the

consumer is reluctant to abandon the spending pattern he has established. Consequently, he compromises by cutting out only part of the extra consumption he established when incomes were rising slowly.

When recovery occurs, consumption will increase along the short-run curve to M. Suppose the level of income passes 100; can one then observe a further movement along the short-run curve, C_S, as income expands further? Not to any significant extent. Let us consider why this assertion is likely to be true.

At a point like M the economy has reached a peak income for the time being. The income of 100 is dependent on the existence of a certain stock of capital accumulated gradually as income increased through time from 50 to 100. If income contracts and then recovers in the course of a cycle, the capital accumulated over the circuit of income will tend to be small. Consequently, the stock of capital is doubtless about the same on the return to M as it was at the departure. In short, 100 is about the largest short-run income possible at the time of recovery to the previous peak income. If income is to increase further, it must do so gradually as a result of capital accumulation, innovation, population growth, and so forth. If the manner of increase of income is gradual and dictated by the slow working of these forces, consumption will move along the long-run function in the direction indicated by the arrow.

Other considerations influencing consumption

INFLATION

According to our arguments to this point, consumption, expressed in money, should increase with income, similarly expressed. However, a little thought suggests that there are certain difficulties connected with this usage. For example, suppose that all prices and all incomes increased in the same proportion. In such a case there would be no reason for the consumer to change the physical quantities of goods purchased. In terms of money, consumption would increase in the same proportion as prices and income. In short, the ratio of consumption to income would remain unchanged. A sequence of such changes would generate a consumption function similar to the long-run function pictured. Such a set of changes

might be generated by an increase in the quantity of money available, and the result would appear to duplicate the long-run function. Yet such changes could occur in the short run.

Some economists prefer to express income and consumption in terms of dollars of constant purchasing power. They feel this will give rise to a more valid relationship. Certainly, it will avoid the picturing of identical real income and consumption as a succession of points instead of a single one.

POPULATION

If National Income is given, and population increases, it is obvious that pressure would exist to expand consumption. When population increases, these is a tendency for the consumption function to shift upward. Accordingly, in order to take account of this effect, some economists attempt to relate per capita consumption to per capita income. There seems little doubt that in a period of population change this is a relevant factor, and its inclusion tends to give a more valid relationship.

TAXES AND CORPORATE SAVING

Consumption is based on Disposable Personal Income. Taxes on corporate and individual incomes, as well as transfer payments, mark off Disposable Personal Income from National Income. Clearly, any change in the saving habits of corporations or the tax structure of government will cause a change in consumption out of a given income. Consequently, it is quite legitimate to correlate consumption and Disposable Personal Income.

EMPIRICAL TESTING

In order to find the best relation between consumption and income, economists have tested various hypotheses. Apparently, it is most valid to use real per capita Disposable Personal Income as the causal factor. This rules out the distorting effects of price, population, and tax changes. Account may be taken also of the difference between short- and long-run functions. In this manner seemingly good results can be obtained in predicting consumption from income. We cannot discuss these tests in detail here. Rather, let us discuss a very simple description of the data.

In this description Disposable Personal Income was used as the independent variable, and consumption as the dependent variable. No account was taken of price or population change. One short-run consumption function was computed for the years 1929–1940;

TABLE 9. Consumption and Disposable Personal Income, 1929–1959[a]
(in Billions of Dollars)

Year	Disposable Personal Income	Consumption	Year	Disposable Personal Income	Consumption
1929	83.1	79.0	1945	150.4	121.7
1930	74.4	71.0	1946	160.6	147.1
1931	63.8	61.3	1947	170.1	165.4
1932	48.7	49.3	1948	189.3	178.3
1933	45.7	46.4	1949	189.7	181.2
1934	52.0	51.9	1950	207.7	195.0
1935	58.3	56.3	1951	227.5	209.8
1936	66.2	62.6	1952	238.7	219.8
1937	71.0	67.3	1953	252.5	232.6
1938	65.7	64.6	1954	256.9	238.0
1939	70.4	67.6	1955	274.4	256.9
1940	76.1	71.9	1956	292.9	269.9
1941	93.0	81.9	1957	307.9	284.8
1942	117.5	89.7	1958	316.5	293.0
1943	133.5	100.5	1959	334.6	311.6
1944	146.8	109.8			

[a] The lines fitted to the data by least squares are listed below, where C denotes consumption and Y_d denotes Disposable Personal Income.

(a) Period 1929–1940; $C = 7.16 + .855\ Y_d$
(b) Period 1947-1955; $C = 16.38 + .862\ Y_d$
(c) Period 1929–1940; 1947–1958· $C = 3.38 + .919\ Y_d$

For an explanation of the method of fitting lines to data by the least squares method, see Croxton and Cowden, *Applied General Statistics*, 2nd. ed , Englewood Cliffs, N.J., Prentice Hall, 1955, Ch. XIX.

The equation listed in (c) was based on data for consumption and Disposable Personal Income as of 1959. Since that time the figures for the last three years have been revised. The revised figures are listed, but the equation was not recalculated; the difference would certainly be negligible.

SOURCE *Survey of Current Business*, National Income Number, July 1958, Table 3, p. 6, February 1959, Table II-2, p. 14.

the years 1941–1946 were excluded because they spanned the war and reconversion period. A second short-run function was computed for the years 1947–1955. The years 1956–1958 were excluded simply because the data for these years did not fit the 1947–1955 pattern. Finally, a long-run function, taking account of all but the war and immediate postwar years, 1941–1946, was computed. For details as to calculation, see Table 9.

For the years 1929–1940 the observations of consumption plotted against income fall very closely about a line passing through the points, as shown in Figure 5. In turn, the observations for the period, 1947–1955, fall about a second line which is approximately parallel to the first line. When two aggregate consumption

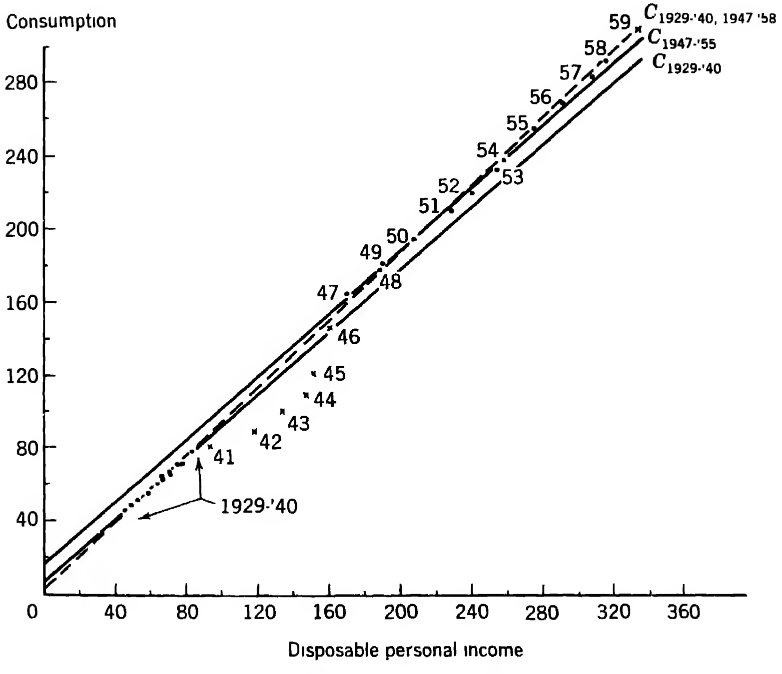


Figure 5. Money Consumption—Income Patterns: 1929-1959. See Table 9. (Based on data gathered by the Department of Commerce.)

functions or lines are parallel, the MPCs of the respective curves are equal. In fact, the indicated value of MPC for each line is approximately .86, the figure for 1929–1940 being .855, while the figure for 1947–1955 is .862, the results being rounded to the third decimal. Accuracy in the third decimal place is not to be expected, since the true values of consumption and income are unknown. Several revisions affecting parts of the data in question have appeared.

Finally, the data for the entire period, excluding the years 1941–1946, are grouped around a straight line which very nearly passes through the origin. In the case of this line the MPC is larger, being approximately .919. At no time, moreover, does an observation fall very far to the right of this long-run consumption line.

The data for 1956–1958 appear to conform to the long-run line. In fact, if the observations for these years are omitted from the data used to compute the long-run consumption line, the estimates of consumption undergo a change of less than \$.3 billion, or less than 1/10 of 1 percent, over the range of incomes recorded. Neither does the inclusion or exclusion of the last three years affect long-run MPC much, the variation being of the order of 1 part in 1000. On the other hand, inclusion of data for the last three years, along with information for the years 1947–1955, markedly affects short-run MPC. Thus the data suggest that the last three years are marked by a movement up the long-run function and not along the last short-run function. The income value for 1959 has also been plotted and it falls almost exactly on the long-run line.

All in all, the results conform very well to the pattern enunciated earlier. However, let us note one point here. If an economist had attempted, as some did, to project the 1929–1940 short-run function into the postwar period, an error of about 10 billion dollars would have been made in the prediction. Such predictions led some economists to prophesy a period of considerable unemployment following World War II. Of course, such predictions proved to be false because they failed to take account of the factors making for a shift in the consumption function.

FACTORS CAUSING AN UPWARD SHIFT IN THE CONSUMPTION FUNCTION

In order to forecast consumption it is necessary to take account of factors which cause an upward shift in the function through time. This is a problem in economic dynamics, and various lines of attack have been proposed. One method simply includes an upward factor of shift-per-time-period. Another, possibly the best, adds a positive factor to the consumption function whenever income exceeds the previous maximum. In years when this happens, consumption increases along a line through the origin. The positive factor added to consumption, as current income

exceeds the preceding maximum (possibly last year's income) increases the "long-run" MPC. Such an explanation is fairly consistent with the observed facts.

First, let us consider why the observed tendency for long-run MPC to exceed short-run MPC holds true. If the individual is allowed only a short time in which to adjust consumption to increased income, spending habits cannot be modified properly. It takes time for an individual to learn to enjoy a larger income. To do this he must buy a better home with more rooms and conveniences, acquire a better car, more household appliances, and visit specialists instead of general practitioners when sick. Furthermore, all this may require moving with a different circle of friends who enjoy the same conveniences. To establish workable consumption patterns at a higher income level requires considerable time and planning to effect. Care must be exercised lest the family plunge too deeply into expenditure and suffer the humiliation of a repossessed automobile or a house too expensive to maintain and which must be disposed of for a more modest home. In the long run, however, appropriate habit patterns which take full, but not excessive, advantage of higher income levels can be effected.

Supplementing this resistance to changes in habit patterns is an uncertainty about the duration of current income levels. When income is below the full employment level, some uncertainty will exist about the permanence of an increase in income. Considerable and quite temporary changes in the level of income occur in the course of a business cycle. To put it another way, rapid and temporary fluctuations in income may occur below the full employment level. Consumers recognize the lack of permanence of such income changes and resist changes in expenditure resulting from these income changes. Only when the income change has lasted long enough to be considered permanent will it occasion the fullest response in consumption. In a situation in which the level of income has reached the full employment level and is gradually expanding, this condition is largely satisfied. Such a state would permit habit patterns to be adjusted gradually also. Accordingly, one would expect a gradually expanding, full-employment economy to move along a long-run consumption line.

At this point it is necessary to refer back to the interpretation given the facts in Figure 5. First, the consumption observations over the entire period, excluding the years 1941–1946, cluster rather closely around the dashed line in the diagram. This line exhibits a larger (.92) MPC than the two respective lines "fitted" to 1929–1940 and 1947–1955 data, respectively; both these lines exhibit MPCs of about .86. To this extent the theory set forth is obviously consistent with the facts. Yet we cannot overlook one point of possible weakness in the theory as stated above. When income increases steadily over a succession of years, consumption should increase in conformity with a long-run consumption line with a high MPC. Although income increased steadily in every year of the 1947–1955 period, the line of closest fit to the 1947–1955 data gives an MPC of .86, not .92.

Evidently, the period of steady increase in incomes generated a consumption line much like that of the 1929–1940 period characterized by alternately contracting and expanding income. Athough the information presented here is far from definitive, our interpretation of it indicates that the time sequence suggested earlier must be compared with the facts quite carefully. In other words, there appear to exist short-run functions which describe the behavior of the consumer in limited time periods. With the passage of time it appears probable that such a short-run consumption line will shift upward. A new and higher consumption line will then describe the behavior of the consumer in the next short period. Finally, there appears to be a long-run line which describes all the data, having the property of being steeper (possessing a higher MPC) than the short-run lines and which cuts the short-run lines from below.

Deflated consumption and income data

In the preceding study the price level varied from period to period and no adjustment was made to income or consumption to offset such changes. The Department of Commerce has made available, however, a set of data on consumption and Disposable Personal Income which expresses each of these series in dollars of constant purchasing power. The study covers the period 1920–1958, and uses the year 1957 as the base period.

The data, as plotted in Figure 6, appear to display two distinct groupings. In the first place, the data for the nineteen-year period, 1922–1940, seem to fall rather closely about a straight line. Likewise, the observations for the interval, 1947–1958, appear to be

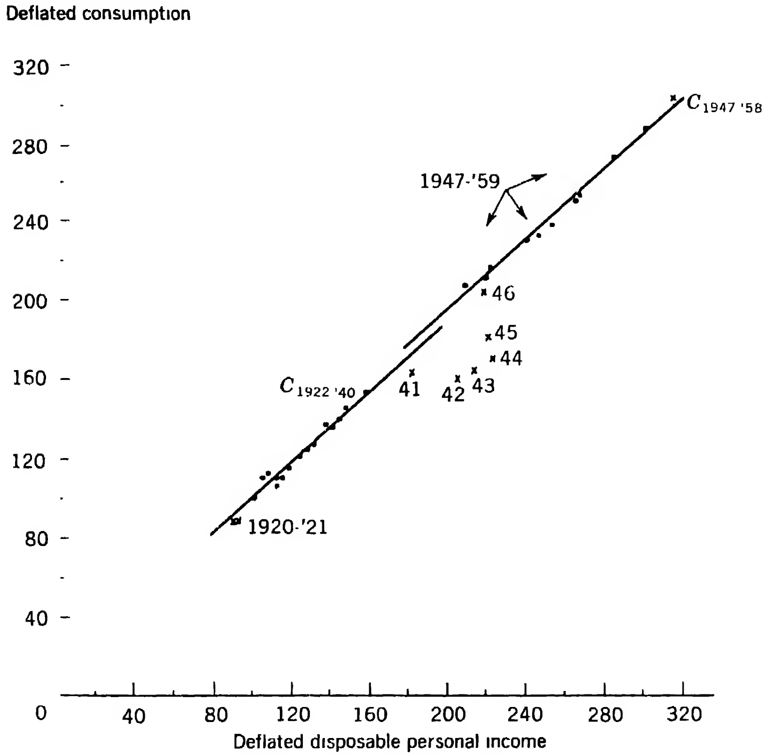


Figure 6. Deflated Consumption—Income Patterns: 1920-1959. See Table 10. (Based on data gathered by the Department of Commerce.)

distributed around a straight line lying above and nearly parallel to the first line. To avoid the distortions of war and reconversion periods the data for 1920–1921 and for 1941–1946 have been excluded in calculating regression lines. The MPC for the earlier period, as measured by the slope of the fitted line is .862, compared to .870 for the later period. Clearly, the MPCs are nearly equal.

The case for a long-run consumption line passing through the origin seems considerably weaker. This line has not been drawn in for this reason, and because its presence would complicate the picture without corresponding benefits. Note that the MPC with the deflated data has about the same value as for the data expressed in money. This indicates that price changes had about the same impact on consumption and income so that a ratio of the change in one to the change in the other has nearly the same size. A second point is that the real consumption line has shifted upward only

TABLE 10. Consumption and Disposable Personal Income, Deflated, 1920–1959 (in Billions of 1957 Dollars)

Year	Disposable Personal Income	Consumption	Year	Disposable Personal Income	Consumption
1920	94.3	88.9	1940	160.8	152.1
1921	90.9	88.5	1941	184.1	162.2
1922	101.8	100.7	1942	207.6	158.5
1923	114.0	105.6	1943	216.1	162.6
1924	115.7	110.1	1944	225.1	168.4
1925	120.1	115.2	1945	222.7	180.2
1926	126.3	120.8	1946	220.6	202.2
1927	130.0	123.5	1947	211.6	205.7
1928	132.2	126.9	1948	222.4	209.6
1929	141.8	134.8	1949	225.0	214.9
1930	132.8	126.6	1950	242.9	228.1
1931	127.7	122.7	1951	249.2	229.9
1932	110.1	111.6	1952	256.1	235.9
1933	107.4	109.4	1953	268.3	247.3
1934	114.7	114.7	1954	270.4	250.4
1935	126.2	121.3	1955	287.7	269.4
1936	142.1	134.2	1956	299.1	277.5
1937	146.7	139.0	1957	305.1	284.4
1938	139.9	136.6	1958	304.3	283.9
1939	150.5	144.3	1959	318.5	298.8[a]

The lines fitted by least squares are listed below. Compare Table 9.

(a) Period 1922–1940: $C = 13.00 + .862\ Y_d$

(b) Period 1947–1958. $C = 16.82 + .870\ Y_d$

(c) Period 1922–1940, 1947–1958: $C = 7.89 + .903\ Y_d$

[a] Derived by determining the percentage increases in DPI and consumption in 1954 dollars, multiplying these by the 1958 values and adding the results to the 1958 values to get the 1959 values in terms of 1957 dollars (instead of in the 1954 dollars in terms of which the 1959 figures are given).

SOURCE: *Survey of Current Business*, March 1959, Table 4, p. 26; February 1960, Table I-5, p. 12, and II-2, p. 14.

slightly. This is a result of the deflation of the data. A proportional change in prices with no change in real income or consumption will cause a like proportional change in money income and consumption. This would be represented in Figure 5 by a movement along a line passing through the origin and the original consumption-income point. Thus the short-run consumption function of Figure 5 tends to shift upward as a result of inflation in contrast to that of Figure 6 which remains unchanged.

In the years 1956–1959 a significant inflation element was present. This may account for the fact that the data for these years fall about the long-run consumption line in Figure 5. This line very nearly passes through the origin and thus meets the above requirement. By the same token the removal of the inflation element from the data may account for the failure of the long-run consumption line calculated from deflated data to cut the consumption axis very close to the origin.

Family budget data

Clearly, an aggregate consumption function is built up out of two basic factors: the individual consumption functions and the distribution of income. Starting with the first of these, we will examine

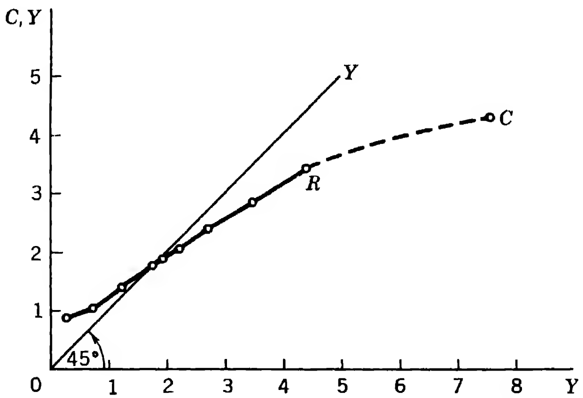


Figure 7. Consumption in Relation to Family Income: 1944. See Table 11. (Based on data gathered by the U.S. Bureau of Labor Statistics.)

TABLE 11. Average Money Income and Consumption of Families of Two or More in Cities, 1944

Disposable Personal Income	Consumption
$ 313	$ 887
776	1053
1043	1407
1779	1788
1950	1877
2259	2051
2757	2410
3480	2838
4408	3439
7595	4305

SOURCE *The Economic Almanack*, New York, Crowell, 1956, p. 382. Material based on a survey conducted by the Bureau of Labor Statistics.

typical family incomes and find corresponding values of consumption. One study of consumption in urban families in 1944 sheds some light on the subject. Corresponding light is thrown on saving habits by a study of saving and income in families averaging 3.3 persons, conducted in 1947.

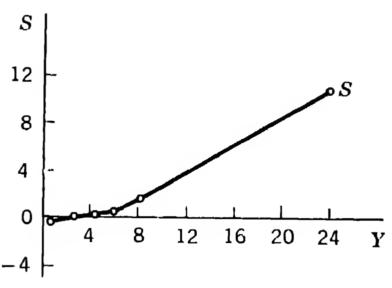


Figure 8. Saving in Relation to Family Income: 1947. (Based on data gathered by Raymond Goldsmith.)

Table 11 and Figure 7 reveal the relation between consumption and income in urban families. A strong linearity is evident in the data up to point R on the C line. Beyond R, it seems obvious that the C line becomes progressively flatter as income increases. Although the intermediate observations are lacking, a dashed line has been inserted between R and the terminal point of the line. Such evidence as this gives an indication that MPC eventually drops as the size of family income increases.

In Figure 8 and Table 12 some data on saving and income in

TABLE 12. Saving and Income in Families of 3.5 Persons, 1947

Income	Saving
$ 710	$ – 296
1620	– 138
2540	– 36
3480	38
4430	174
6000	616
8300	1737
24000	10699

SOURCE: Goldsmith, *A Study of Saving in the United States*, Vol. III, Princeton, Princeton Univ. Press, 1955–1956, p. 186, Table H 23.

families of 3.5 persons for 1947 are presented. In this case the saving line becomes progressively steeper as income increases. By the same token the consumption curve doubtless becomes flatter, as in Figure 7. When the marginal propensity to save increases, the marginal propensity to consume decreases. In short, the saving data presented lead to the same conclusion as the consumption data. As income increases, consumption sooner or later begins to increase more slowly—MPC declines.

Character of budget studies and national income data

In family budget studies the data all refer to one particular period of time. Clearly, no changes can take place in the character of the functional relations between quantities under these conditions. In National Income data each observation refers to a separate period of time. The economist cannot be sure that the consumption function does not undergo change between one set of observations and the next. In general, the shorter the period for which the National Income data are used, the more nearly will they be related to budget data collected within the same over-all time span.

SHORT-RUN RELATION BETWEEN BUDGET DATA AND NATIONAL INCOME DATA

Let us assume that the period of time under consideration is so short that a change in the consumption function cannot occur.

Assume also that enough uncertainty about future income exists that the reaction to income changes is a conservative one, marked by a relatively low MPC. These conditions are fulfilled most plausibly by an economy operating in a condition of unemployment characteristic of the lower phase of the business cycle.

Consider the possible bearing of unemployment on the situation. When unemployment exists, increased income payments will flow mainly to newly employed persons. If money wages are rigid prior to the full employment level the per capita incomes should not show a perceptible increase. Thus a significant movement along family consumption lines should not occur. Up to full employment the main determinant of the social MPC would be the APCs of newly employed persons. If these APCs show no systematic change as employment expands, the MPC should conform to their typical value. In turn, this would lead to an approximately linear consumption line.

LONG-RUN RELATION BETWEEN BUDGET DATA AND NATIONAL INCOME DATA

One might suppose that the consumption function would tend to become flatter at incomes exceeding the full employment level. Let us recall that income can expand only gradually through time once full employment is reached. Under such conditions various forces are playing on the consumer to cause him to consume more out of a given income. In the first place consumer wants and tastes evolve with time and with the attainment of new living standards. Encouraging the change of tastes is the flow of new products catering to and encouraging the advance in the scale of living. At the same time, emulation pushes the consumer into purchasing things which other consumers possess. When the television craze strikes a neighborhood of people with similar incomes, capitulation by all is but a matter of time. In the past generation or two how many unused pianos could be seen cluttering up living rooms? In short, the attainment of full employment and the emergence of a period of growth leads also to upward shifts in family consumption lines. Such upward shifts may well cancel the possibility of social consumption lines which level off at higher incomes.

To summarize the relation between budget data and National Income data we may say that in the short run the presence of unemployment implies that an increment of income will be distributed in considerable part to the unemployed. As explained above, social MPC is then governed by the APCs of the newly employed. If no systematic change in the APCs takes place, social MPC should be approximately constant and the consumption line linear in form. In the long run a gradual expansion of incomes at full employment generates rising living standards which cause upward shifts in family budget lines. Such upward shifts lead to linear movements along a long-run consumption line. These two explanations may help to bridge the gap between the evidence offered by the two types of empirical data on consumption functions.

Individual and aggregate consumption functions

By starting from the theory of the consumer we can establish from plausible assumptions the existence of a household consumption function. Assume that an individual with a limited budget seeks to allocate his income between consumption and saving so as to maximize satisfaction. By considering alternative distributions of his income between these two uses, the individual finds a point at which a subtraction of $1 from consumption and its addition to saving fails to improve his satisfaction. At this point consumpton and saving out of a given income are determined. At each income level a corresponding allocation is reached. By listing consumption at each level of income we find an individual consumption function. Empirical evidence supports those who believe that such a function exists.

In reality, the individual consumption function holds little interest. What we are seeking is a relation between aggregate consumption and aggregate income. Suppose we have two such schedules for two different households. Can we then add these schedules together—after the manner of demand schedules? The answer in general is "no."

To illustrate, suppose we are given the two consumption schedules listed in Table 13. Note that there are two income schedules and two consumption schedules. There is no rule to tell

us what income of Household A to add to the corresponding income of Household B; correspondingly, there is no way to determine the two values of consumption to add. Additional information about

TABLE 13. Consumption Schedules of Two Households

Household A		Household B	
Income	Consumption	Income	Consumption
$100	$110	$100	$ 95
200	175	200	150
300	240	300	205
400	305	400	260
500	370	500	315
600	435	600	370

the relation between A's income and B's income is needed. Suppose (Table 14) B's income is always $100 more than A's. We can then find the desired function by arranging the data properly in rows. In adding the data some of the information from Table 13 is discarded.

TABLE 14. Addition of Consumption Schedules of Two Households (B's income $100 more than A's at every income level)

Income of Household			Consumption of Household		
A	B	A + B	A	B	A + B
$100	$200	$300	$110	$150	$260
200	300	500	175	205	380
300	400	700	240	260	500
400	500	900	305	315	620
500	600	1100	370	370	740

The consumption function is the relation between the third and the sixth columns. Is this a unique relation? Again, the answer is "no." The function depends on the way in which the income of B changes compared to that of A. Suppose that A's income is always $100 more than B's, reversing the previous situation. In that case the results would be those shown in Table 15.

In Table 15 the consumption is exactly \$10 higher at every level of income than in Table 14. Of course, these are special cases. The level of income depends on the distribution of income between the

TABLE 15. Addition of Consumption Schedules of Two Households (A's income \$100 more than B's at every income level)

Income of Household			Consumption of Household		
A	B	A + B	A	B	A + B
\$200	\$100	\$300	\$175	\$ 95	\$270
300	200	500	240	150	390
400	300	700	305	205	510
500	400	900	370	260	630
600	500	1100	435	315	750

various income recipients. There are a few special cases covering the distribution of income that may be mentioned without proof.

EFFECTS OF INCOME DISTRIBUTION ON THE CONSUMPTION FUNCTION

Case 1: If the marginal propensity to consume of all households is equal, the aggregate marginal propensity to consume has the same value as that of each household. No change in the distribution of income will affect the amount consumed at any level of income.

Case 2: If the marginal propensity to consume of every household diminishes with income; if the distribution of income is so adjusted that each household's MPC is equal to a common value, then this common value is the MPC of society; moreover, the total consumption is a maximum for this level of income. This marks an upper limit to which consumption at a given income level can go by way of income redistribution.

Case 3: The social MPC may be negative or greater than one. If income is increased by \$1, and the distribution changed in such a way that the "rich" get richer and the "poor" get poorer, consumption may decline. If the distribution is changed the other way, the "rich" getting poorer and the "poor" getting richer, the increment in consumption may exceed the increment in income. By

"rich" we mean those with low MPCs, a condition usually associated with high incomes; by "poor" we mean those with high MPCs, a condition usually associated with low incomes.

Case 4: If every individual has an MPC which lies between zero and one and no individual's income decreases when National Income increases, the social MPC must lie between zero and one.

Case 4a: The same conclusion applies if we combine into groups those individuals whose incomes increase by the same amount. In this case if the average of MPCs for each such group lies between zero and one and the income of every group increases, the social MPC lies between zero and one.

Case 4b: We may combine into groups all those with the same MPC. In combining individuals some of whose incomes have increased while those of others have decreased, the effects of negative changes in incomes can be offset against the positive changes which predominate. So long as each group experiences a positive increase in income and all groups have MPCs lying between zero and one, the social MPC lies between zero and one.

CHAPTER 4

Investment and income determination

Investment

To set the stage for a discussion of investment let us review a few basic economic concepts. First, *production* consists in the creation of utilities. Whereas such utilities may consist in the useful material things which we call *goods*, they may consist also in the immaterial satisfactions created in rendering *services*. In this spirit, a lawyer's reassurance that it is safe to go ahead with a proposed contract yields utility; such utility is on a par with the satisfaction provided by the purchase of a fountain pen or a lathe to be used in a factory. Whereas production gives rise in some cases to a flow of services, in other cases it gives rise to a stream of material goods. Over time, as a result of a process of accumulation, a stock of material goods comes into being. Such a stock of goods is known as *wealth*.

Of the stock of useful material goods known as wealth, certain items yield direct satisfactions to individuals. For example, a tennis racket is capable of yielding direct satisfaction to the owner when he uses it in play. Items of wealth which yield direct satisfaction to the consumer are known as *consumer goods*. On the other hand, certain material goods are acquired with a view to the future satisfactions which they may yield. For example, the purpose of the management of a railroad in buying a locomotive is not to enjoy the thrill of seeing a powerful and shiny piece of new equipment. Rather, the purchase is designed to create more railroad services which will yield a stream of future revenues. Let us refer to such produced goods used in further production as *capital goods*. Expenditure on new capital goods per time period is known as *investment*. If a railroad buys a new locomotive for $100,000, after maintaining all other equipment in its usual condition, it has undertaken investment of this amount.

EXPECTED FUTURE RETURNS

Investment is undertaken in anticipation of a stream of future incomes accruing from the capital acquired. At the time when the locomotive is purchased it is expected to yield a stream of net income over a twenty- or thirty-year period. In certain cases of investment the future return is not very distant. For example, an automobile dealer may purchase a car for $2500 from the manufacturer. If all goes well, he will sell the car in a week or so for perhaps $3200, thereby realizing a quick return on his investment. When the full return is expected within a year, the investment is said to be embodied in *circulating capital*. When the return is expected over a period of future years, the investment is said to be embodied in *fixed capital*.

In the array of forces determining investment in capital, the most important element is the set of future returns expected from the investment. Obviously, such future returns must be compared with costs. Moreover, some adjustment must be made for the fact that costs are incurred in the expectation of returns accruing in the future. If the firm is to make an investment, it must formulate anticipations about the returns expected from the use of capital in the years lying ahead. Thus, a trucking concern cannot rely wholly on current traffic volume to determine the net income from a truck for the next five years. Rather, it must project conditions into the future for a fairly extended period. For this reason, it hardly seems plausible for the firm to base its investment on current income. By the same sort of reasoning, applied to the economy, it does not seem proper to assume that investment is based on current income.

In the light of this discussion it should be evident why investment is regarded as governed by anticipations of future income. Since these anticipations are governed by the operation of certain long-run conditions pertaining to economic growth, they are independent of current income to a large extent. In view of the probable lack of dependence of investment on current income, investment may be treated as autonomous. By "autonomous" we mean self-governing; investment is therefore treated as independent of current income but capable of change whenever anticipations change. If anticipations of future returns improve, investment will increase. However,

changes in income, unaccompanied by other factors, are not considered to usher in such changes.

Such an approach to the determination of investment is merely a first approximation. Although the relation between current income and investment may be weak or uncertain, it will be considered at a later stage. Likewise, the interest rate which affects the costliness of investment can be shown to have a bearing on the situation. At an elementary level, however, it seems most appropriate to treat investment as autonomous. To summarize, this means that investment is treated as: (1) capable of change in the short run in response to changes in anticipations; (2) independent of current income.

Income determination

Let us assume for the time being a purely private economy, closed to imports and exports. In such an economy there are neither taxes nor government expenditures. To reach equilibrium in an aggregative sense, the economy must attain a condition in which aggregate demand and aggregate supply are equal. First, let us consider the meaning of "aggregate supply."

INCOME AND SUPPLY

In price theory "supply" refers to a schedule of the amounts of a particular good which will be offered for sale at a corresponding schedule of prices. If 4 million bushels of wheat are offered for sale at $1 a bushel, 6 million are offered at $1.50, and so forth, the set of all such combinations of quantity offered and price encompasses "supply." To go into the matter a little more deeply, producers are only willing to offer 4 million bushels of wheat if the $1 per bushel they receive is an adequate return for the factors necessary to provide the wheat. In some sense $1 measures the unit cost of bringing 4 million bushels to market. In the short run, this unit cost corresponds to marginal cost. In the long run this unit cost corresponds to the average cost of this quantity. In either case the price received includes a sufficient return to enable the factors to be hired to the end that the output be produced. Evidently, the term "supply" is closely linked with the payment of those costs to the factors which will induce them to render their services for the period in question.

An appropriate conception of aggregate supply defines it as that sum of money whose payment will induce the forthcoming of those factors necessary to produce a given level of output. As we have stated on several previous occasions, the output in question consists of a flow of many different kinds of goods having only the common denominator of money. In this case we seek to measure that flow by the factor cost of producing it. Since National Income can be regarded as the sum of the payments made to the factors of production for the use of their services, aggregate supply can be measured by National Income regarded as factor cost. Included in the returns which are counted are wages, interest, rent, and profit. Of course, only that amount of profit which is necessary to induce the forthcoming of the output is included. Even monopoly profits are included, since the firm will contract output unless it receives the monopoly price.

INCOME AND DEMAND

It has previously been shown that National Income may be interpreted as the value of output. In turn, the value of output is the same thing as the total expenditure on output. In the discussion which follows we shall refer to income in this sense as "expenditure." Without going into a detailed discussion of demand, it is rather obvious that the expenditure of society on output measures aggregate demand. In fact, expenditure on output is aggregate demand expressed in money. For this reason it is reasonable to interpret income in the sense of money expenditure as the aggregate demand for output. Henceforth "money expenditure" and "aggregate demand" will be used synonymously.

EQUILIBRIUM OF AGGREGATE SUPPLY AND AGGREGATE DEMAND

In price theory an industry attains equilibrium when that price is set which will exactly equate the quantity which buyers wish to purchase with the quantity which sellers are willing to offer. More crudely, equilibrium is reached when quantity demanded equals quantity supplied. In National Income theory the appropriate condition is that aggregate demand be equal to aggregate supply. If such a condition can be reached, the market for goods as a whole will be cleared. Using the terminology established, aggregate

supply and demand are equated when expenditure or demand equals income or supply. Since income measures output at factor cost, producers will receive an amount which will permit them to meet their factor costs. In turn, they will be able to provide the flow of goods currently being produced.

SHORT-RUN EQUILIBRIUM

Let us assume that businessmen have expectations favorable enough to justify 80 billion dollars worth of investment. Since a short-run period is in prospect, the current level of income is not expected to prevail indefinitely. In view of this condition the consumption function is of the short-run type. In conformity with this assumption, consumption exceeds income at levels of income below 100 billion dollars. For the same reason MPC is rather low, being only .6.

In Table 16 income regarded as factor cost, Y, is listed in the

TABLE 16. Income Determination in the Short Run

Y	C	I	$C + I - E$	
0	40	80	120	Expansion
100	100	80	180	,,
200	160	80	240	,,
300	220	80	300	Equilibrium
400	280	80	360	Contraction
500	340	80	420	,,

left-hand column. In the next column consumption, C, is listed, starting with 40 billion at zero income and increasing by 60 with each increase of 100 in income, Y. In the third column autonomous investment of 80 is listed. Since investment is independent of income by our previous discussion, its value is 80 at any level of income. Finally, expenditure, E, is the sum of C and I, a total given in the last column.

Referring to Table 16, it is evident that when the level of income is 300 billion, total expenditure, $C + I$, is exactly equal to income or supply. The market is cleared, and the receipts taken in are

exactly sufficient to remunerate the factors for the work done in producing the output.

If income happened to be at a level of 100 billion, expenditure on output would be 180 billion. This means that a demand is being expressed for 80 billion more goods than are currently available, measuring supply by factor cost. To meet this excess of demand over supply, producers will speed up production. In order to do so producers will hire more factors, especially labor in the short run, and will pay out extra income. As income increases, consumption also increases; when supply or income reaches 200 billion, expenditure will reach 240 billion. At this level of income, expenditure (aggregate demand) still exceeds income (aggregate supply). Aggregate supply or income will continue to increase as long as expenditure exceeds income. When income reaches 300, aggregate supply will be exactly equal to aggregate demand. The market will be exactly cleared, and no occasion for a further change in production or income will exist.

If income is in excess of 300, say at 500, aggregate supply at 500 exceeds the corresponding aggregate demand or expenditure of 420. In the face of larger supply than can be sold, producers cut output. By the same token less of the factors will be hired, and income falls. As income falls, consumption and expenditure fall too. When income has reached 400, it still exceeds expenditure of 360, and overproduction continues. To correct this, firms cut output further until income reaches a level of 300. At this point equilibrium between aggregate supply or income and aggregate demand or expenditure exists.

To summarize the discussion, an income level smaller than the value at equilibrium occasions an excess of expenditure over income, an excess demand, which causes income to expand. At the opposite extreme, an income level larger than the value at equilibrium causes an excess of income over expenditure, an excess supply, which causes income to contract. The expansion or contraction of income to make it equal to expenditure tends to create an equilibrium of aggregate supply and aggregate demand.

Let us consider how the situation may be represented in a diagram. In Figure 9 we measure income, Y, along the horizontal axis; we measure consumption, consumption plus investment, and

income, on the vertical axis. Consumption is plotted against income, giving the C line. Next, $C + I$, expenditure, is plotted against income. If this were all the graphical apparatus we possessed, it would be difficult to find the point of equilibrium E. This is the point at which $C + I$ equals Y. To find this point it is desirable to draw in the Y line. As explained earlier, this is simply the locus of points having the property of representing an equal value of income along either axis.

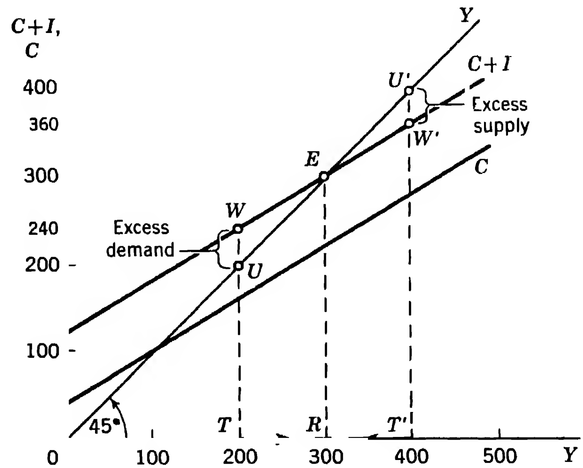


Figure 9. Income Determination—Total Spending Method.

With the aid of the Y line we are in a position to define the equilibrium of aggregate supply and aggregate demand. In fact, this equilibrium is found at point E where the $C + I$ line cuts the Y line. Here expenditure or aggregate demand just equals income or aggregate supply. Point E defines an income of OR, 300, and an equal value of expenditure. Note that OR equals RE in virtue of the 45° angle EOR. If income is $OT = TU = 200$, expenditure is $TW = 240$. This gives an excess demand of UW or 40, leading to an expansion of income, as indicated by the arrow along the horizontal axis. On the other hand, an income $OT' = T'U' = 400$ generates expenditure of only 360 or $T'W'$. This leads to excess supply of $U'W'$ or 40 which generates a contraction of income from OT' or 400 in the direction indicated by the arrow.

The foregoing explanation is couched entirely in terms of "real" output and demand. Although the preceding apparatus may be given a purely monetary explanation, it seems more appropriate to use this in connection with an alternative explanation. This explanation depicts income determination as the outcome of the equalization of saving and investment.

SAVING AND INVESTMENT

In the present analysis direct taxes levied on income are assumed to be zero. Clearly, this removes one drain on personal income. However, in virtue of the retention of earnings by corporations, as well as other items, income cannot be identified with Disposable Personal Income. In this context we define saving, S, by the equation, $S = Y - C$. Such an equality represents a "definitional equation." The expression on the right, $Y - C$, serves to define the meaning of the term on the left, S. Since C is functionally dependent on Y, S is functionally dependent on Y also. In Table 17, which uses the data of Table 16, the value of S at any level of income is derived simply by subtracting C from Y in the same row. This gives the following saving schedule, shown as S in Table 17.

TABLE 17. Saving and Investment

Y	C	$S = Y - C$	I
0	40	− 40	80 Expansion
100	100	0	80 ,,
200	160	40	80 ,,
300	220	80	80 Equilibrium
400	280	120	80 Contraction
500	340	160	80 ,,

As explained previously, I is treated as an autonomous quantity, independent of income but capable of varying in accordance with expectations. In the short run these expectations are assumed to remain constant, and for the same reason investment is assumed to remain at 80. If equilibrium is to prevail, the level of income must move to that point at which saving is equal to the fixed level of investment, 80. Why is this true?

When income is received by the individual, he consumes part and saves the remainder. That part consumed leads to corresponding production; in order to meet the demand of consumers, corresponding production is maintained, factors are retained, and incomes are paid to those factors. In sum, consumption generates corresponding income. On the other hand, saving does not correspond to, or in any way constitute, demand. Unless a demand appears to fill the gap left by saving, all the income will not correspond to demand. If demand falls short of income, income will be diminished. Clearly, a further source of demand fed by saving is the investment of business firms. If the "inflow" of investment offsets exactly the "drain" of saving from income, income will be maintained.

In Table 17 saving and investment are equal only at an income of 300. At this income the savings drain of 80 is precisely offset by a corresponding inflow of investment. So long as income remains at this level nothing occurs to change income. However, if income is 200, investment is 80, while saving is only 40. If we assume this to be possible, the inflow of investment funds more than offsets the drain of savings. Evidently, the level of income rises and will continue to do so, for so long as investment exceeds saving. As an analogy, let us think of a pool with water draining out at the rate of 40 cubic feet per time period. At the same time, water is flowing in at the rate of 80 cubic feet per time period. Clearly, the level of water in the pool must rise.

INVESTMENT EXCEEDS SAVING

What is not so evident is that investment *can* exceed saving. When firms seek to invest, they draw first on internal sources. For example, corporations rely heavily on the use of retained earnings. If this source fails to meet their needs, firms seek to draw on the savings of individuals. By the selling of securities such savings can be channeled to the business firms that require the funds. If all savings, business and individual, are put into use and still more funds are required, from what source will they come?

Some may say that firms will borrow from anyone with money to lend. Such a remark draws a quick rejoinder, "What money?" All saving out of current income has been utilized by firms. What

about borrowing from people with capital assets of one kind or another? If these assets are stocks and bonds, or other titles to income, they cannot be used for lending purposes. If the assets are in the form of currency or bank deposits, the case is more encouraging. In fact, such funds can be turned over to businessmen for investment purposes. However, both individuals and firms seek to maintain a level of cash sufficient to carry on expenditures and to satisfy other needs. The mere desire of businessmen for funds is not likely to cause these funds to be released. At this stage of analysis it seems desirable to assume that these balances cannot be utilized.

Failing all these sources, what is left? Only one source (other than unneeded bank balances) can be found, and that is the banks. By borrowing money from the banks business firms engender the creation of new money. As the new money is used to cover the investment needs not met by saving, the level of income rises. Reverting to our analogy of the pool, water, which drains out the bottom of the pool, is recirculated by a pump, and, in the process, is supplemented by a new source of water. As the enlarged quantity of water enters the pool, the level of income rises. Evidently, the creation of new money by the banks in the form of deposits, when used to buy capital goods increases both demand and income.

SAVING EXCEEDS INVESTMENT

If income happened to be at a high level, say 400, investment would be 80, saving would be 120, and the reverse situation would occur. In this case more of a leakage from the income pool occurs in the form of saving than investment can replace. Clearly, the level of the pool will fall, as more drains out than is returned to it. The deficiency in demand created by saving is not fully made up by investment. Consequently, income cannot be sustained at its current level, and will fall.

Again, a question arises as to the disposal of the excess of savings over investment. If 120 is saved and 80 is invested, what happens to the 40 of savings not needed to expand the supply of capital? Since business firms do not need the money, it cannot conveniently be lent out to them. On the other hand, individuals had adequate balances before, and there is no reason to suppose that they will want more now. For this reason an addition to idle bank balances would not be

a desirable goal for the consumer. If individuals or firms which have saved and do not require the money have debts at the banks, they may retire bank loans. In the act of so doing they will relieve themselves of the interest *costs* they must pay on loans and thereby add to *net* income. Clearly, discharging a bank debt is an alternative to investing in new securities.

Only one question remains. What happens to the money used to repay bank loans? Very simply, the money is destroyed. The act of paying off the loan reduces the balance of the firm or individual; as a check is made out against the deposit balance to pay off the loan, that deposit balance is correspondingly reduced. Since deposit balances are employed as "money" in our system, the reduction of bank balances is a destruction of money. To summarize, the excess saving funnels off a certain part of the income pool to the banks. In the act of paying off loans, firms and individuals cause the money funneled out of circulation to be destroyed. Evidently, this is going to reduce income. After income falls by a certain amount, saving will be reduced and the net drain of money from the income pool will cease.

ILLUSTRATIONS OF SAVINGS-INVESTMENT EQUILIBRIUM

In illustrating the notions set forth, two convenient diagrams may be used. First is the straightforward graph shown in Figure 10,

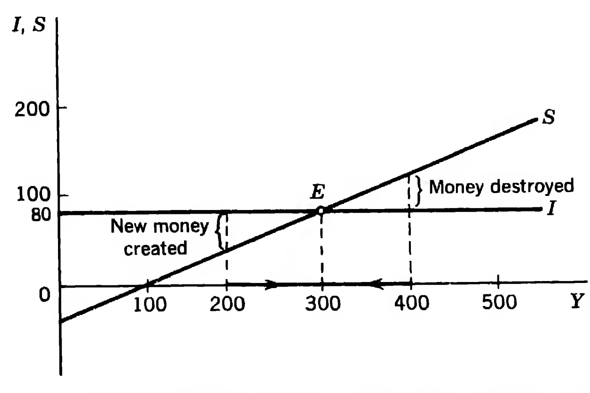


Figure 10. Income Determination—Saving-Investment Method.

exhibiting saving and investment plotted against income; the data are drawn from Table 17. If income is at the level 200, investment exceeds saving, the gap being made up by the creation of new money. As explained verbally, this pushes income in the direction of the equilibrium at 300. If the level of income is 400, saving exceeds investment; the excess saving is used to pay off bank loans, causing a corresponding amount of money to be destroyed. As we

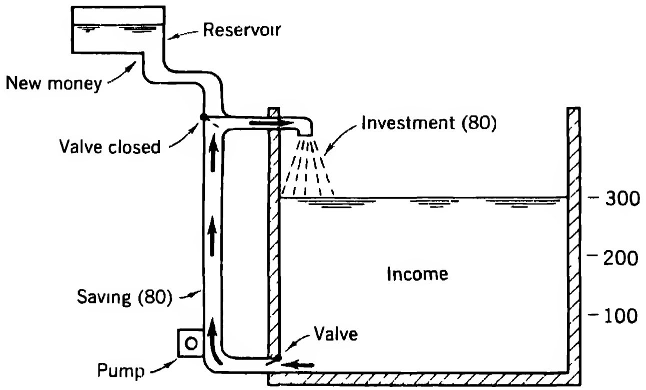


Figure 11. Flow Interpretation of Income Equilibrium.

explained, this situation leads to a decline in income from the 400 level toward 300. At 300, exactly, equilibrium prevails, as shown at *E*. Money is neither being created nor destroyed.

By extending the analogy of a pool we can provide an alternative to the foregoing interpretation of income determination. In Figure 11 a cross section of a pool is depicted. At the bottom of the pool is a valve responding in direct proportion to pressure. As stated earlier, the level of water in the pool is taken to be income. At any given height of water in the pool, the water pressure causes the valve to open a certain distance. As it does so, water escapes from the pool into a pipe. As the level of water rises, the valve opens farther and more water escapes per time period. The escaping water is, in this analogy, saving.

At the top of the pool is a spigot through which water enters the pool. Continuing the analogy, the water flowing in is taken to be investment. To collect the necessary water needed to supply the

requirements for investment, savings are pumped from the pipe at the bottom of the pool. If the quantity flowing out at the bottom exactly meets investment needs, this amount is simply pumped back into the pool through the spigot. Clearly, no change in the level of water in the pool can occur.

Conceivably, the required investment could exceed the saving flow. In this event recourse must be had to the reservoir pictured.

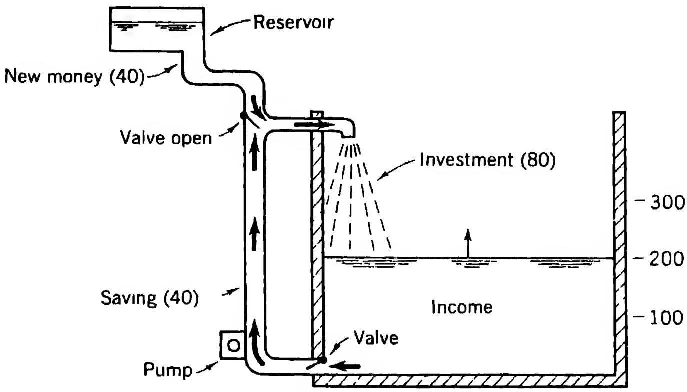


Figure 12. Flow Interpretation of Income Increase.

Obviously, this reservoir corresponds to the money-creating potential of the banks. If investment requirements exceed the saving drain, a valve leading to the reservoir opens and new money enters the income pool. In this manner the level of income will rise. In Figure 12 with an assumed income level of 200, the pressure on the outlet valve is low and the outflow of saving is only 40. Since investment requirements equal 80, a call on the reservoir for 40 of new money is made. In consequence, the total inflow, increased by the new money, exceeds the outflow and the income level rises.

RECONCILIATION OF THE TWO METHODS

From the foregoing discussion it is evident that income determination can be viewed in two lights: as a problem in supply and demand, or as a problem in income flows. The first approach is readily represented by the relation between the total demand, $C + I$, and total supply, Y, in Figure 9. On the other hand, the flow

approach can be represented either by a straightforward saving investment diagram as in Figure 10 or by a model of a pool with recirculating water. In reconciling the approaches let us consider the relation between the diagrams depicting $C + I$ in relation to Y with the one depicting I and S.

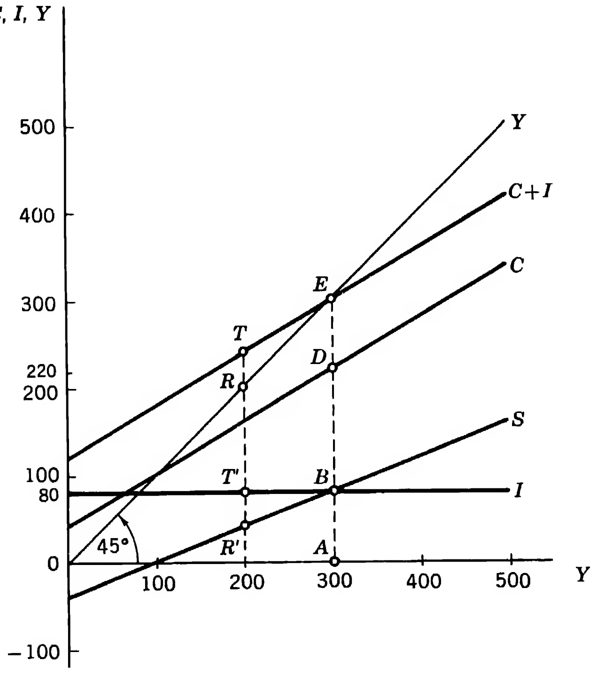


Figure 13. Reconciliation of Total Spending and Saving-Investment Methods.

When income, Y, and expenditure, $C + I$, are in equilibrium, $Y = C + I$. Subtracting C from both sides, we find an equivalent statement of the equilibrium to be $Y - C = I$. In short, in equilibrium, the gap between the Y and C line is exactly equal to I. However, I is the gap between the C and $C + I$ line. Accordingly, equilibrium prevails when the gap between C and Y is equal to the gap between C and $C + I$. In other words, the gap between Y and C is filled by I. This says that saving, $Y - C$, is equal to investment.

Economically, it means that the gap between aggregate supply (Y) and consumption (C) at that level of income is exactly taken off the market by investment demand, I.

In Figure 13 the saving gap, $Y - C$, is equal to DE, or 80, at an income of OA (300). This gap is exactly filled by I, the vertical distance between C and $C + I$, which is measured by DE (80). Investment is equal to 80 throughout the range of income and is marked off as distance AB; by construction, AB is equal to DE which also measures I. The S line is constructed by marking off the vertical distance, $Y - C$, at various levels of income and connecting the points thus defined. At E the distance $Y - C$ amounts to DE (80) and is marked off as the equal distance AB. As explained above, this also equals investment, so that equilibrium prevails at income OA.

APPROACHES COMPARED

It should be clear that the monetary analysis applied to saving and investment can be used with the income–expenditure analysis. This can be shown in the combined diagram (Figure 13). At an income of 200 investment exceeds saving by $R'T'$, but $(C + I) - Y = I - S$. In short, the difference between $C + I$ and Y is the same as the difference between I and S. The distance $R'T'$ must be equal to RT in Figure 13 and, in either case, constitutes an excess demand for goods. It represents the excess of total spending over income. The excess comes from monetary sources, especially from the creation of new money. All the monetary-flow analysis applied to saving and investment can now be used in connection with the difference between the $C + I$ and Y lines. Trial will show that excess supply or a deficiency of demand develops at incomes in excess of 300. This deficiency is associated with the destruction of money.

Full employment

Since the level of income tends toward a definite equilibrium value determined by economic forces, it is appropriate to consider several questions concerning this income level. Does equilibrium income correspond to capacity production? At the equilibrium

income are all the factors—especially labor—fully employed? According to the classical economists, the economy always tended to provide full employment. Here, briefly, is their argument.

Suppose that less than full employment prevails. If businessmen expand aggregate supply, generating additional income of 100, corresponding additional employment is created. Part of the additional aggregate supply or income, say, 60, will be consumed; the rest, 40, is saved. If a man saves, however, he does so with a view to earning a return. He will, therefore, seek to acquire some capital with which he can earn an income. When the 40 which is saved is spent on capital goods, additional spending of 40 results. In virtue of this fact, total expenditure on output will be 100, which is exactly sufficient to take the extra production from the market. Since production generates equal demand, production can be pushed to capacity and employment to the full-employment level. The argument given above is a variation by David Ricardo of an argument first advanced by J. B. Say, and known as "Say's Law."

KEYNES AND SAY'S LAW

According to J. M. Keynes, Say's Law is untrue. The crux of Keynes's argument has to do with the character of investment. In Say's time a good deal of saving was done by individuals who invested these funds directly in their own businesses. However, in the modern era saving and investment are carried out by different classes of people. Investment is undertaken to a large extent by corporations which draw part of their funds from their own retained earnings and part from securities sold to many individuals. Furthermore, the desire of corporations and other firms to invest is governed by considerations of profitability, whereas individual savings are governed by the desire for security or income. In general, the motives to save and to invest cannot be expected to coincide in magnitude.

As a first approximation, investment is regarded as autonomous or independent of income. If income is at a level lower than that corresponding to capacity, and production is then expanded, consumption will increase by a fraction of the extra income. With additional income or aggregate supply of 100, consumption increases by 60, leaving 40 units unsold. Since investment is autonomous,

no additional amount is undertaken as income expands. Indeed, in the presence of excess capacity there would be little need to expand capacity. In consequence, producers would fail to sell part (40) of the supply (100), necessitating a contraction to the previous level. In short, the equilibrium level established may correspond to less than full employment or overfull employment.

SAY'S LAW AND THE LONG RUN

Classical economists typically thought in long-run terms. To contrast the ideas of the present day and those of the classical writers we may use long-run consumption and saving functions. Any gap between Y and C, any saving, is channeled automatically into investment, according to the classical argument. Consequently, any excess of aggregate supply, Y, over C is automatically made up by investment. For this reason the Y and $C + I$ lines coincide, as shown in Figure 14(A). Similarly, in Figure 14(B),

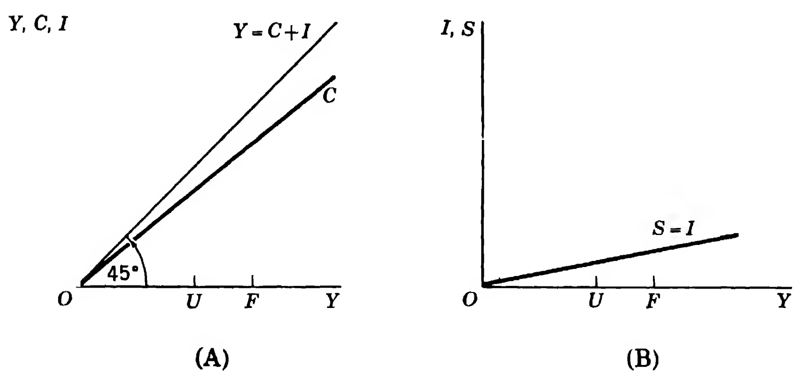
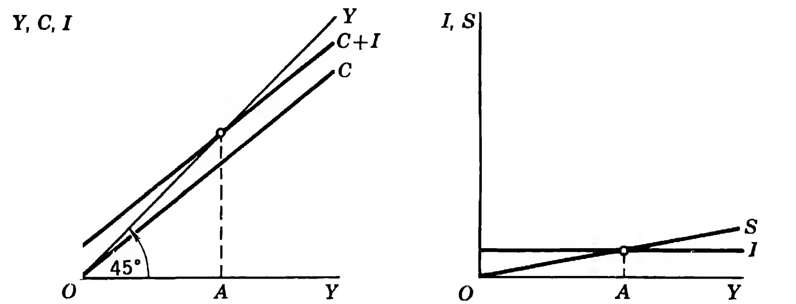


Figure 14. Say's Law Interpreted Graphically.

showing the saving–investment approach, saving and investment are represented as equal at any level of income. In such a situation any level of income is an equilibrium level.

In Figure 15 the long-run consumption and saving functions are represented, but investment is treated as autonomous. In Figure 15(A) the single equilibrium income OA is found where the $C + I$ line cuts the Y line. Similarly in Figure 15(B) the equilibrium

income level is found where the *S* line cuts the *I* line. Evidently, both the assumptions and the conclusions differ from the classical case.



(A) (B)

Figure 15. Long-Run Keynesian View of Income Determination.

Stability of equilibrium

What we have been dealing with in the preceding pages is a very simple model of the economic system. In essence, the model seeks to picture the way in which National Income is determined. In the analysis of a model of this sort one first seeks an equilibrium position in which the several forces operating are in balance. Next, it is necessary to discover whether this equilibrium is stable. If one of the variables is changed, the equilibrium is said to be stable when the variables assume their former values. If the variables change to some new set of values, or change indefinitely, the initial situation is described as a position of unstable equilibrium.

A popular mechanical analogy used to illustrate stability is that of an orange in a bowl. If the orange is at the bottom, it is in equilibrium. If displaced to one side, it will tend to return to the initial position. The orange is in a position of stable equilibrium, then, when it is at rest at the bottom of the bowl. If the orange is placed on a table and then displaced to the right, it will stay in the new position. In such a situation the orange is in a position of neutral equilibrium. Finally, the orange may be placed on top of an inverted bowl. If the orange is moved to one side, it will continue in the

same direction. The initial position of the orange at the top of the bowl is one of unstable equilibrium.

In an economy whose functioning can be accurately described by Say's Law, the equilibrium of income and expenditure can only be neutral. Any displacement of income from its initial position leads to a new equilibrium position. In such a situation no one level of income is favored. Any level of income from zero to full employment is equally a position of equilibrium.

In Figure 9 (p. 65) a typical short-run situation is pictured, equilibrium being found where the $C + I$ line cuts the Y line, giving OR as the equilibrium income. In this situation an increase in income will cause aggregate supply, Y, to exceed demand, $C + I$. This will cause a contraction of income. On the other hand, a contraction of income will cause demand, $C + I$, to exceed aggregate

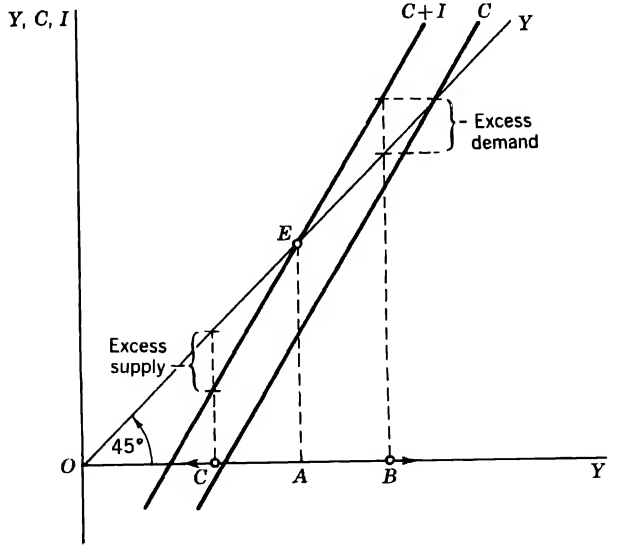


Figure 16. Unstable Equilibrium of Income.

supply, Y, and thus occasion an increase in aggregate supply and income, Y. Evidently, the situation is stable, since income exhibits a persistent tendency to return to the equilibrium point.

By way of contrast consider the situation pictured in Figure 16.

Under these conditions a tendency of income to exceed the equilibrium value, pictured as a movement to the right of A, sets up an excess demand. In turn, an excess demand leads to a further increase in income, which implies a tendency of income to "run away" toward indefinitely large values. By corresponding reasoning a fall in the level of income sets up forces leading to a further downward movement. Clearly, the situation pictured here corresponds to that of the ball on the inverted bowl poised in unstable equilibrium.

Although our economy is marked by ups and downs, it is not marked by indefinite expansion or contraction. For this reason the presumption is that the system is stable. The condition of stability is that demand, $C + I$, increases less rapidly than aggregate supply or income, Y, as income or Y increases. Such a condition implies that the $C + I$ line must be flatter than the Y line.[1] In algebraic

[1] We can demonstrate that fulfillment of the stability condition,

$$\Delta(C + I) < \Delta Y \text{ or } \frac{\Delta(C + I)}{\Delta Y} < 1,$$

implies that the $C + I$ line is flatter than the Y line.

Let the Y line make a 45° angle with the OY axis. Drop a vertical line from the intersection of the Y line with the $C + I$ line at R to OY cutting in R'. Extend a line from R to the right, parallel to OY, for an arbitrary distance, terminating in T. Draw a line through T, perpendicular to RT, intersecting OY in T', the $C + I$ line in U, and the Y line in V.

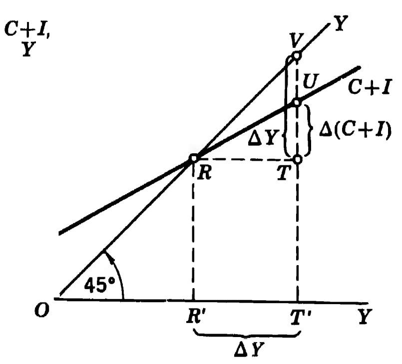


Illustration of Algebraic Condition for Stable Equilibrium.

Equilibrium occurs at R with income OR'. Let income increase by the arbitrary quantity $R'T'$ equals RT. Let us show first that ΔY which equals $R'T' = RT$, by definition, is also equal to TV. Since any line, such as the Y line, makes equal angles with any two parallel lines, RT and OY, angle VRT = angle VOT' = 45°. By construction, angle RTV is 90°. Since any triangle contains 180°, by subtraction angle RVT is 45°. Since angles RVT and VRT are equal, the sides opposite these angles must also be equal or $RT = TV$. Consequently, we have shown that $\Delta Y = TV$.

As income increases from OR' to OT', expenditure, $C + I$, increases from $R'R$ to $T'U$. From this it is evident that $\Delta(C + I) = T'U - R'R = T'U - T'T = TU$.

terms this implies that $\Delta(C + I) < \Delta Y$ or $\Delta(C + I)/\Delta Y < 1$. Since I is autonomous and does not vary with income, only C changes with Y in this first approximation. Accordingly, the condition of stability is:

$$\frac{\Delta(C + I)}{\Delta Y} = \frac{\Delta C}{\Delta Y} < 1.$$

This states that the marginal propensity to consume must be less than 1. There is considerable empirical evidence to indicate that this is, indeed, the case.

The multiplier

Since investment depends on profit anticipations, it is capable of changing independently in accordance with changes in the business climate. In a modern industrial society capital production plays a rather important role. It is relevant to inquire, then, what effect a change in investment will produce on the level of income.

An increase of a given amount in investment expenditure will justify an increase of several times that amount in income. Suppose investment increases by 40. Such an increase in demand generates corresponding income. When the additional demand is manifested, production levels will be raised. To carry out this program businessmen will find it necessary to hire additional labor and other variable factors, resulting in the payment of additional income. When the increased income reaches individuals, consumption will increase also. In every case, then, increased investment is accompanied by increased consumption expenditure. Income (aggregate supply) will increase to just that extent at which the total increase in demand

The stability condition states that $\Delta(C + I)/\Delta Y < 1$. Substituting the values of $\Delta(C + I)$ and ΔY which we have found, this gives the following relation:

$$\frac{\Delta(C + I)}{\Delta Y} = \frac{TU}{TV} < 1.$$

By inspection of the diagram it is clear that $TU/TV < 1$ or $TU < TV$ if, and only if, U lies below V. When this is true, the $C + I$ line is flatter than the Y line. This represents the case of stable equilibrium pictured in Figures 15 and 9. If $TU/TV > 1$ or $TU > TV$, U lies above V and the $C + I$ line is steeper than the Y line. This situation is pictured in Figure 16. If $TU/TV = 1$ or $TU = TV$, U coincides with V and the $C + I$ line coincides with the Y line. This situation fulfills the conditions of Say's Law pictured in Figure 14.

is equal to the increase in income (aggregate supply). With an MPC of .6, consumption increases by 60, when income increases by 100. This leaves a gap of unsold goods equal to 40, which is just taken from the market by the additional investment demand of 40. Thus, the total increase in demand, 40 of I and 60 of C, was exactly equal to the increased income (supply) of 100.

TABLE 18. Investment Changes and the Income Level

Y	C	$S = Y - C$	I	$C + I$	$I + \Delta I$	$C + I + \Delta I$
100	100	0	80	180	120	220
200	160	40	80	240	120	280
300[a]	220	80	80	300	120	340
400[b]	280	120	80	360	120	400
500	340	160	80	420	120	460

[a] Old income level.
[b] New income level

This process can be illustrated by the same basic example used earlier in this chapter (see Table 17, p. 66). Referring to Table 18, we see that the original equilibrium occurs at an income of 300 where $C + I = Y$ or $I = S$. When the level of investment changes from 80 to 120, $I > S$, and $C + I$, which is now 340, exceeds a Y of 300, so that $C + I > Y$. Expansion of income occurs until investment and saving, expenditure and income, are equal at 400. We may note that the increase in income is 100, compared to an increase of 40 in investment. The ratio of the increase in income to the increase of investment is:

$$K = \frac{\Delta Y}{\Delta I} = \frac{100}{40} = 2.5.$$

This ratio indicates that an increase in investment of 1 billion produces an increase in income of 2.5 billion. This implies that an increase in investment produces a multiplied effect on income.

In Figure 17(A) income originally is equal to OR. As investment increases, the $C + I$ line shifts upward to the $C + I + \Delta I$ position. As this happens, income increases to OS. In numerical terms, income increases from 300 to 400 as investment increases by 40 from

80 to 120. In Figure 17(A) the rise in investment is shown by the upward shift of the $C + I$ line, which is measured by the vertical distance CD. In turn, this is the distance between 360 and 400 on the vertical axis, an amount equal to 40.

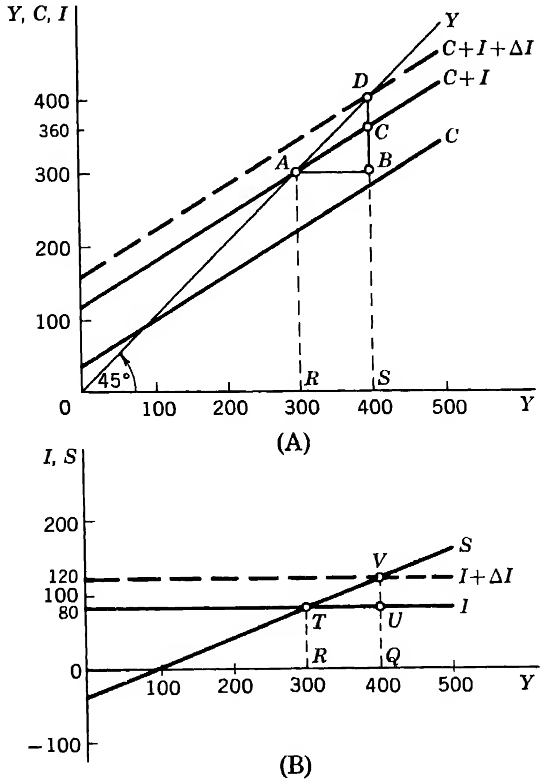


Figure 17. Multiplier Principle Illustrated in Two Ways.

In Figure 17(B) the effect of a change in investment on income is shown by a saving–investment diagram. Originally, equilibrium is found at T where the saving and investment curves intersect. This equilibrium determines an income of 300 or OR. When investment rises from 80 to 120, a new equilibrium is found at V which determines a higher level of income amounting to 400 or OQ. Evidently, the rise in investment has caused a rise in the level of income.

THE MULTIPLIER FORMULA

In the preceding discussion we have shown that an increase in investment causes income to rise. Proceeding from the last approach, we take it to be true that saving equals investment. If investment changes, income must change by an amount sufficient to cause saving to equal investment. If investment changes, saving must change by the same amount. In Figure 17(B), $\Delta I = UV$, and when income rises by RQ, saving increases along the curve from T to V, increasing sufficiently that $\Delta S = UV$.

Let us now proceed to the proof of the multiplier formula. By definition:

$$K = \frac{\Delta Y}{\Delta I} = \frac{\Delta Y}{\Delta S}, \text{ by the preceding discussion,}$$

$$= \frac{\Delta Y}{\Delta Y - \Delta C}, \text{ since } \Delta Y = \Delta C + \Delta S,$$

$$= \frac{1}{1 - \dfrac{\Delta C}{\Delta Y}}, \text{ after dividing numerator and denominator by } \Delta Y,$$

$$= \frac{1}{1 - \text{MPC}}$$

The equation, $\Delta Y/\Delta I = 1/(1 - \text{MPC})$, is the conventional formula for the multiplier. If MPC is .6, the multiplier is $K = 1/(1 - .6) = 1/.4 = 2.5$. This means that an increase of 1 billion in investment will increase income by 2.5 billion dollars.

GRAPHICAL INTERPRETATION OF MULTIPLIER FORMULA (OPTIONAL)[2]

The increase in income in Figure 17(A) is $RS = AB$. Since parallel lines which intersect a given line make equal angles with that line, angle DAB = angle DOS. Now the Y line is constructed by taking values of income on the horizontal axis and laying out the same distance vertically. So by laying off OR on the horizontal axis and then laying off OR in a vertical direction from point R, we find

[2] This and other optional sections may be read by students who would like more detailed explanations.

point A. Evidently $OR = RA$ by construction. By the same token $OS = SD$. Since angle ODS and angle ADB are the same, since angle DAB and angle DOS are also the same, and angle DBA and angle DSO are both right angles and therefore the same, triangles DOS and DAB are similar. Therefore, the sides are proportional. Hence, remembering that $OS = SD$, we have,

$$1 = \frac{SD}{OS} = \frac{BD}{AB} \cdot \quad \text{Therefore, } AB = BD.$$

The multiplier is the ratio, $\Delta Y/\Delta I$, the quantity ΔI being shown in the figure as CD. We can therefore interpret the multiplier geometrically as:

$$K = \frac{\Delta Y}{\Delta I} = \frac{RS}{CD} = \frac{AB}{CD} = \frac{BD}{CD} \cdot$$

The last ratio is the most convenient. If we desire, we may use the figure to derive the multiplier formula. To do this, we must first note that the C, $C + I$, and $C + I + \Delta I$ lines are parallel. This means that a change in C is measured by an upward movement on any one of the three. As income increases by $RS = AB$, consumption rises by BC. Armed with this information we can derive the multiplier formula graphically, as follows:

$$K = \frac{BD}{CD} = \frac{BD}{BD - BC} = \frac{1}{1 - \dfrac{BC}{BD}}$$

$$= \frac{1}{1 - \dfrac{BC}{AB}} = \frac{1}{1 - \dfrac{\Delta C}{\Delta Y}} = \frac{1}{1 - \text{MPC}} \cdot \qquad (4\text{–}1)$$

In Figure 17(B), which exhibits a saving–investment approach, a different interpretation is appropriate. In the initial situation the equilibrium level of income is OR. When investment increases by the amount UV, income increases by RQ. Evidently, the multiplier value is:

$$K = \frac{\Delta Y}{\Delta I} = \frac{RQ}{UV} = \frac{TU}{UV} = 1 \Big/ \frac{UV}{TU} \cdot$$

Note that UV represents the increase in saving as income rises by TU. Evidently $UV/TU = \Delta S/\Delta Y$ and represents the ratio of an

increment of saving to the increment of income that caused it. By analogy to MPC we can define the ratio $\Delta S/\Delta Y$ as the marginal propensity to save. Therefore we can write the above formula:

$$K = \frac{1}{(UV/TU)} = \frac{1}{(\Delta S/\Delta Y)} = \frac{1}{\text{MPS}}. \qquad (4\text{–}2)$$

In words, the multiplier is the reciprocal of the marginal propensity to save.

Since the two approaches lead to different formulas, 4–1 and 4–2, some sort of reconciliation is necessary. First, in a society with no taxes, an increment of income is divided between consumption and saving. Writing this symbolically, we secure:

$$\Delta Y = \Delta C + \Delta S.$$

Dividing both sides by Y, we find that:

$$1 = \frac{\Delta C}{\Delta Y} + \frac{\Delta S}{\Delta Y},$$

a result which can be stated as: $1 = \text{MPC} + \text{MPS}$. This means simply that the fractions of a dollar respectively consumed and saved add up to a dollar. With this in mind it is quite evident that:

$$K = \frac{1}{1 - \text{MPC}} = \frac{1}{\text{MPS}}, \qquad (4\text{–}3)$$

indicating that formulas (4–1) and (4–2) are interchangeable.

THE MULTIPLIER FUNCTION

The principle underlying multiplier theory is that an equilibrium income is established when investment and the consumption function are given. In this situation there is just one quantity capable of varying independently, to wit, investment. If profit anticipations change, investment will change and with it the level of income. This means that the principal cause of variations in income is to be found in changes in investment. To be brief, income is functionally dependent on investment. It remains merely to identify the functional relationship.

First, let us consider Table 19. (*Note:* The data are extracted from Table 17. Several new investment columns have been added.)

Suppose investment is −40. This means that businessmen decide to retain 40 of the flow of depreciation funds for some financial purpose, and permit capital equipment to run down by this amount.

TABLE 19. Income Changes with Varying Investment

Y	S	I_0	I_1	I_2	I_3	I_4	I_5
0	−40	−40	0	40	80	120	160
100	0	−40	0	40	80	120	160
200	40	−40	0	40	80	120	160
300	80	−40	0	40	80	120	160
400	120	−40	0	40	80	120	160
500	160	−40	0	40	80	120	160

At this level of investment, to wit *dis*investment of 40, saving and investment will be equated at zero income. Note that consumption is 40, since what is dissaved at zero income must be consumed by definition. Accordingly, all members of society do not die off immediately.

If investment is zero, S and I are equated at an income level of 100. If investment is 40, S and I are equated at an income level of 200, and so forth. Let us define the multiplier function as that relation which gives the level of income resulting from any given level of investment. Summarizing the information we have just derived, we may write the schedule shown in Table 20. Examining this schedule, we find that if we replaced I by S, we would have the income-saving schedule shown in Table 19. Only one difference is

TABLE 20. Multiplier Schedule

$I(=S)$	Y
−40	0
0	100
40	200
80	300
120	400
160	500

evident. The independent variable in the saving schedule is income, whereas in this case it is investment. What is noteworthy in Table 20 is simply an inversion of the positions of the dependent variable, saving, and the independent variable, income. Obviously, saving is replaced by investment, to which it must be equal, in Table 20.

In Figure 18, the principle is illustrated as follows. First, we draw in the usual saving curve, plotted from Table 19. Suppose the

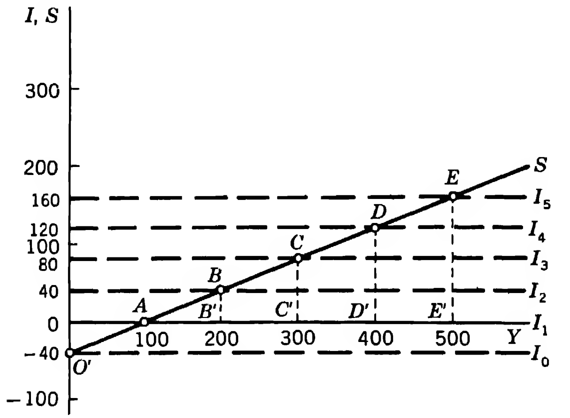


Figure 18. Graphical Derivation of the Multiplier Function.

level of investment is -40; this value of investment is represented by a horizontal straight line placed below but parallel to the income axis. In this case income must be zero to equate saving and investment. If the level of investment is zero and the investment line coincides with the Y axis, the level of income must be OA to equate saving and zero investment. When the level of investment rises to I_2, equilibrium is reached at B with income OB'. As investment increases, the points of equilibrium rise along the saving curve in points C, D, and E. The whole process may be defined by a schedule, as in Table 21.

By examination of the schedule and figure, Table 21 can be seen to correspond to Table 20. In the schedule of points each point referred to is the point defined by the corresponding level of Y and I. Thus, point C is reached by going out to income OC', and up $C'C$

investment to C. Since each point represents the relation between investment and corresponding income, the plot of all such points

TABLE 21. Multiplier Schedule

I	Y	Point
$I_0 = OO'$[a]	O	O'
$I_1 = O$	OA	A
$I_2 = B'B$	OB'	B
$I_3 = C'C$	OC'	C
$I_4 = D'D$	OD'	D
$I_5 = E'E$	OE'	E

[a] A negative amount.

represents the multiplier function in the form of a curve. All such points as O', A, B, C, D, and E fall along the saving curve. Evidently, the multiplier function is to be identified with the saving curve.

Only one difference between saving and multiplier function is evident. In the multiplier function investment is the cause, whereas income is the effect. In the case of the saving curve, income is the cause and saving is the effect.

The following statements serve to summarize the discussion.

1. *Definition:* The multiplier function is the schedule of levels of income generated by a corresponding schedule of values of investment.

2. *Conclusion:* The multiplier function is the saving function, investment being substituted for saving, and the positions of dependent and independent variables being interchanged.

Income tends to reach a stable equilibrium when investment is autonomous and MPC is less than 1. If investment changes, income will change in a definite way indicated by the multiplier function, a reversed saving–income relation.

Government and income determination

Let us consider some of the ways in which government taxing and spending influence income. Instead of focusing attention on various useful rules concerning taxing and spending which might be

developed, the discussion deals mostly with elementary theory. Let us begin with a consideration of the influence of income taxes. Let us suppose that the income tax is one which falls on both corporations and individuals. In the following discussion we will refer to National Income plus transfer payments less income taxes as private income. Symbolically, we denote National Income by the letter Y, and private income by Y_p.

INCOME TAX AND MPC

Suppose that we consider the relation between consumption and National Income. Let us note first that transfer payments may be assumed to be independent of income, or constant. In dealing with income changes, then, we may ignore transfer payments and take the difference between National Income and private income to be taxes.

Assume that MPC out of private income, MPC_p, is .8. Let the marginal propensity to tax additional income, MPT, be .25. Obviously, that part of additional income not taxed is $1 - MPT = 1 - .25 = .75$. From this information we know that of an additional dollar of National Income .75 is passed back to income recipients (individuals and corporations). Of this .8 is spent, making added consumption equal to $.8(.75) = .6 = MPC$. As a result of the tax drain MPC out of National Income, MPC, is going to be less than MPC out of private income, MPC_p. In this case it is .6 as compared with .8. To be precise, the relation between the two is:

$$MPC = MPC_p(1 - MPT).$$

An assumption is necessary to the effect that the distribution of income after tax is the same as the distribution of income was at the same income level before the tax.

TAXATION AND THE CONSUMPTION LINE

Let C be plotted against National Income in Figure 19. When an income tax is imposed, the consumption line is lowered and flattened. As was explained in the chapter on consumption, the flatness of the C line is determined by the MPC: the lower the MPC, the flatter the C line. In the previous paragraph we showed that MPC is less than MPC_p, indicating that MPC is reduced by the tax,

a fact which flattens out the C line. Accordingly, C_{AT}, indicating the consumption line after tax, is flatter than C_{BT}, the consumption line before the tax. Moreover, part of National Income being taxed away, less private income is available after tax, and consumption

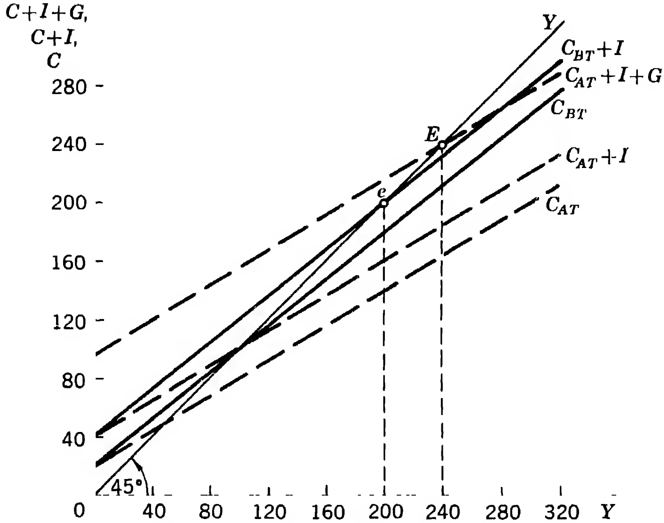


Figure 19. Effect of Government Taxing and Spending.

declines. Evidently, the C line falls—from the C_{BT} position to the C_{AT} position. Since these assertions may be hard to accept from the limited explanation given, let us illustrate them with a numerical example. The relation between Y and C_{BT} schedules represents the consumption function before the tax was imposed. By assumption society consumes 20 at zero private income. With additional income of 40, society consumes an extra 32 which is .8 of the additional income. Accordingly, at a National Income of 40 before tax, society consumes its initial 20 plus an additional 32, making a total of 52. Beginning with a C_{BT} of 20 at zero income and adding 32 for every 40 of income, we can build up C_{BT} in column 2 of Table 22. In turn, this plots into a C_{BT} line in Figure 19. When no tax is present, this represents the consumption function with C plotted against private income.

Consider the build up of the consumption after tax column under C_{AT}. Since C equals 20 at zero Y_p, and $Y = Y_p$ at zero Y, taxes being assumed to be zero at zero income, $C = 20$ at zero Y. In short, consumption is unchanged at 20 when National Income is zero, because the tax is zero, being proportional to income. As Y increases by 40, Y_p increases by $.75(40) = 30$, the tax bite on 40 being $.25(40) = 10$.

TABLE 22. Consumption Schedules Before and After Tax

Y	C_{BT}	C_{AT}
0	20	20
40	52	44
80	84	68
120	116	92
160	148	116
200	180	140

As Y_p increases by 30, C_{AT} increases by $.8(30) = 24$. Evidently, C_{AT} increases by $.8(.75)40 = .6(40) = 24$, where .6 is MPC. To restate this verbally, MPC out of National Income is only .6 owing to the tax bite, causing consumption after tax to increase by only 24, when National Income increases by 40. At an income of 40, the additional C of 24 is added to the initial 20, giving a total of 44. Adding 24 every time Y increases by 40, we build up the consumption after tax, the C_{AT} column.

As stated above, Table 22 plots into Figure 19. Our assumption was that the tax did not change the distribution of income. The C_{BT} line is drawn up on the assumption that no tax exists, and is therefore the same as a C line plotted against private income, Y_p. Under a condition of zero tax, Y and Y_p are the same. In order to find the level of National Income it is necessary to consider investment and government spending. As in the preceding treatment, investment is treated as an autonomous quantity—independent of income, but capable of short-term variation. Next, consider the nature of government spending.

GOVERNMENT SPENDING

In the short run, government spending may be regarded as a constant. That is, government spending is fixed for the budget

period by the legislature or Congress. Within the budget-fixing period, government spending will be approximately constant. Unlike investment, which is autonomous and capable of variation in the short run, government spending can be regarded as absolutely fixed during the budget period. In short, government spending is *not* to be regarded as an independent variable except in periods when the budget changes. In a formal sense the distinction makes no difference. In both cases this spending is taken as independent of income.

The composite result of government entry onto the scene is twofold. First, the consumption function is lowered as a result of the tax. Second, the government spending is added to consumption and investment to derive a new total. It is not our purpose here to discuss the outcome of this composite change, but simply to explain the change in a formal sense.

By extending Table 22 giving the two consumption functions—assuming a value of 20 for I and of 56 for government spending, G—we get all the desired information shown in Table 23. From

TABLE 23. Incorporation of Government Taxing and Spending into Income Determination

Y	C_{BT}	I	$C_{BT}+I$	C_{AT}	$C_{AT}+I$	G	$C_{AT}+I+G$
0	20	20	40	20	40	56	96
40	52	20	72	44	64	56	120
80	84	20	104	68	88	56	144
120	116	20	136	92	112	56	168
160[a]	148	20	168	116	136	56	192
200	180	20	200	140	160	56	216
240[b]	212	20	232	164	184	56	240
280	244	20	264	188	208	56	264
320	276	20	296	212	232	56	288

[a] Equilibrium income before the tax was imposed.
[b] Equilibrium income after the tax was imposed.

Table 23 and its plot in Figure 19, it is evident that income was established at 200 before the tax. As usual, let us suppose that the tax rate is .25. After the tax, and with government spending set at 56, the level of income becomes 240. Note that the tax, T, is $.25(240) = 60$, while G is 56, so that the surplus, S, is $S = T - G = 60 - 56 = 4$. In consequence, the rise in income which

accompanied the additional taxing and spending did *not* arise from a deficit in the government budget. With so many possible combinations of tax and spending situations it is dangerous to draw conclusions about the consequences of budgetary policy. We will not discuss this matter further here. In Figure 19 the formal modification of the graph is shown. Before taxing and spending the equilibrium was at *e*; afterwards it is at E. As a result, equilibrium income rises from 200 to 240.

THE GOVERNMENT MULTIPLIER

If the government adds to its expenditure *without modifying the tax rate*, the level of government spending rises with no change in the consumption function. Accordingly, total spending rises by the amount of government spending at the old level of income. In the same way that changes in investment cause income changes, so do changes in government spending. Formally, the analysis is the same.

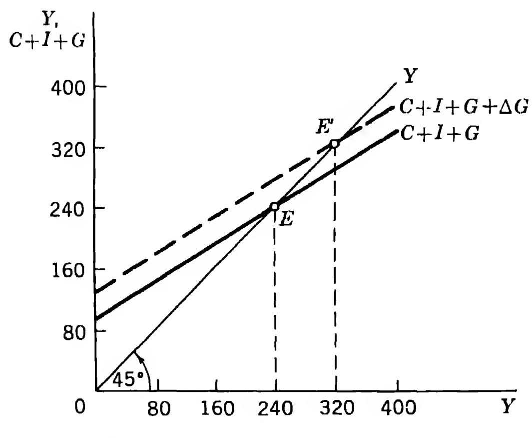


Figure 20. Multiplier Effects of Government Spending.

The same multiplier formula applies. In Figure 20 a rise in government spending with the same tax rate causes the $C + I + G$ line to shift upward parallel to itself. In consequence, the equilibrium shifts upward from E to E'. The diagrammatic analysis is exactly similar to that of Figure 17(A).

MODIFICATION IN FORMULAS

In the equilibrium analysis, income is now determined by the equation: $Y = C + I + G$, which expresses the condition that aggregate supply, Y, be cleared from the market by demand. Furthermore, when government enters onto the scene, Y, conceived of as income, is allocated into three parts: consumption, saving, and taxes. To express or define this fact, we can write the equation: $Y = C + S + T$. Setting the two expressions for Y equal to one another, we arrive at the condition: $C + I + G = C + S + T$, or, canceling C on both sides, $I + G = S + T$. This relation indicates that, in equilibrium, investment and government spending must

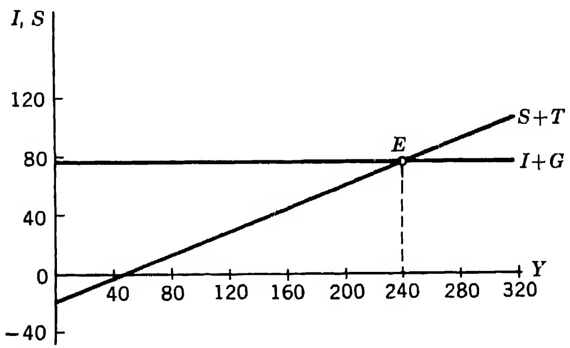


Figure 21. Equilibrium with Government Spending and Taxing.

exactly offset the drains from the income stream caused by saving and taxing. In terms of the diagrammatic analysis we simply substitute a saving plus taxing line, denoted by $S + T$, for our old saving line. We also substitute an investment plus government spending line, denoted by $I + G$, for our old investment line. The equilibrium level of income is determined in Figure 21 by the intersection of $S + T$ with $I + G$. In Table 24 the corresponding numerical information is set down.

MULTIPLIER FUNCTION

It is easy to see that the multiplier function for all autonomous spending, including government, at periods of change in the budget,

TABLE 24. Income Determination With Government Taxing and Spending[a]

Y	I	G	$I + G$	C_{AT}	$S + T = Y - C_{AT}$
0	20	56	76	20	−20
40	20	56	76	44	− 4
80	20	56	76	68	12
120	20	56	76	92	28
160	20	56	76	116	44
200	20	56	76	140	60
240	20	56	76	164	76
280	20	56	76	188	92
320	20	56	76	212	108

[a] I and G are autonomous. The C_{AT} schedule is derived in Table 22. $S + T$ is simply the excess of Y over C_{AT}.

is an inverted saving plus taxing function. Thus a slight change in terminology accounts for the government influence.

Investment as a function of income

In our previous discussion of income determination, investment has been treated as strictly autonomous. As a first approximation it seems sensible and fairly realistic to treat investment as determined strictly by anticipations of future income, and therefore independent of current income. As a second approximation it seems appropriate to take account of the possibility that investment may improve with a rise in current income. Such an assumption is based on the notion that anticipations of future income will improve if current income increases.

FORMAL MODIFICATIONS

The only formal changes necessary in the analysis consist in changes in the slope of the I line. Since I is assumed to increase with income, the I line rises with income. As shown in Figure 22 this variation of investment with income causes the $C + I$ line to diverge from the C line as income increases. In Figure 23 (p. 98), which takes the saving-investment approach, the I line rises from left to right instead of assuming a position parallel to the income axis. Other than these changes in the shapes of the respective curves the

analysis is much the same. Later, modifications in the multiplier analysis will be shown. Since I varies with income, the condition of stability is now:

$$\frac{\Delta(C+I)}{\Delta Y} < 1,$$

where I varies as well as C. As we indicated in the discussion of stability, this condition states that the $C+I$ line must cut the Y line from above to the left.

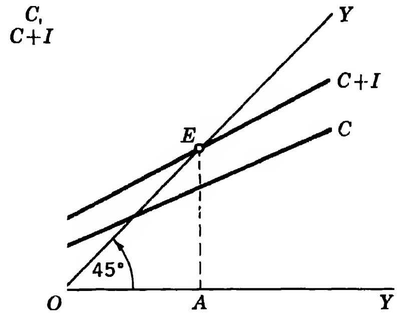


Figure 22. Income Determination When Investment Varies with Income.

STABILITY CONDITION: SAVING AND INVESTMENT

In earlier discussions the stability conditions took the form of a relationship between the behavior of total spending, $C+I$, and income (cost of output), Y. For stability to exist, expenditure $(C+I)$ must vary less than income (Y) when income changes. In certain cases it is convenient to cast the stability condition into a form which compares saving with investment. For example, the "Paradox of Thrift," a doctrine advanced by Keynes, necessitates the use of a saving–investment stability rule (see below, pp. 99 ff.).

In transforming the aforementioned rule for stability it is necessary to recognize the rule of marginal allocation of income. When no taxes exist, an additional dollar of income may be allocated either to consumption or to saving. Since saving is defined as unconsumed income, that part which is not consumed is saved. For example, suppose the level of income rises by \$1.00, the MPC being .6; the level of consumption will then increase \$.60. Subtracting the \$.60 from \$1.00 to get the unconsumed part of income, we arrive at \$.40 as the quantity saved. From this it is quite easy to see that the Marginal Propensity to Save, the MPS, is .4. Furthermore, the sum of the fractions consumed and saved equals 1. In algebraic form this may be written:

$$1 = \text{MPC} + \text{MPS}.$$

Specifically, $1 = .6 + .4$ in the foregoing example. In an optional section this relation has already been used (p. 84).

A Digression on Equations and Inequalities

Before proceeding further with the discussion it is appropriate to make a remark about equations and inequalities. In our discussion the quantities involved are real numbers such as dollars, hours of labor, or units of output. If a common amount is subtracted from both sides of such an equation, a new one is obtained. In fact, both sides of the equality, having been reduced by the same amount, are still equal to each other. Starting with the equation, $6 = 2 + 4$, for example, we may subtract 1 from either side, giving $5 = 1 + 4$ or $5 = 2 + 3$.

Although this may seem painfully obvious, the reader may not be as fully aware that the principle applies also to inequalities. Suppose the inequality is:

$$5 \text{ is less than } 2 \text{ plus } 4, \text{ or } 5 < 2 + 4.$$

Beginning with this inequality it is proper to subtract 1 from each side, giving:

$$4 < 1 + 4 \text{ or } 4 < 2 + 3.$$

STABILITY CONDITION RESUMED

Since we are reassured that it is proper to subtract a like amount from both sides of an equation or inequality, we may subtract MPC from both sides of the allocation equation, $1 = \text{MPC} + \text{MPS}$. This gives:

$$\begin{aligned} 1 - \text{MPC} &= \text{MPC} - \text{MPC} + \text{MPS} \\ &= \text{MPS}. \end{aligned}$$

Let us refer to the ratio of an increment of investment to the increment of income that caused it as the Marginal Propensity to Invest or MPI. If income increases by 100, leading to additional consumption which ultimately stimulates 20 of investment, $\text{MPI} = \Delta I/\Delta Y = 20/100 = .2$. Clearly, there is a close formal analogy between the MPC and MPI, both expressing the marginal influence of income on a category of spending.

According to the original stability condition it is necessary that:

$$\frac{\Delta(C + I)}{\Delta Y} \quad 1.$$

In other words the increment of spending should be less than the increment of income that induced it. Since there are two categories of spending, C and I, we can write the above inequality as:

$$\frac{\Delta C}{\Delta Y} + \frac{\Delta I}{\Delta Y} = \text{MPC} + \text{MPI} < 1.$$

If MPC is .6 and MPI is .2, MPC + MPI is .8 which is less than 1. On the other hand, if MPC is .8 and MPI is .3, .8 + .3 = 1.1 which is not less than 1. In the latter case, if receipt of $1.00 of additional income prompts $1.10 of spending, the $1.10 of spending will generate $1.10 of income. With the higher income, expenditure would increase still more. Evidently, in such a situation income would run away.

Since we can subtract a common number from both sides of an inequality, take MPC from each side. This gives the inequality:

$$\text{MPI} < 1 - \text{MPC}.$$

From the transformed allocation equation we know that 1 − MPC = MPS. Consequently, the stability condition may be written:

$$\text{MPI} < \text{MPS}.$$

If stability is to prevail, an extra dollar of income must prompt less investment than saving. If MPI is .3 and MPS is .4, stability is assured. But if MPI is .4 and MPS is .3, the situation is unstable.

Since this interpretation of stability is new, a word on its meaning is pertinent. In the first place, the condition in question has precisely the same meaning as the stability condition given earlier, that

$$\frac{\Delta(C + I)}{\Delta Y} < 1.$$

Verbally, the earlier condition is interpreted as meaning that the slope of the $C + I$ line is flatter than the Y line. In Figures 9 and 16 (p. 65 and p. 77) a graphical explanation is provided which shows why the existence of the condition cited provides stability.

Since the new condition is merely a reinterpretation of the old, it may be justified by appeal to the same argument.

Perhaps a direct interpretation of the implications of the condition would provide additional insight. If MPI < MPS the I line is flatter than the S line.[1] Consider the implications of this fact as revealed in Figure 23. If income is OM, the economy finds itself in a position of equilibrium. Suppose the level of income were OR'. With this level of income saving is $R'V'$ while investment is only $R'T'$, giving excess saving of $T'V'$. In this event more money is being withdrawn from the income stream than is being returned. By our analogy to a pool, the level of income must fall, and income declines from OR' toward OM. Recall that the excess saving can best be used to retire bank loans. In turn, this implies that bank credit

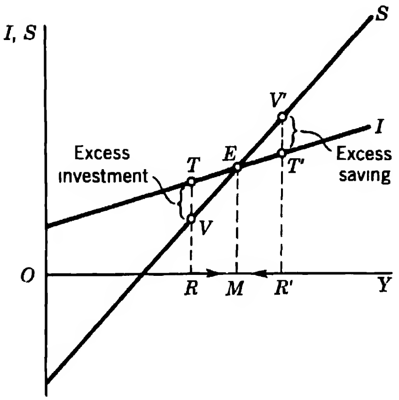
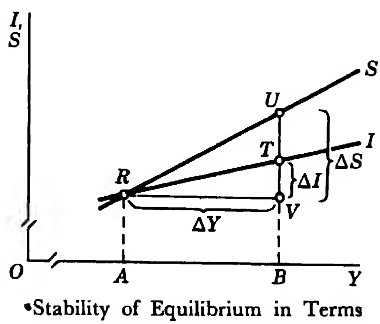


Figure 23. Stable Saving-Investment Equilibrium with Varying Investment.

[1] By definition $MPI = \Delta I/\Delta Y = VT/RV$, while $\text{MPS} = \Delta S/\Delta Y = VU/RV$. Since $VT < VU$, $VT/RV < VU/RV$ or MPI < MPS. As long as point T lies below U, the condition is fulfilled. At the same time this implies that the I line is flatter than the S line.



•Stability of Equilibrium in Terms of I and S Curves.

(or money) is being destroyed, a process definitely associated with declining income.

If income is OR, saving is only RV, while investment is RT, giving excess investment of VT. More money is being pumped into the income stream than is being withdrawn in saving. By our pool analogy the level of water or income must rise. In addition, the excess investment of VT can only be provided from new bank credit (or release of idle balances). Consequently, VT represents an expansion producing an increase in the effective quantity of money.

Such is the direct argument which explains the saving–investment form of the stability condition. Neutral and unstable equilibrium can be explained by similar diagrams with saving and investment curves in different positions.

THE PARADOX OF THRIFT

To illustrate the use of the modified apparatus it will be appropriate to mention a problem noticed by Keynes, and which has been called the "Paradox of Thrift." Since Keynes was fond of putting himself in opposition to conventional views, we will do well to state the classical position on this matter. According to the classical writers, the progress of society was assured by the accumulation of capital. In order to accumulate capital it was first necessary for society to save in order to provide the funds for investment. In the long run the additional capital thus provided would increase income by adding to the productive resources of society. For the short run, with which they were not greatly concerned, the classical writers held the position that the extra saving was turned over to capitalists for investment or invested directly by the savers. In this way the additional drain from the income stream was offset by corresponding investment.

Keynes's argument is concerned only with the short run. His thesis runs as follows: if consumers seek to save a larger amount out of any given level of income in a stable situation, *saving will decline*, along with income. In short, the attempt of consumers to save more will reduce saving. This is the paradox.

Suppose savers attempt to save more at the existing equilibrium

level of income. Clearly, if saving and investment formerly were equal, saving now exceeds investment. In terms of money flows the drain of savings from the income stream exceeds the compensating injections of investment. For this reason the level of income will fall. In a stable situation saving falls faster than investment as income falls. Eventually saving and investment become equal at a lower level of income. Thus an increased propensity to save led to reduced saving.[2]

It is quite easy to see this in diagrammatic form, as in Figure 24. In the original situation equilibrium is found at an income *OB* with a saving of *OA*. When the propensity to save increases, the saving function shifts upward. The new equilibrium

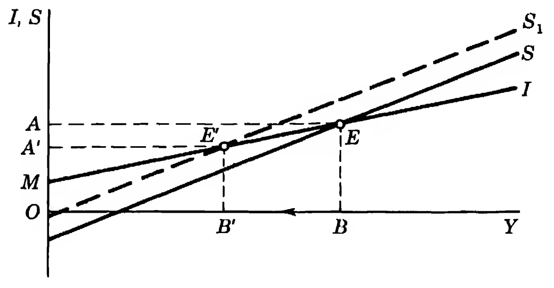


Figure 24. The Paradox of Thrift.

is found at *E'* with reduced income of *OB'* and reduced saving of *OA'*. This result indicates that increased thriftiness reduces income.

If *OB* corresponds to an underemployment equilibrium to begin with, the resulting fall in income will reduce income, output and employment; the same applies if *OB* corresponds precisely to full employment. Obviously, this is an unfortunate result in the circumstances. This possibility led Keynes to remark that saving,

[2] If this does not seem plausible, consider the diagram in the preceding footnote (p. 98). *S* exceeds *I* by *TU* at income *OB*. As income drops by *VR*, saving drops by *VU* while investment drops by *VT*. The fact that saving drops more than investment, ($\Delta S = VU > VT = \Delta I$) permits the excess of saving over investment to be eliminated and a new equilibrium to be established at *R*. In the new equilibrium saving is less than it was in the beginning.

while a private virtue, is a public vice. To elaborate, saving leads to the desirable end of promoting individual security, but at public expense by increasing unemployment.

Multiplier and super multiplier

When investment is strictly autonomous, its magnitude is unaffected by a change in the level of income. In such a case an autonomous or once-and-for-all increase in the level of investment will produce a twofold effect on expenditure. First is the increase represented by the investment expenditure on capital goods by businessmen. Second is the consumption expenditure generated by increased incomes flowing from increased activity in the capital goods market. Such consumption may be described as the *induced consumption* generated by the additional income and ultimately by the increment of investment.

When investment experiences an increase as a consequence of a rise in the level of income, this increase is said to constitute *induced investment.* In this case since a rise in the level of income also occasions induced consumption, there are two categories of induced spending. These may be combined for analytical purposes into a total entitled *induced expenditure.* In conclusion, if investment responds to income changes, an increment of autonomous investment, by raising income, generates induced expenditure on consumption and investment. In the case described, both autonomous and induced investment play significant parts in the analysis.

Under the modified conditions a somewhat different effect on income is produced by an autonomous increase in investment. In place of induced consumption we have induced expenditure of two kinds. Likewise, in place of a simple Marginal Propensity to Consume we have the Marginal Propensity to Expenditure; this is the sum of the Marginal Propensity to Consume and the Marginal Propensity to Invest. If the MPC is .6 and the MPI is .2, the Marginal Propensity to Expenditure, MPX, is .8. In equation form,

$$\text{MPX} = \text{MPC} + \text{MPI}.$$

Omitting the detailed argument, the Marginal Propensity to

Expenditure plays the same role in multiplier theory under the new conditions that the Marginal Propensity to Consume did under the old. Accordingly, the "compound multiplier" formula applicable to a situation with induced investment is written

$$K_1 = \frac{1}{1 - \text{MPX}}.$$

Since MPX = MPC + MPI, we can also write this in the form

$$K_1 = \frac{1}{1 - (\text{MPC} + \text{MPI})} = \frac{1}{1 - \text{MPC} - \text{MPI}}.^{3}$$

INTERPRETATION

The interpretation of the compound multiplier is essentially the same as that of the simple multiplier. By definition, the value of the compound multiplier is the number of dollars that income will rise if an autonomous increase of $1.00 in investment occurs. Phrased in another way, the value is the ratio of an increase in income to the initiating increase in autonomous investment.

[3] *Proof:* In equilibrium, "supply," or Y, must equal "demand," $C + I$, where I is the sum of autonomous investment and induced investment, $I_a + I_i$. Algebraically, $Y = C + I_a + I_i$ is the condition that the market be cleared. This equation must hold for increments caused initially by a change in I_a, autonomous investment. Consequently, after income has been adjusted so that the market is exactly cleared of goods:

(a) $\Delta Y = \Delta C + \Delta I_a + \Delta I_i$, (a′) $\Delta Y - \Delta C - \Delta I_i = \Delta I_a$.

This is the basic information needed, the rest being mere algebraic manipulation.

By definition the compound multiplier is the number of dollars of income generated by one additional dollar of autonomous investment. In algebraic form,

(b) $$K_1 = \frac{\Delta Y}{\Delta I_a}.$$

Substituting from (a′) in the definition we get:

$$K_1 = \frac{\Delta Y}{\Delta I_a} = \frac{\Delta Y}{\Delta Y - \Delta C - \Delta I_i}$$

$$= \frac{1}{1 - \dfrac{\Delta C}{\Delta Y} - \dfrac{\Delta I_i}{\Delta Y}},$$

after dividing numerator and denominator by ΔY. Noting that $\Delta I_i/\Delta Y$ is the MPI and that $\Delta C/\Delta Y + \Delta I_i/\Delta Y = \text{MPX}$, we may write the above result in the forms

$$K_1 = \frac{1}{1 - \text{MPC} - \text{MPI}} = \frac{1}{1 - \text{MPX}}$$

Suppose that MPX = MPC + MPI = .6 + .2 = .8. In this event a \$1.00 increase in the level of income leads to a \$.60 increase in consumption and a \$.20 increase in investment. Clearly, the increased investment is stimulated by rising demand. Under these assumptions the value of the compound multiplier is:

$$K_1 = \frac{1}{1 - .6 - .2} = \frac{1}{1 - .8} = \frac{1}{.2} = 5.$$

When investment increases autonomously by \$1.00, income ultimately rises by \$5.00; of this, \$5.00 × .6 = \$3.00 consists of additional consumption; \$5.00 × .2 = \$1.00 consists of additional induced investment; \$1.00 consists of the original autonomous investment.

SIMPLE AND COMPOUND MULTIPLIER

If the compound multiplier is used, does it follow that the simple multiplier is no longer valid? The answer to this question is necessarily rather subtle. If the conditions appropriate to the compound multiplier hold true, it is definitely the appropriate one to use. The "base" of the multiplier, $\Delta Y/\Delta I$, is the quantity ΔI. If the ratio is to have its full meaning, ΔI should represent an autonomous increase in investment. When induced investment is absent, all investment is autonomous, and ΔI fulfills the required condition.

When part of investment is induced, we *can* calculate the simple multiplier. In the foregoing illustration, for example,

$$K = \frac{\Delta Y}{\Delta I} = \frac{\$5}{\$2} = 2.5.$$

Since the "base" of \$2.00 contains induced investment, the ratio can no longer be interpreted in the old way. Now the figure 2.5 merely represents the ratio of increased income to increased investment; it is *not* the rise in the level of income caused by a \$1.00 autonomous increase of investment. In addition, the figure 2.5 is no longer a ratio between a dependent variable, ΔY, and an independent variable, ΔI; as we are aware, ΔI = \$2.00 contains the \$1.00 of autonomous investment and the dependent \$1.00 of induced investment.

In conclusion, only the use of the compound multiplier affords a sharp distinction between cause and effect when conditions appropriate to its use prevail. In contrast, the simple multiplier remains valid in a formal sense but does not afford the distinction between cause and effect.

MAGNITUDE OF THE MULTIPLIERS

Perhaps the simplest way to make comparisons is to break down the subject into cases. (1) In a case where no induced investment is present all investment is autonomous and the MPI is zero; the compound multiplier reduces to the simple multiplier. In this event a common value and a common meaning can be given to the multipliers. If the MPC is .6 and the MPI is zero, "the" multiplier is 2.5. (2) If induced investment is present, the compound multiplier gives a higher value than the simple multiplier, as the previous analysis indicates. If the MPC is .6 and the MPI is .2, the simple multiplier is 2.5, whereas the compound multiplier is 5. Here the difference in numerical values arises from a difference in the interpretation of the basic information.

More interesting than the formal difference in the multipliers under the second case is the change in the economic situation which shifts the problem from case 1 to case 2. In a depression the economy finds itself with more capacity to produce than is called for by the state of demand. To be more elaborate, a low level of demand calls forth a low level of production which falls short of the capacity of the economy to produce. Under these conditions a temporary condition of "excess capacity" exists. When an increase in the level of demand occurs under these conditions, businessmen are prone to utilize excess capacity to meet demand; induced investment is likely to be in the neighborhood of zero.

If autonomous forces continue to raise the level of demand to such an extent that excess capacity becomes rather small, business men will be prompted to invest in new capital goods. As the level of income rises to the point where production begins to press on capacity, induced investment will commence. At this juncture the business community takes fire and adds to the general expansion. Under these conditions the MPI is positive and the compound multiplier increases in value. In short, the leverage value of

additional autonomous spending increases greatly as the economy approaches capacity.

In the study of economic cycles this fluctuation in the value of the compound multiplier may be given a central role. In fact, the expansion phase of the cycle may be largely accounted for by the explosive effects of induced investment. On the other hand, the depression phase may be accounted for by the doldrums experienced when nothing can induce businessmen to invest under conditions of excess capacity.

OBJECTIONS TO THE COMPOUND MULTIPLIER

Almost all economic data are provided in the form of time series or dated information. Owing to this fact, it is quite difficult to distinguish autonomous from induced investment. Suppose income increases from \$300 billion to \$330 billion in a certain year. At the same time investment rises from \$40 billion to \$50 billion. Is it possible to break down the increment of \$10 billion investment into an autonomous and an induced part? It may be the case that all is autonomous, that the multiplier is 3, and that the consequent increase in income is \$30 billion. On the other hand, \$7.5 billion may be autonomous, \$2.5 billion induced by the increase in income, and the compound multiplier may be the ratio \$30 billion/\$7.5 billion = 4. In fact, it is often difficult to distinguish between cause and effect in time series of economic information. For the same reason it is difficult to set up a model, like the compound multiplier, which reflects the true relation between the (unknown) cause and the effect.

CHAPTER 5

Interest, money, and investment

The liquidity preference theory of interest and money

The liquidity preference theory was developed by a single man, J. M. Keynes, and can be regarded as his invention. According to the theory, the rate of interest is determined by the demand for and the supply of money. The supply of money consists of two elements: the quantity of currency and coin and the quantity of demand deposits. Since demand deposits account for over 90 percent of the volume of transactions carried on in the United States, it is fairly reasonable to discuss the supply of money in terms of bank deposits alone. As books on money and banking explain, banks create bank deposits in the act of making loans or buying securities. For this reason it is quite proper to say that the banking system through its actions in creating loans and deposits determines the quantity of money. Under our banking laws and institutions, the Board of Governors of the Federal Reserve System and the twelve Federal Reserve Banks have a strong influence on the quantity of deposit money created. Of course, they are part of the banking system in the larger sense.

On the other side, the demand for money consists in the desire to hold money for certain purposes. Since money is the most "liquid" of all assets, this desire for money was entitled by Keynes "liquidity preference." In very broad terms, the payment of interest is necessary because society prefers assets which are liquid to those which are not. Money, as the asset which can most readily be used for any desired purpose, comes to command a premium over other assets. The measure of the preference for cash over other assets is the rate of interest. In other words, interest is the price that has to be paid by the borrower to the lender in return for the fact that the lender surrenders "liquidity" when he exchanges cash for any other asset. To be specific, a share of stock or

a bale of cotton cannot be used directly to purchase other goods or assets. First they must be liquidated. To compensate the lender for this absence of liquidity, a rate of interest is paid.

Motives for liquidity

THE PRECAUTIONARY MOTIVE

Every normal individual or business finds it desirable to keep a liquid reserve for unforeseen contingencies. Such contingencies may be of a negative sort such as the possibility of a sudden illness, the loss of employment, or the unanticipated need to discharge some debt. On the other hand, they may be more positive, as in the appearance of a sudden bargain in consumer goods, or the opening up of a new investment opportunity for the individual.

THE TRANSACTIONS MOTIVE

In order to bridge the gap between receipts and disbursements, both the individual and the firm need to keep a margin of cash. If it were possible to synchronize receipts and disbursements precisely, no cash balance would be required. In practice, disbursements will exceed receipts at some times and fall short at others. Thus the salaried man receives income once a month in a lump sum and spends his money throughout the month until the next salary check is received. Whereas income is received discontinuously, expenditure is more or less continuous. To keep the two in balance, the individual must maintain a cash balance at all times.

To illustrate, suppose that the individual receives $400 a month, that he spends at an even rate through the month, and that he always spends his entire income. Part of this spending may be on insurance, securities, or other titles to capital assets. Under these conditions the individual will start the month with $400, end with nothing, and maintain an average balance of $200. It is the $200 that we are thinking of when we refer to the individual's demand for money under the transactions motive. It is the average amount he wants to hold in his cash balance. The inequalities in demand through the month tend to average out with offsetting inequalities of other demanders such as businessmen. As the quantity held by

consumers declines, businesses accumulate balances from sales to consumers. In turn, the balances which they are accumulating will permit them to meet their wage bills at the beginning of the next month. Since the needs of consumers and business firms are complementary, their respective demands tend to form a stable aggregate.

THE FINANCIAL MOTIVE

In the preceding section the notion emerged that business firms exhibit a transactions demand for money. In effect, such a stock of money acts as a buffer between receipts and disbursements; it is the monetary form of circulating capital whose physical counterpart consists in stocks of goods. Corresponding to the firm's need for monetary balances to satisfy circulating capital requirements is a demand for balances to finance the construction of fixed capital facilities. Such a demand was termed by Keynes the demand for "finance."

Basically, there is a close resemblance between the finance and transactions motives. When a firm decides to undertake a fixed investment of considerable size, it will need money. This is not the same thing as saying that it must raise a sum sufficient to pay the entire expense of the construction. In fact, the money referred to is a cash balance out of which the expenses of construction are paid as they accrue. Periodically, sums must be deposited to the account of the firm to maintain balances, but it is the balance required, and not the sums deposited, with which we are concerned.

As the investment undertaken by the firm is completed, its demand for "finance" will cease. As this firm ceases to invest, other firms will be taking on new construction and will absorb the money released by the first firm. In this way the demand for finance tends to be maintained at a steady level which is related to the volume of investment.

To illustrate, suppose a firm wishes to construct a $10,000,000 plant over a ten-month period. It may then enter into a contract with an investment bank which contracts to sell its securities at leisure and meantime to provide $1,000,000 monthly for the next ten months. For simplicity, assume that the firm spends the money at an even rate for supplies, labor, and equipment, and that it

precisely exhausts the monthly payment at the end of the month. Since the firm begins each month with \$1,000,000 and ends with nothing, the average balance held by the firm for purposes of finance is \$500,000.

THE SPECULATIVE MOTIVE

Among the commonly recognized functions of money is its use as a store of value. In this respect money is but one of a number of instruments used by man as a cushion against variations in his income flow. An alternative to storing value in the form of money is the acquisition of capital goods. Since capital goods employed in productive uses can yield a return sufficient to provide for replacement, such capital is physically maintainable. For this reason, the acquisition of capital goods can be used as a means of fixing income in a permanent physical form. In recent times, individuals have generally acquired capital by purchasing securities. Such securities constitute a claim against the capital of a firm. In virtue of the physical capital it represents and the yield it affords, each security bears a price. In time of need the security may be sold and its value converted into money.

Although securities serve as a store of value, money is preferred for this purpose. In explanation, variations in security prices lead to alterations in values stored by such means. Thus a share of stock whose price is \$100 this week may fall in value to \$80 or rise to \$120 the following week. Since the price of a security fluctuates through time, the holder cannot be certain of the available value represented by this asset. On the other hand, if an individual holds \$100 in the bank, he is certain of his power to command this monetary sum. Since goods are acquired in our society by disbursing money, it is necessary to hold some stock of money to meet the need for assured command over goods.

In many cases, an individual acquires things which he intends to pay for later. During the credit period, he must prepare to make payment by accumulating cash or some asset convertible into cash. Since securities fluctuate in monetary value, holding them with a view to liquidation as the due date approaches is not entirely satisfactory. Again, securities cannot serve as a completely satisfactory substitute for money. In seeking a means to store value

with a view to discharging debts, an individual usually selects money with its stable value.

To persuade an individual to part with money, the seller of securities must offer a premium in the form of interest. The premium required by the purchaser before surrendering liquidity may be entitled the *rate of liquidity preference*. Suppose an individual is indifferent between \$100 in cash and \$104 in securities, including interest; his (marginal) rate of liquidity preference is then 4 percent. If the rate of interest is 6 percent, the premium paid is more than sufficient to induce him to part with liquidity. In this event he may be willing to part with a second \$100 of cash for a bit more, say \$105. Again, the interest rate of 6 percent more than compensates for the (marginal) liquidity preference rate of 5 percent. If the third \$100 given up is equivalent to \$106, his (marginal) rate of liquidity preference is 6 percent, a value just equal to the rate of interest. At this point, the rate of interest just compensates for the loss of liquidity preference, and the individual will find it advantageous to lend no more than \$300.

Consider now the following question. On what does a person base his preference for cash over securities? First he must form an estimate of the expected future price of the security. Suppose the price could take on the values \$90 or \$135, with probabilities $\frac{2}{3}$ and $\frac{1}{3}$, respectively. By taking the arithmetic mean of these, he finds that his "expectation" is that the price will be: $E = \$90 \times \frac{2}{3} + \$135 \times \frac{1}{3} = \$105$. However, an "expectation" of a price of \$105 is not the same as the certainty of \$105 which would be signified by the single expectation of \$105 with a probability of 1. When he is certain of \$105, the individual entertains no belief except the single thought that the price will be \$105. Only when he holds \$105 of money can he reasonably hold this belief.

As we have already stated, it is plausible to assume that the certainty of \$105 in cash is preferable to the mere "expectation" of \$105 which is the weighted average of a pair of possible values. In order to make explicit the distinction between expected values and cash, we may speak of the "certainty equivalent" of an expected asset value. Since the certainty of \$105 in cash is preferable to an expectation of the same amount, a cash sum smaller than \$105 will be equivalent to the "expectation" of \$105. Suppose a person is

indifferent between the "expectation" of \$105, as defined above, and \$100 in cash (\$100 with a probability of 1). In this event a premium of \$5 or 5 percent measures the preference for cash or the aversion to bearing risk. In short, this rate measures liquidity preference.

In the Keynesian theory, the preference for money over securities is taken as an axiom. Since it is inherently plausible, we may accept it as a self-evident proposition and carry on from this point. For the sake of those who want a further account of the matter, an optional section at the end of the chapter develops some additional implications.

GRAPHICAL REPRESENTATION OF LIQUIDITY PREFERENCE

As we have explained, the demand for money consists of four elements: (1) the precautionary motive; (2) the transactions motive; (3) the finance motive; (4) the speculative motive. The first two motives are similar in that they are related to income rather than to the rate of interest. Therefore, they can be combined into a single element called L_{p+t}, where L stands for the liquidity preference demand for cash, p stands for the precautionary motive, and t stands for the transactions motive. The finance motive, depending as it does on the demand for investment funds, is related both to interest and to income. This demand is referred to as L_f, where the subscript stands for finance. Finally, the speculative motive, referred to as L_s, is related principally to the rate of interest.

In Figure 25 these demands are represented by the several L curves. The precautionary and transactions motives are represented by the vertical line L_{p+t}. Next, the combined demands from precautionary and transactions motives, together with the finance motive, are represented in the $L_{p+t} + L_f$ curve. Since this curve represents the sum of two sets of demands, the finance demand is the difference between the total exhibited by this curve and the precautionary-transactions demand, L_{p+t}. Finally, the speculative demand is added to the others to give the total demand curve for money, $L = L_{p+t} + L_f + L_s$. The speculative demand is shown as the difference between the L curve and the $L_{p+t} + L_f$ curve. To illustrate, the precautionary and transactions demands

equal AT at interest rate OA; the finance demand is TU, while the speculative demand is UV; the sum of these is AV. In drawing up this diagram we assume that income is constant; its influence on the demand for money is not explicitly illustrated.

As the rate of interest drops, the L curve flattens out—a result attributable to the operation of the speculative motive. When this rate of interest falls to a low figure, institutional considerations, to

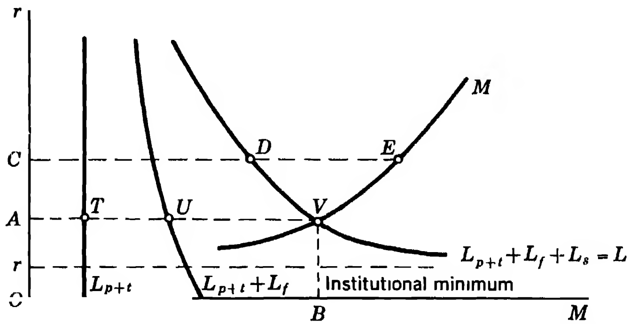


Figure 25. Liquidity Preference Theory of Interest Rate Determination.

be discussed shortly, preclude a further fall in the rate of interest. Accordingly, the lower the interest rate falls, the less is the probability of a further fall and the greater is the probability of a rise which will bring down the prices of securities. Consequently, the demand for money increases greatly as the minimum rate determined by institutional considerations is approached. In the figure, this minimum is represented as a horizontal dashed line and is appropriately labeled.

Supply and its graphical representation

Since 90 percent of all monetary transactions in the United States are performed by checks drawn against bank deposits, it seems appropriate to analyze the supply of money in terms of bank deposits. In turn, the quantity of bank deposits created is determined by the institutional conditions which govern banking. On

the one hand we must consider the nature of an individual bank, confronted with the necessity of maintaining its ability to meet the needs of depositors and at the same time earning a return on its investment. On the other hand, we must consider the Federal Reserve System whose principal function is to control the quantity of money in the public interest.

In order to emphasize the private sector of the economy, the conditions of supply are based on characteristics attributed to individual banks. To be specific, let us assume that banks must earn a return on loans sufficient to meet the attendant variable expenses. In addition, assume that such variable expenses change in proportion to the volume of loans. In this event, a horizontal marginal cost curve for servicing loans will exist. Suppose this cost is 1 percent of the value of the loan. Such a cost will constitute a minimum expense which the bank must cover to make the loan worth while. Such a minimum cost may be referred to as "the institutional minimum" to the rate of interest, since banks will not lend money at lower rates.

In addition to covering variable cost, the bank must maintain an adequate reserve position. Here we encounter once again the sort of considerations which underlie the speculative motive. It seems reasonable to attribute to the banks a preference for cash over the uncertain value of less liquid assets. If the bank is to make the loan, it should cover the variable cost of the loan plus the preference for cash over less liquid assets. In consequence, the supply price asked by the bank consists of a sum of these two elements, marginal cost of lending and liquidity preference.

As the rate of interest rises, the bank can afford to extend more loans, offsetting increased liquidity preference with the higher interest rate. Ultimately, the point may be reached at which loans and deposits are expanded to the legal limit. If the reserve ratio set by the Board of Governors is 20 percent, the quantity of loans and deposits cannot be expanded to more than five times the available cash reserves. In short, given the reserve ratio, banks are subject to an upper limit on deposits. No matter how high the interest rate, deposits cannot exceed this limit.

If the Board of Governors wishes to modify the credit available, it may do so in a variety of ways. However, a discussion of this

matter is beyond the scope of the present text. As far as we are concerned, the common effect of such actions is to shift the supply curve of money. Since the actions of the Board arise from the general business context, the resulting changes in the money supply are not directly related to the interest rate.

Summing up the supply side we find: (1) a minimum rate of interest set by the marginal cost of loans; (2) a maximum quantity of deposits set by reserve requirements; (3) a supply price which is the sum of the marginal cost of lending and a liquidity preference factor. The supply curve of money shown in Figure 25, meets these specifications.

INTEREST RATE DETERMINATION

Like other prices, the rate of interest is determined at the point where the supply (of money) is equal to the demand. Equilibrium is found in Figure 25 at point *V* where the *M* curve intersects the *L* curve. At a rate of interest *OA*, the quantities supplied and demanded take on the common value *AV*. Consequently, the distance *OA* measures the equilibrium rate of interest which clears the money market.

If the interest rate were higher, the quantity supplied would exceed the quantity demanded. If the rate were *OC*, the quantity of money banks are willing to create would be *CE*, and the quantity the public wishes to hold would be *CD*. As this happens an excess of quantity supplied over quantity demanded equal to *DE* develops. In this situation, some banks are willing to offer more money than the public wishes to hold or use. To tempt a greater use of money, banks will cut the interest rate. Eventually, the rate falls until the quantity which people wish to hold is just equal to the amount which the banks are willing to offer.

If the rate of interest falls below the equilibrium rate, *OA*, the situation is reversed. We leave the working out of this case as an exercise to the reader.

A modification of the theory

The theory as presented above is substantially the original Keynesian one with a few interpretations on the supply side. In

certain respects the theory presented is incomplete or inadequate. From the viewpoint of practical application the distinctions between "finance," "the precautionary motive," and the "transactions motive" are somewhat artificial. (The speculative motive does appear to single out a distinguishable aspect of the demand for money.) What we need, at this juncture, is a working hypothesis concerning the relation between the demand for money and its principal determinants. This should replace the somewhat vague discussion in terms of motives.

With the discussion of motives as a background, we may set forth the following hypothesis describing the demand for money.

The demand for money is functionally dependent on: the level of income at factor cost, its distribution among recipients and among originating sources; the level of expenditure, its distribution into categories such as investment, consumption, and government spending; the rate of interest.

Obviously, this assumption makes the demand for money depend on structural factors. It also allows for a disequilibrium between income and expenditure. As a first approximation it is convenient and no doubt fairly realistic to make the assumptions given and discussed below.

INCOME DISTRIBUTION ASSUMPTION

The demand for money is independent of the following factors: (1) a change in the distribution of factor income among recipients; (2) a change in the distribution of factor income among originating sources.

The first part of the assumption implies that a redistribution of income among people will not significantly affect the aggregate of balances held. The second part implies that a shift in the distribution of production among firms or other originating sources will not affect the demand for balances. In the absence of this assumption it would be necessary to analyze the structure of income as well as its level. While it is quite obvious that structural changes may alter the demand for money, the hypothesis is essential to any application of the theory in terms of aggregates.

EXPENDITURE DISTRIBUTION ASSUMPTION

The demand for money is independent of the distribution of expenditure between investment, consumption, and government spending.

To see the weight of this assumption consider the following possible situation. Suppose investment increases, causing a greater demand for money ("finance"), while government spending contracts to the same extent. By the above hypothesis some government balances would no longer be needed and their use would be taken up by businessmen. Again, this hypothesis avoids the difficulties attendant on the analysis of structure.

Finally, in most of the problems analyzed income and expenditure are assumed to be equal. Consequently, we can generally invoke a third condition.

EQUILIBRIUM-IN-THE-GOODS-MARKET ASSUMPTION

Income (at factor cost) equals expenditure.

If the three assumptions are combined with our general statement about the demand for money we reach a final simplified statement.

HYPOTHESIS CONCERNING THE DEMAND FOR MONEY

The demand for money depends on the level of income (expenditure) and the rate of interest.

By implication the demand for money is assumed to be independent (to a first approximation) of the distribution of income (expenditure) into categories. With this assumption we can vary the size of categories, so long as totals remain unchanged, without changing the demand for money. On the surface, the analysis under the revised approach is the same. However, the application of the apparatus is modified considerably, the new system having far simpler and more elegant properties. In the further application of interest theory we will adhere strictly to the last hypothesis. The discussion of the Keynesian motives for liquidity can be taken as leading up to this final hypothesis.

SUMMARY OF INTEREST RATE DETERMINATION

According to the liquidity preference theory, the rate of interest is determined by the demand for and the supply of money. On the one hand, individuals and firms are motivated to hold bank balances by the desire for liquidity. By this means, they enjoy a

measure of security not enjoyed by those who hold income earning assets alone. On the other hand, the supply of money offered by the banks is governed by the necessity of compensating for two "sacrifices": the cost of lending and the reluctance of the bank to give up the necessary cash reserve. When the rate of interest moves to that point at which the quantity of money supplied is equal to the quantity demanded, equilibrium is achieved. Such an equilibrium determines the interest rate and the quantity of money created by the banking system.

Interest and investment

In our treatment of investment in Chapter 4, we stressed the fact that the purchase of capital goods depends on the prospective returns obtainable from their use. Let us consider now the nature of these propective returns and how they may be weighed against the cost of obtaining them. The viewpoint adopted is both macroscopic and short run. Since the principal assumptions and results relate to the economy as a whole, the macroscopic approach is necessary. Also the treatment is short run, and therefore does not deal with the consequences of capital accumulation over a succession of time periods. Instead, it is designed to cast light on the effects of variations in the rate of capital accumulation (investment) in a short time period.

When investing in capital the firm is faced with the fact that whereas costs are incurred immediately, returns are deferred. In fact, returns may be expected to flow in over the life of the capital in question. In some cases the return is not long delayed. For example, money invested in stock by a grocery store can be expected to flow back within a few weeks. Capital which affords a return within a period as short as a year is known as *circulating capital*. On the other hand, money invested in a canal, dam, toll road or toll bridge may flow in over a fifty-year period. When the returns are deferred as long as ten years the investment may be entitled *fixed capital*. Such items as trucks, the stock of a jewelry store, automobile tires, and the like, with returns deferred from 1 to 10 years may be entitled *semicirculating capital*.

MARGINAL EFFICIENCY OF CAPITAL

Before acquiring a piece of capital the firm must compare the cost of borrowing with the return expected. In Keynesian analysis the cost of borrowing is taken to be the rate of interest, whereas the annual percentage return on capital is known as *the marginal efficiency of capital.* Generally, it is necessary to use compound interest concepts and tables to find the value of the marginal efficiency of capital. To illustrate the idea it will suffice at this point to give a particular example.

Suppose a man owns an empty garage. He finds that he can buy a suitable keg and fill it with grape juice for \$100. After being stored in the garage for a year the grape juice matures into wine. Suppose the sale value of the keg of wine is \$120; the growth in the value of the invested capital of \$100 is \$20, and the percentage return is \$20/\$100 or 20 percent. The last value is the percentage return on the invested capital of \$100 and is, for this case, "the marginal efficiency of capital." Subsequently, we will express the quoted phrase by abbreviating it to MEC, when this seems appropriate.

When the marginal efficiency of capital exceeds the rate of interest, it will pay a firm to borrow money in order to acquire capital. For example, if the rate of interest is 5 percent in the foregoing illustration, it will pay the individual to borrow at 5 percent in order to invest at 20 percent. At this point we are justified in making the following statement:

If the marginal efficiency of capital exceeds the rate of interest, it will pay a firm to borrow and invest.

CAPITAL STOCK AND THE SHORT-RUN ASSUMPTION

If the student is familiar with the method of marginal productivity, he will be tempted to extend the preceding statement. Taking the stock of capital to be a variable, the student may well argue that it will pay firms to increase the stock of machines until the marginal efficiency of capital and the rate of interest are equal. However, such an argument is quite inconsistent with the handling of time periods convenient and pertinent for our purpose. In fact, the distinctive character of this analysis arises from its concern with

the "short run." From elementary economics we recall that the short run is a period of time long enough to permit production to be adjusted to demand, but so short that the stock of capital is virtually constant. In accord with this approach we assume that the period of time under consideration is too short to permit the stock of capital to change significantly.

To acquire the feel of the situation, let us examine the meaning of the assumption that the capital stock is constant. First, consider an apparent paradox afforded by the short-run assumption. Our goal in this section is to uncover those forces which determine the level of investment. In turn, investment is the quantity of new capital which businessmen wish to acquire in a given period of time. If businessmen are adding to the stock of capital (investing), how can we assume that the stock of capital is constant?

To facilitate discussion, let us assume that time is divided into a succession of intervals entitled "time periods." In economics, it is customary to assign the length of the time intervals with a view to facilitating the analysis. When such a limitation is imposed the analysis loses breadth of application even as it gains some desired quality of convenience.

Let us now define "investment" as the increase in capital stock noted during a single period. When investment is positive, the capital stock experiences an increase during the period in question. By exercising the power to define the time period, we may reduce its duration as much as we desire. If the time period is contracted to a very short interval, no significant change in the capital stock will be registered in a single period. If the time period is chosen to be infinitesimally short, the change in the stock of capital is effectively zero. In brief, the choice of a short time interval as our period renders the capital stock a constant.

By contracting the time period, we reduce capital stock to a constant. Perhaps the following hydraulic analogy will clarify the assertion just made. We may liken the stock of capital in the economic system to the level of water in a swimming pool; we may liken investment to the volume of water flowing into the pool in a unit time interval from a hydrant or spout. Suppose that the pool when half full contains 240,000 gallons and that the inflow is 4 gallons per minute. In one minute the volume of water added

would amount to only 1 part in 60,000. If the object of the inflow is to fill the pool, a minute's contribution can be neglected. Even an hour's inflow of 240 gallons would constitute but one part in 1000 and raise the water level a corresponding, negligible amount.

Turning again to the economic system, let us suppose the capital stock of the United States to be 1 trillion dollars, while the quarterly accumulation of capital is 7 billion. In such a case the quarterly increase in the capital stock would amount to 7 parts in 1000 or $\frac{7}{10}$ of 1 percent. Roughly speaking, these figures express the current relations between capital stock and investment. In view of the small percentage change noted, it is not unreasonable to say that the stock of capital is approximately constant for a time period equal to a quarter of a year. From a practical viewpoint, the "short-run" period chosen might be equal to or longer than a quarter.

Let us return to our analogy of the pool. If the rate of flow of water through the spigot varies moderately, what is the effect of the change on the water level? In a short period of time the effect is negligible. To illustrate, trebling the flow of water from 4 to 12 gallons per minute would imply that the flow had increased from 1 part in 60,000 per minute to 3 parts in 60,000 per minute. Neither rate of flow will affect the water level significantly in a minute's time. In short, the volume of water is substantially independent of the rate of inflow in a brief space of time. Arguing in the same way, we may conclude that the capital stock is independent of the rate of investment in the short run. For this reason, the marginal efficiency of capital cannot be regarded as a function of variations in the stock of capital in the short run. In fact, such a relation can only be established with reference to a longer period of time.

CAPITAL MARKET IN RELATION TO GOODS MARKET AND MONEY MARKET

The present analysis is designed to uncover the forces determining investment. To carry out this design with simplicity, it is necessary to reduce the number of variables under consideration. By an appropriate contraction of the time period, we render the stock of capital effectively constant. At the same time the rate of investment remains variable. In order to bring the other variables into perspective, let us briefly note the areas of analysis already

considered. On the one hand, the market for all output operates to equate the total demand for goods ($C + I$) with the total supply (Y, income at factor cost) by variation of National Income, Y. On the other, the quantity of money which the banks are willing to create is matched with the quantity which the public wishes to hold by a proper adjustment of the rate of interest. Should we treat income and interest as variables or constants in the discussion of investment determination?

In price theory a technique is used which is pertinent in this connection. When discussing the determination of the price of a particular good, it is customary to abstract from other markets. What this really means is that the prices in other markets are assumed to remain constant during the working out of the equilibrium for the market in question. Actually, economists recognize this constancy to be a mere abstraction, used because it greatly simplifies the analysis. To summarize, the method of "partial equilibrium" just described proceeds on the assumption that the critical variable in every other market remains constant.

Applied to the present problem, this technique requires us to regard as constant the level of income in the market for goods and the rate of interest in the money market. By assuming these variables to be provisionally constant we can concentrate on the simpler and more obvious features of short-run investment determination. In a later optional section of this chapter both these provisional assumptions are dropped. In the chapter on cycles (Chapter 10), a model is given which makes capital stock variable also. Meanwhile, we may enjoy the advantages of an elementary analysis which explores fundamentals.

SUMMARY OF ASSUMPTIONS

Since most of the analysis in the text is short run in character, it is appropriate to extend this assumption to problems of investment. Such an assumption implies that the capital stock is constant, while investment is variable. For convenience of analysis, the determining factors in other markets, namely, income and interest, are assumed to remain constant. Our problem is to find the level of investment which will occur with given values of income and interest and a constant capital stock in the short run.

Investment determination in the short run

Assume that the marginal efficiency of capital exceeds the rate of interest. In this situation, it will pay businessmen to borrow at banks and invest in new capital. Since the period of time under consideration is short, the capital stock does not vary significantly. In the limited time period under consideration, businessmen have no reason to fear an immediate saturation of investment opportunities. Accordingly, an excess of the MEC over the rate of interest prompts them to speed up the pace of investment to cash in on the profit-making opportunity.

With an increased demand for capital goods comes a higher rate of production and a higher supply price. When the price of capital goods rises, the MEC falls. If the expected future returns are unchanged, a lower rate of discount is required to equate the sum of the discounted returns with the higher cost. This lower rate of discount is the reduced MEC.

If the MEC exceeds the rate of interest even by a small margin, the pace of investment will be increased to reap the profit opportunities which exist. Since this increases the price of capital goods and lowers the MEC, the reaction carries the MEC toward equality with the rate of interest. When the rate of investment is one which increases the supply price of capital to afford an MEC equal to the rate of interest, equilibrium obtains.

DIAGRAMMATIC TREATMENT

Without filling in details we know that the greater the rate of investment, the higher is the supply price of capital. In turn, the higher the supply price of capital, the lower is the MEC. In brief, the MEC varies inversely with the rate of investment. By finding the MEC at every rate of investment we secure the schedule of the MEC. By plotting such a schedule as in Figure 26(B) we obtain the curve of the MEC.

In the left-hand portion of the figure is shown the determination of the interest rate by supply and demand in the money market. Extending the interest rate found over into the right-hand diagram by a horizontal line, we find a point of equality between the interest rate and the MEC at *E*. Dropping a vertical dashed line

to the horizontal axis, we find investment to be *OA*. Thus investment is determined by the condition of equality between the rate of interest and the MEC.

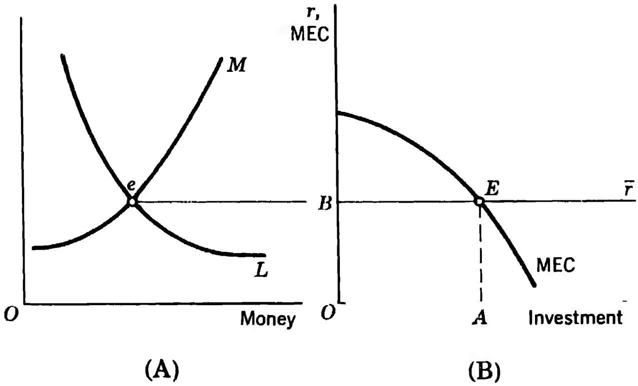


Figure 26. The Interest Rate and Investment Determination.

THE MEC CURVE AND THE INVESTMENT DEMAND CURVE

In price theory, the demand schedule for a good is defined as:

The schedule of the quantities of a good which buyers will seek to purchase at a corresponding schedule of prices.

For example, the demand schedule for oranges in a well-defined market consists of the number of dozens of oranges buyers wish to purchase at a set of corresponding prices per dozen. In demand schedules of this kind, the quantity purchased consists of a number of units of a commodity or service, such as dozens of oranges. Corresponding to such a quantity of the commodity is the price which gives rise to a demand for this amount.

In problems of investment demand, the question of paramount concern is the relation between the rate of interest and the dollar value of the investment business men wish to undertake. On the one hand, the rate of interest plays the role of price in this analysis; on the other, the dollar value of investment plays the role of quantity demanded. Both "price" and "quantity" have unusual dimensions in the theory of investment demand, "price" being

expressed as a percentage instead of as dollars per unit of output, "quantity" being a total dollar value instead of the number of units of a capital good. Having these distinctions in mind, we may state the definition of the investment demand schedule.

The investment demand schedule is the schedule of the dollar values of new capital demanded (investment) at a corresponding schedule of rates of interest.

With this definition in hand, let us see whether we can build up or identify this schedule. Our procedure will be geometrical.

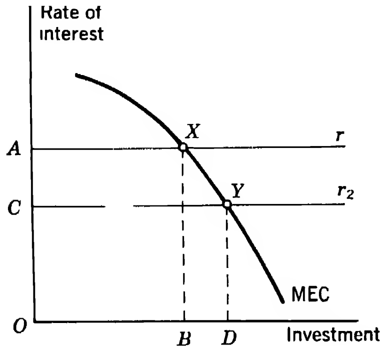


Figure 27. Investment Demand Schedule and the MEC Curve.

In Figure 27 draw in the MEC curve. Assume that the rate of interest is OA with a constant value shown by the line r. Given this rate of interest, businessmen will push investment to the point of intersection of the MEC curve with the rate of interest line r, at X. Dropping a vertical dashed line to the investment axis, we find the investment level to be OB. From this we conclude that a rate of interest OA gives rise to investment OB. Suppose now that conditions in the money market lead to a lower rate of interest OC, represented by interest line r_2. In the new situation businessmen will step up investment until the MEC comes into equality with r_2 at Y. Again dropping a vertical dashed line to the horizontal axis, we find investment to be OD.

At this juncture we have found the following information which may be summarized in schedule form.

TABLE 25. Investment Demand Schedule, or MEC Schedule

Interest Rate	Investment (Dollar Value)	Plot of Interest Rate Against Investment
OA	OB	X
OC	OD	Y

Note that under the interest rate and investment columns we have two representative pairs of values in the desired investment demand schedule. By its nature, any demand schedule plots into a point lying on a corresponding demand curve; but each point on the demand schedule which we have derived lies on the MEC curve. From this fact we may conclude that the investment demand curve *is* the MEC curve. This is the desired conclusion.

With the knowledge that the MEC curve is the investment demand schedule, we can make several obvious points. Determination of investment can be regarded merely as a matter of picking the right interest rate and choosing the corresponding investment level from the MEC (investment demand) curve. When the MEC curve is regarded as an investment demand curve, the interest rate is taken to be the independent variable and investment the dependent variable. This agrees with the usual treatment of price as independent variable and quantity as dependent variable in consumer demand problems.

At this juncture the essential features of investment determination have been treated. However, the description given verbally may be filled in and given added precision with the aid of a somewhat more elaborate diagrammatic treatment. For those who wish added detail and precision we have included the following optional section.

Diagrammatic treatment of investment demand—(optional)

As the preceding analysis indicates, the MEC curve is the essential piece of apparatus in the problem of investment determination. In the last section we showed that the MEC curve is the investment demand schedule. Let us now derive somewhat more rigorously the investment demand schedule, thereby finding the MEC schedule. As we have just pointed out, the interest rate is regarded as the cause and investment demand the effect. Recognition of this status for the interest rate is essential in the argument that follows.

To begin with, we start with an arbitrary value of the rate of interest, r, given from the money market. Next, we must be given

the set of returns on the capital good, expected in the several years of its life, to wit, a_1 in the first year, a_2 in the second, . . . , a_n in the n^{th}. These returns are net of the supplementary cost of using the machine, but are not net of depreciation allowance. If r is the rate of interest, the value of the piece of capital is equal to the sum of the discounted future returns, and is given by the formula:

$$V = \frac{a_1}{(1+r)} + \frac{a_2}{(1+r)^2} + \cdots + \frac{a_n}{(1+r)^n}.$$

We could develop our argument in terms of this formula, but a special case lends itself better to numerical illustration. Assume that the machine has perpetual life and yields a constant return equal in amount to a. In this special case the value of the capital is expressed by the simpler formula:

$$V = \frac{a}{r},$$

where a is the constant annual return and r is the rate of interest.

In the discussion to follow the expected annual return, a, remains constant, because its value depends on expectations and income, both of which are assumed to remain constant throughout the argument. Note that no deduction is being made on account of depreciation to arrive at the return a so that this return does not depend on the cost of the machine. The rate of interest may vary, however. As a result of changes in the rate of interest, r, with constant a, the value of the capital good, V, undergoes corresponding changes. Suppose a in our example happens to be \$40. Then a table of values of the capital good may be constructed.

TABLE 26. Schedule of Value of Capital at Varying Interest Rates

a	r	V
\$40	1%	\$4000
40	2	2000
40	4	1000
40	6	667
40	8	500

The sense of the calculation may be recalled easily. Suppose a machine yields a $40 annual return, the rate of interest being 2 percent. The maximum price the buyer will pay for the machine is one which yields the going rate of interest. If the price of the machine is just $2000, the annual return of $40 affords a yield of exactly 2 percent. At this price the asset yields exactly the going rate. If the price of the asset were below $2000, a return of $40 would afford a yield in excess of 2 percent, and this asset would come into demand. Eventually, bidding would raise the price to $2000. If the price rises above $2000, a reverse process would occur for opposite reasons.

In Figure 28(A), *V* is plotted as a function of *r* from the second and third columns of Table 26. Given any value of *r*, the value of

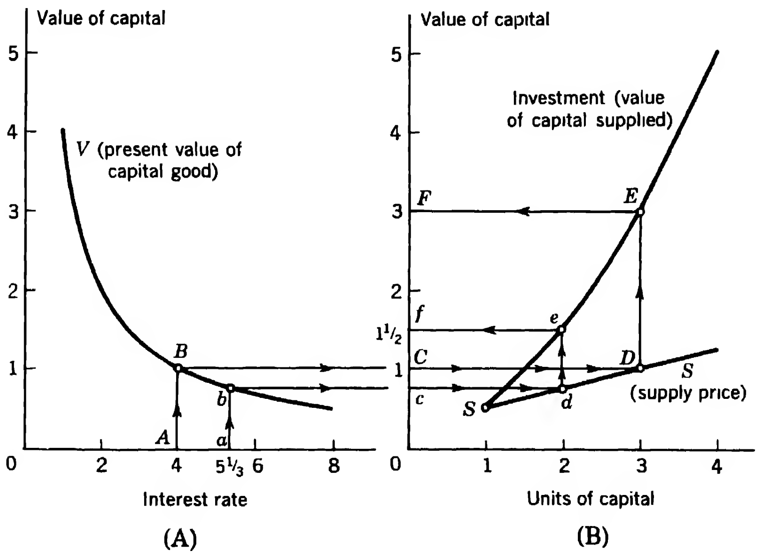


Figure 28. Alternative Derivation of Investment Demand Schedule.

V may be determined from the *V* curve. If the rate of interest is given as 4 percent, we start at 4 percent or *A* on the interest scale of Figure 28(A). Drawing a vertical line from *A* or 4 percent up to the *V* curve at *B* and over to the vertical axis of the right hand diagram at *C*, we find the numerical value of the capital good. The

right-hand diagram has the same vertical scale as the left-hand diagram, measuring the price of the machine in thousands of dollars. At point C, the price of the capital good is seen to be \$1000.

In the right-hand diagram the conditions of supply in the capital goods market are displayed. For simplicity we assume that a single

TABLE 27. Supply Schedule of Capital and Total Investment

(1) Units of Capital Offered	(2) Supply Price (In Thousands)	(3) = (1) × (2) Investment (In Thousands)
1	\$.50	\$.50
2	.75	1.50
3	1.00	3.00
4	1.25	5.00

type of capital good is being produced and measure its production in physical units along the horizontal axis. On the vertical axis we may measure two quantities, the price of the capital good and the total cost of a given number of units of the capital good. In fact, the vertical axis is simply scaled off in thousands of dollars.

From a knowledge of cost conditions for the capital good, we may find the supply curve SS, displaying the number of units of the capital good producers are willing to offer at a given interest rate and capitalized value of the good V. Such a curve represents investment in money value at each corresponding value of V or quantity of capital, and is labeled the Investment curve. Consider its derivation.

In Table 27, the first two columns give the units of capital produced and the supply price, respectively. Together they constitute the supply schedule. From this data we would like to determine investment supplied at each price of the capital good. To secure this, we simply multiply the units of capital offered by the supply price to secure investment or total cost of capital goods supplied, as listed in column three. Column three plotted against column one gives the Investment curve in Figure 28(B). With this information we can proceed to find the investment demand schedule.

What we wish to do is determine investment at a given rate of interest. By recording the level of investment at a given interest rate we make up part of our investment demand schedule. Let us start with the interest rate of 4 percent at *A* on the horizontal axis in Figure 28(A). Following the arrow upward from *A*, we arrive at *B* on the *V* curve. Moving horizontally from *B* to *C*, we find the discounted value of the machine to be *OC* or \$1000 by reading the vertical scale in the right-hand diagram. In effect, the horizontal line *BCD* which represents the value of the machine as \$1000 is a perfectly elastic demand curve for capital. It is perfectly elastic because the time period is too short to work out investment opportunities by capital accumulation.

At point *D* where the value line *BCD* intersects the supply curve of capital goods, the value of the capital equals its supply price, and equilibrium prevails. In alternative phraseology, the present value of capital, *V*, is equal to the supply price found on the *SS* curve. Extending a vertical line upward from the supply curve at *D*, we reach the investment curve at *E*. Extending a line horizontally to the left toward the price–cost axis, we arrive at *F* and note that the value of investment with an interest rate of 4 percent is \$3000. Starting again from an interest rate of $5\frac{1}{3}$ percent on the horizontal scale in the left-hand diagram, we follow a similar route, *abcdef*, indicated by the arrows. At *f* the value of investment corresponding to a $5\frac{1}{3}$ percent interest rate is found to be \$1500. This information may be summarized in a table.

TABLE 28. Investment Demand Schedule or MEC Schedule

Rate of Interest	Investment (In Thousands)
4%	\$3
$5\frac{1}{3}$	$1\frac{1}{2}$

Such a table represents the investment demand schedule, or the beginning of one. Since the investment demand schedule with interest as the independent variable coincides with the MEC schedule, the former can also be taken as the MEC schedule. If we adopt this position, however, we must label the first column MEC

In fact, the MEC would be regarded as the dependent variable which depends functionally on investment. As a matter of strict logic, it leads to more consistent reasoning to employ the labeling and handling of independent and dependent variables displayed in the table and preceding argument. In any event, the MEC or investment demand schedule has been derived from more underlying data by the method indicated. From a practical viewpoint, considering the elementary design of the text, this explanation *exhausts the fundamentals* of a complex subject.

Investment during inflation

Up to this point we have assumed that no systematic change in the price level takes place. During inflation, however, the prices of goods and services tend to rise. At the same time the rate of interest may prove to be "sticky" or even constant. Such a situation leads to a noteworthy condition in the capital market.

REAL MARGINAL EFFICIENCY OF CAPITAL

In a period of inflation, the price level is rising. The simplest case occurs when the prices of all goods and services rise in the same proportion, with certain exceptions to be noted. Suppose this rate of increase to be 2 percent per time period. To find the value of an article at the end of the period, it is necessary to "deflate" its value with the aid of a price index. In this case, the price index for all goods is 102 at the end of the period. To find the price of an article in dollars of the same purchasing power as prevailed at the beginning of the period, we divide the price at the end of the period by the price index of 102. To be more precise, we divide by 1.02.

To illustrate the problem, suppose a firm sets aside a quantity of green wood bought for $100 and allows it to season for a year. At the end of the year, the wood sells for $108.12. Nominally, the MEC in this use is 8.12 percent, representing the growth in value of $8.12 expressed as a percentage of $100. Let us assume, however, that immediately after purchase the green wood and all other commodities rise 2 percent in value and remain at this level throughout the period. To find the real MEC, we must make an adjustment for the change in the price level.

Following the procedure outlined, we divide the terminal sum by the price index (in proportion form) to get \$108.12/1.02 = \$106. The deflated sum of \$106 represents the value of the seasoned wood expressed in dollars of the same purchasing power that prevailed at the beginning of the period. Comparing this with the sum invested, we find that the *real* MEC is 6 percent. Let us define the *real* MEC as that rate of return earned on capital over and above the monetary sum which preserves the purchasing power of the sum invested.

REAL RATE OF INTEREST

Suppose the sum of \$100 invested in green wood was borrowed from a bank at 6 percent. If the price level rises, the purchasing power of the money repaid will decline. At the end of the period, the borrower repays \$106, but the purchasing power of this sum is reduced. Deflating the sum repaid by the price index, we find the sum in dollars of constant purchasing power to be \$106/1.02 = \$103.92. In real terms the interest earned is 3.92 percent (roughly, 4 percent). We shall call the percentage rate paid on borrowed money the *nominal rate of interest.* In contrast, let us refer to the percentage return earned after making an adjustment to maintain the purchasing power of the sum lent as *the real rate of interest.*

EFFECT OF PRICE CHANGES ON INVESTMENT

When the price level is constant, the economy moves toward that level at which the MEC equals the rate of interest. In this case, the real and nominal rates coincide. In the above case, if the price trend were absent, wood bought at \$100 would rise in value to \$106 at the end of the year, giving a nominal and real MEC of 6 percent. When the price change impinges, the value of the wood rises as well as the terminal value received, and the real MEC remains 6 percent. Assume that businessmen investing capital recognize and compensate for all price changes.

If the banks are lending money at 6 percent before the price change, and the real and nominal rates are thus 6 percent, the businessman borrowing from the bank just manages to break even. Suppose now that the price change begins but that, because of a

money illusion affecting the money market, no change occurs in the nominal rate of interest. As indicated above, the real rate of interest drops to roughly 4 percent. In short, the real rate of interest drops by approximately the rise in the price level. Now the real interest rate has fallen below the real MEC, which remains constant. Under these conditions additional investment will be worth while. By this, we mean investment expressed in dollars of constant purchasing power. Investment will increase until the real MEC is forced down to equality with the real rate of interest.

In Figure 29, the quantity of capital invested, expressed in dollars of constant purchasing power, is scaled off along the horizontal axis. On the vertical axis the real rate of interest is shown

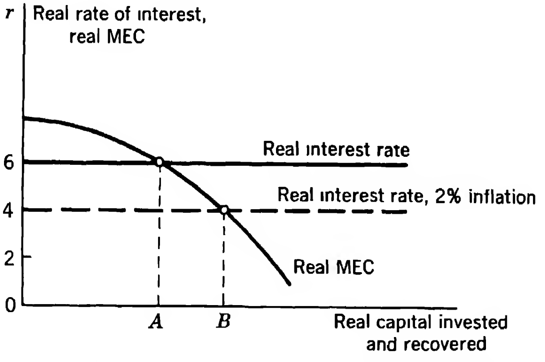


Figure 29. Effect of Inflation on Investment.

together with the real MEC. Before price inflation begins, equilibrium occurs at an investment of OA and a 6 percent real rate of interest and MEC. With a 2 percent annual rate of inflation, the real rate of interest drops, owing to the money illusion or institutional rigidity in the money market. The real MEC curve remains in the same position. To equate the real MEC with the lower real interest rate, businessmen extend the rate of investment from OA to OB. In short, the deflated value of investment undergoes an increase.

EXPLANATION OF CONSTANT NOMINAL RATE OF INTEREST

In the argument given, the nominal rate of interest is pictured as constant while prices of goods are rising. On first consideration this appears to be a highly arbitrary assumption. Actually, there are several circumstances which may bring about such a situation. One is the existence of a slow response of the money market to price changes. If those who hold money do not notice the change and if banks also fail to make an adjustment for this circumstance, the nominal rate of interest will not change. This condition amounts to a money illusion which affects the money market.

A more plausible explanation lies in the existence of a definite policy formulated by the monetary authorities and designed to stabilize the nominal rate of interest. Under such circumstances the money supply is manipulated to bring about the constant nominal rate desired. Again, this might be a consequence of rigidity in the thinking of the monetary authorities which arises from a failure to consider the change in the price level.

REPERCUSSIONS OF INFLATION

When the inflation commences, it sets in motion an increased demand for capital. With income and consumption taken provisionally as constants, unconsumed production or the available flow of capital goods remains constant. As this fixed quantity of capital meets an increased demand for capital, the price of capital goods should rise. Evidently, a further upward push is given to the price of this particular category of goods.

We might trace out the interactions of the forces set in motion. Evidently an increase in investment will cause an increase in income through the multiplier. In turn, this puts further pressure on the price of goods as a whole. Also, a rise in investment directly increases the demand for "finance" to undertake new investment. As investment increases, the induced rise in income causes an increased transactions demand for cash. Both of these factors put an upward pressure on the interest rate. We will not attempt to work out all the implications of these changes.

The real balance effect and Say's law

Patinkin[1] has recently argued that an important mechanism of adjustment by the economic system was overlooked by Keynes. This mechanism is the real balance effect, already discussed in the chapter on consumption. Here is the gist of the argument.

Suppose investment falls relatively to saving. Assume that a high degree of price flexibility exists in the goods market. A decline in the demand for goods will tend to reduce the price of goods. In turn, this increases the real value or purchasing power of the cash balances possessed by the individual. In such circumstances the individual may wish to spend a fraction of his additional or surplus *real* balances on consumption. Such an increase in consumption will tend to counterbalance the initial decline in investment. If a given decline in the price level does not do the trick, the process can continue further until the desired increase in consumption is accomplished.

MAGNITUDE OF THE REAL BALANCE EFFECT

Unquestionably, there is a theoretical basis for the real balance effect. In National Income analysis, however, we lose the spirit of the subject if we fail to take account of the practical aspects of the matter. Consider a slightly doctored case, designed to conform roughly to the facts of the American economy. Consider a representative family with a yearly income of \$4500, consumption of \$3600, and a balance in cash of \$300.[2] Suppose the level of gross investment in the economy is \$65 billion, while the net investment is \$35 billion. In this situation an autonomous decline in the level of investment takes place. Assume that output remains constant but that prices decline.

We must be rather realistic about the possibilities of a price decline. Within the bounds of recent experience, however, we may select the most extreme case. Between the 1929 peak and the 1933

[1] Don Patinkin, *Money, Interest, and Prices*, Evanston, Illinois. Row, Peterson, 1956, p. 21.

[2] "Preliminary Findings of the 1959 Survey of Consumer Finances," *Federal Reserve Bulletin*, March 1959, p. 252. By interpolation we get a median cash balance of about \$300 for 1959.

trough consumer prices fell sufficiently to cause an increase of one-third in the real balance from \$300 to \$400, using dollars of constant purchasing power. Assume that the marginal propensity to spend out of real balances is .9, leading to additional spending of \$90. We assume that this decline of prices takes place in a single year instead of four and that the spending out of extra real balances applies to the same year. Assume that the level of National Income is \$350 billion, with consumption of \$280 billion.

If the individual family consumes an additional \$90, this represents an increase of $2\frac{1}{2}$ percent of the consumption of \$3600. Applying this percentage to total consumption of \$280 billion, we find the increase to be \$7 billion. Between 1929 and 1933 gross investment fell off slightly more than 90 percent. A similar decline would amount to \$58.5 billion in the present example. This is on the order of 8 times the estimated real balance effect on consumption. Of course, we have no reason to assume that the entire decline in the level of investment was autonomous.

What are we to conclude from this example? The great disparity between the variation in investment and the real balance effect suggests that the latter cannot be expected to play a major part in cushioning depressions, though it may be a minor stabilizer of the economy.

In fact, we have no reliable evidence that this real balance effect works in the way suggested. It is a theoretical possibility that the marginal propensity to consume out of surplus real balances may be negative, implying that a fall in prices would tend to depress consumption. Owing to the possibilities inherent in the dynamics of a price decline investigation of this subject opens a veritable "Pandora's box." As we pointed out, a price decline sustained for some time leads to a feeling that the decline will continue. In this case money balances are a good buy, and the consumer may well decide to add to them. This implies that all and more than the surplus real balances will go into money balances, and that consumer goods are inferior. It is this author's belief that a price decline large enough to provide a significant real balance effect is going to have to take place over a fairly long time interval. If the decline takes place for three or four years, a situation favorable to a negative real balance effect tends to occur.

CONCLUSION TO REAL BALANCE EFFECT

The real balance effect is a welcome addition to the theoretical apparatus of the economist. It represents another step in the process of broadening and generalizing our approach to the study of national income. However, from the numerical example just considered we receive the strong suggestion that the effect, at best, can provide only a minor increase in total spending. At worst, it can be associated with a decline in spending with destabilizing effects.

Does our study of the real balance effect justify the conclusion that the economy has a strong tendency always to move toward and to remain at the full employment level? In short, does this argument justify our concluding that Say's Law is still an effective principle with new underpinnings? There are three reasons for refusing to accept this position with our present state of knowledge: (1) strong empirical evidence to sustain this position has not been presented; (2) it seems likely that the magnitude of the real balance effect is small compared to variations in the level of investment; (3) in periods of declining prices the real balance effect may be to reduce consumption spending.

Summary of essential results

At this juncture a review of the entire theory is in order. As a matter of fact, readers who wish to pass on to the dynamic portions of the analysis will find the necessary equipment to be essentially complete. In the static theory which follows, the determination of employment is discussed, but this subject is essentially subsidiary to the preceding matter.

First, let us list the variables that fall within the scope of the analysis. In order of introduction they are: income, consumption, saving, investment, interest rate, and money supply. For the sake of simplicity the total-expenditure approach may be employed in discussing the determination of the variables.

DETERMINING RELATIONSHIPS

Each variable is considered to be determined by some relationship or economic condition. When the condition is fulfilled, the variable is considered to be determined.

1. Consumption is functionally related to income in such a way that fixing the level of income determines a corresponding level of consumption. This relation is known as the *consumption function.*
2. Saving is the excess of income over consumption. This relationship defines saving and is not a new functional relationship, if the consumption function is counted.
3. Income is determined by the condition that income at factor cost (supply of goods) be exactly equal to the expenditure on output (consumption plus investment equals total demand for goods). Alternatively, income is determined by the equality of saving and investment.
4. The rate of interest is determined by the condition that the demand for money be equal to the supply of money.
   a. The demand for money is determined by liquidity preference, or a preference for cash over other assets, which preference must be overcome by the payment of interest.
   b. The supply of money is conditioned by the need of banks to cover variable cost and the desire to maintain liquidity. In order to induce banks to lend, a rate of interest sufficient to cover both elements must be paid.
5. The quantity of money is the common amount supplied and demanded when the rate of interest reaches the equilibrium value.
6. Investment is extended to that point at which the MEC is equal to the rate of interest.

In essence, conditions (1), (3), (4), (5), and (6) contain the basis of Keynesian income analysis. So far, employment has been ignored. If wages are rigid, income will be roughly proportional to employment up to that income which corresponds to full employment. At higher incomes employment will remain stable. In view of the likely prevalence of rigid wages, this seems to be a proper interpretation of the actual situation. In a later section the assumption will be modified and the level of employment considered as a separate variable.

By these simple methods, we represent the essential doctrines of National Income economics. In certain respects, however, the discussion is incomplete. When analyzing the value of a particular

variable, it has been our practice to assume that variables not under consideration remain constant. For example, when analyzing the demand for money and the rate of interest, it is convenient to assume that the level of income remains constant. Indeed, if the level of income should rise, the transactions demand would increase, causing a change in the rate of interest. In short, the rate of interest is influenced by income, and the arbitrary assumption that income is constant is a matter of analytical convenience. To complete the analysis, we need to show how the different variables and markets interact. By this means we can remove the artificial boundary lines between the income-determining forces, and the money market and the capital market. To this end the following optional section is added.

General equilibrium (optional)

In essence, there may be said to be two basic relationships in the preceding analysis. First, the income–expenditure or saving–investment equation, and second the equality of the demand for and the supply of money. Rather than attack this problem in complete generality, we will make some fairly realistic simplifying conditions. First, the effect of the rate of interest on the amount saved is assumed to be rather weak. Second, the effect of an increase in the level of income on the supply of money is assumed to be weak. Both these assumptions appear to be fairly reasonable. For illustrative purposes, the resulting analysis will be quite satisfactory, though a bit simplified.

In Figure 30(A), the saving curve in relation to income may be drawn in one fixed position, since the influence of interest on saving is assumed to be negligible. At any given rate of interest, the level of investment rises with the level of income. At a 4 percent rate of interest the investment curve takes the position shown by the solid line, I_2. If the rate of interest is raised to 6 percent, the amount of investment undertaken at any level of income will be reduced. This is true because the rate of investment must be reduced to raise the marginal efficiency of capital to equality with the higher rate of interest. For this reason the investment curve at a 6 percent rate exhibits less investment at every income level;

thus, I_1 lies below I_2. By similar reasoning a lower interest rate increases investment, causing the I_3 curve, drawn up at a 2 percent rate, to lie above the I_2 curve, drawn up at 4 percent.

In Figure 30(C), immediately below (A), the horizontal axis again measures income. The rate of interest is represented on the vertical axis. We are now in a position to derive a curve, to be

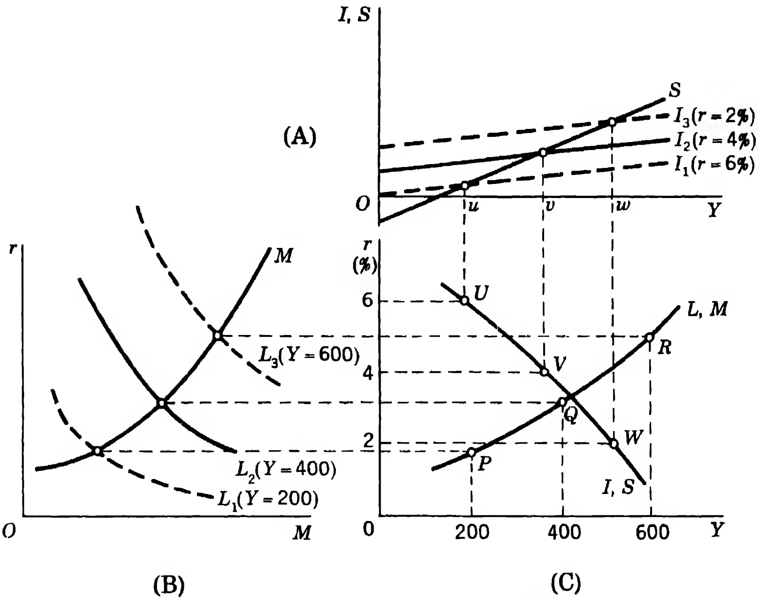


Figure 30. Derivation of *I,S* and *L,M* Curves.

entitled the *I,S* curve for obvious reasons, giving the level of income at which investment is equal to saving at every possible rate of interest. If the rate of interest is given as 6 percent, we can mark a corresponding point on the vertical axis of Figure 30(C) and extend a horizontal dashed line to the right. In Figure 30(A), the intersection of the investment curve I_1, generated by a 6 percent rate of interest, with the saving curve determines an income *Ou*. From point *u*, representing this income, a dashed line is extended downward to intersect with the line extended across from 6 percent. These intersect at point *U*. By finding the income which equates

saving with investment at 4 percent, we determine the point V in a similar way. Finally, the point W corresponds to the equilibrium of saving and investment at a 2 percent rate. Connecting points U, V, and W with an appropriate curved line, we find the I,S curve.

In much the same way we derive the L,M curve, giving the rate of interest which equates the demand for money, L, with the supply, M, at any given level of income. If the level of income is \$400 (billion), the liquidity preference curve L_2 in Figure 30(B), representing the schedule of the quantities of money demanded at various rates of interest, is generated. If the level of income rises to \$600, the transactions and finance motives give rise to a greater demand for money, and the L curve shifts to the higher position, L_3. On the other hand, if the income level falls to \$200, the demand for money falls to the position represented by L_1.

Suppose the level of income is \$200. Mark out this number on the income axis in Figure 30(C), and extend a dashed line vertically upward from the point. The income \$200 generates a demand for money represented by the L curve, L_1. At the point where the L_1 curve intersects the M curve, extend a dashed line horizontally to the right. This will intersect the line drawn upward from \$200 in Figure 30(C) at point P. In a similar way the points Q and R can be determined. Drawing an appropriate curve through these points gives rise to the L,M curve.

A simultaneous equilibrium in the saving–investment (income–expenditure) market and the money market is attained at the point where the I,S and L,M curves cross. In short, the rate of interest and the level of income defined by the intersection lead to a simultaneous equilibrium in each market. It remains only to show that such an equilibrium may be reached.

In Figure 31 the summary diagram showing the I,S and L,M curves is reproduced. A little additional terminology may be helpful. Let us refer to the natural rate of interest as that rate which equates saving and investment or total supply (Y) with total demand ($C + I$). On the other hand, let us refer to the money rate as that rate which equates the supply of money with the demand, and provides equilibrium in the money market. At income A_0 the natural rate is A_0F_0, while the money rate has the lower value A_0D_0. The excess of the natural rate over the money rate affords the

businessman an opportunity to borrow in the money market and invest in the capital or savings–investment market. Suppose the natural rate is 6 percent while the money rate is 4 percent, the level of income being $300. In effect, this means that the rate of return in the capital (savings–investment) market is 2 percent above the rate in the money market. Under these circumstances an individual may borrow at 4 percent in the money market (most probably from a bank) and invest the borrowed funds in the capital market at 6 percent. This means actually buying capital goods on which the 6 percent return can be obtained. Clearly, the level of income will be increased by the stream of new money and by the leverage effect of the new investment on income via the multiplier. Thus income tends to increase from A_0 to A_1.

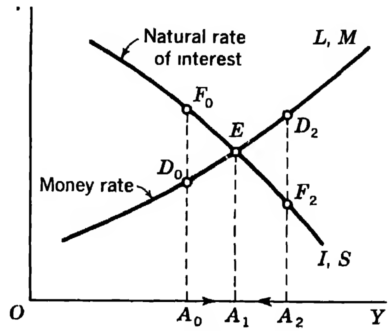


Figure 31. Simultaneous Determination of Interest and Income.

On the other hand, at income A_2 the money rate of A_2D_2 exceeds the natural rate of A_2F_2 by D_2F_2. The difference D_2F_2 represents to the businessman or saver an opportunity to discharge debts in the money market. If a saver has an opportunity to choose between discharging a bank debt which costs him 6 percent and making an investment in capital which earns him 4 percent, he will clearly choose the former. Obviously, funds will flood into the money market to discharge debts with the result of contracting bank loans and thereby reducing bank deposits or "money." Investment will then have to be cut back for lack of funds. As the quantity of money is reduced and investment is curtailed, income contracts by the multiplier. Consequently, income tends to move back from A_2 toward A_1. At income A_1 the equilibrium rate of interest, A_1E, is found, representing an equalization between the natural rate and the money rate. If this rate prevails in both markets, no incentive exists for a flow of funds between the two markets.

To sum up, there is an equilibrium income and interest rate which, if reached, will equate saving and investment, the demand for and the supply of money, all at the same time. As the foregoing argument shows, there are some good reasons for supposing that a tendency exists for income and the interest rate to move in the desired directions. The reasons advanced are not entirely conclusive for two reasons. First, it is not entirely certain that the I,S and L,M curves will have the precise form shown. Second, the various lags in human action and their effects on the situation have not been explored. Within these limitations it is possible to say that a tendency toward general equilibrium exists.

Liquidity preference and diminishing utility of money (optional)

According to the liquidity preference theory, people prefer the certainty of a given sum of money to the (uncertain) expectation of a like amount. If the chance of realizing more than the expectation is just as great as the chance of realizing less, why should these possibilities not exactly balance in the mind of the individual? Expressed in terms of money the expected gains and losses do balance off in a certain expected money value. Under our assumptions, however, the expectation of gain in terms of satisfactions is less than counterbalanced by the corresponding expectation of loss, as evidenced in the preference for cash. In order to facilitate the comparison of gains and losses, it is necessary to attach to them a measure of satisfaction or utility. By pursuing the implications of liquidity preference, we may discover something about the implied nature of consumer satisfactions or utility. If the existence of liquidity preference implies some familiar form of behavior of utility, then the concept in question is rendered more plausible.

Suppose the expected price of a security is \$105, while the "certainty equivalent" is \$100, giving a rate of liquidity preference equal to 5 percent. To be more explicit, suppose the expected price to be represented by a $\frac{1}{3}$ probability of a price of \$135 and a $\frac{2}{3}$ probability of a price of \$90. Then we can write:

$$E_{\$} = \$105 = \$135 \times \tfrac{1}{3} + \$90 \times \tfrac{2}{3} = \$45 + \$60$$

where $E_\$$ stands for the expected price of the security. In spite of the fact that, on balance, a price of \$105 is expected, the individual will be just as well satisfied with \$100 in cash. What does this imply about the utility schedule for money?

To proceed further, it is necessary to discuss the measurement of utility. Certain assumptions are necessary. First, assume that the total utility associated with money increases with the quantity possessed. In accordance with this rule, we may assign the numbers 2 and 5 as the utilities of \$90 and \$135 respectively. However, any pair of numbers which increase when the quantity of money increases may be used. For example, the pair 3 and 8 would do quite as well.

Second, we assume that the utility of the security to a person is measured by the mathematical expectation or weighted average of the utilities of the possible values of the security. In our case, the expected utility, E_u, is given as:

$$E_u = 2 \times \tfrac{2}{3} + 5 \times \tfrac{1}{3} = \tfrac{9}{3} = 3.$$

By our assumption, 3 measures the utility of the security or the expected utility of the security. This assumption assigns an easily calculable utility to the security.

In addition to the two assumptions stated, we have the liquidity preference assumption. According to this assumption, a security with a given expected value (including any yearly dividend) is equivalent to a smaller quantity of cash. In our own example it is assumed that the security with an expected value of \$105 is worth \$100 in cash to the individual. This completes the list of assumptions required.

By rearranging the assumptions and writing them in a condensed sequence, it will be easy to draw a conclusion. First, we write the liquidity preference assumption as applied to our example. Second, we write a combination of the first two assumptions for the example in question. (1) \$100 in cash is equivalent to an expectation of \$105. (2) An expectation of \$105, afforded by the purchase of a security, provides a level of satisfaction indicated by the number 3 (E_u above equals 3.)

From these assumptions it follows that: (3) \$100 in cash provides a satisfaction of 3.

By the aid of the assumptions given above, we have identified the utility attached to a sum between \$90 and \$135. With this additional information in hand, we can write the following utility schedule.

TABLE 29. Utility and Marginal Utility of Money

(1) Cash	(2) Utility of Cash	(3) Change in Cash	(4) Change in Utility of Cash	(5) = (4) ÷ (3) Marginal Utility of \$1 Cash
\$90	2			
100	3	10	1	.100
135	5	35	2	.057

In columns (1) and (2) we find the utility schedule; the first and last entries were given, the middle derived. In column (5), the marginal utility of cash is derived by dividing the change in the utility of cash by the change in cash, or column (4) divided by column (3). As the last column reveals, the marginal utility of money declines as the amount of cash possessed increases. This is exactly what we have been looking for—a plausible implication of liquidity preference. If we look at things from this viewpoint, the liquidity preference of an asset holder may be reinterpreted.

Why is the security whose expected value is \$105 worth only \$100 in cash? The answer may be construed to lie in diminishing marginal utility. Since additional sums of money yield a diminishing marginal utility, their utilities are subject to a psychological discount. In other words, an individual is more interested in holding on to what he has than in acquiring more. Consequently, the possibility of an expected rise in price, discounted by diminishing utility, fails to offset the possibility of a loss. This lack of balance is expressed in equivalence between \$100 in cash and a security with an expected value, $E_\$$, equal to \$105.

In modern demand theory, the foregoing assumptions relating to the measurement of utility are accepted as plausible. If one is willing to assume a single unique utility schedule to begin with, the above reasoning process may be reversed. We can then arrive at

liquidity preference as a consequence of a utility schedule for money subject to diminishing marginal utility.

In conclusion, there seems to be a very close connection between liquidity preference and diminishing marginal utility of money. The reader may find in this connection a further basis for believing in the existence of liquidity preference.

CHAPTER 6

Employment theory

Prior to 1929 there was an accepted theory of employment which ran as follows: Employment is determined by the supply of and demand for labor. A decline in employment implies that wage rates are too high relative to prices of goods. A suitable cut in wages relative to prices of goods will restore the wage-price balance and bring about a correction of the reduced level of employment.

This view of employment determination was held more or less independently of the position on Say's Law. The latter principle assured economists that output and production could always be extended to the limits of capacity. Both Say's Law and the theory of employment led to similar conclusions: that production and employment could always be extended to the full amounts of these variables.

In the 1930's came a deep depression which cast doubt on both beliefs. According to data on the American economy, one person in four seeking work was unable to find it in the year 1933. Such a level of unemployment was not plausibly explained by wage rigidity. Neither did the currently accepted supply and demand theory of employment fit the facts. At this point, in the middle 1930's, J. M. Keynes put forward an explanation of employment based on the demand for goods. This theory is designed to explain unemployment. Before discussing the theory itself let us classify the various situations in which an economy may find itself at any given time.

Depression, full employment, and boom

Speaking very broadly, we can describe an economy as being either in depression, at full employment, or in a state of boom. A period of depression is characterized primarily by low demand for goods with attendant effects on employment and output. At a higher level of demand for goods, employment and output improve

and reach a level at which those who seek employment are able to find it, and output is limited by a scarcity of available resources. Such a situation may be described as "full employment." Finally, a still higher level of demand for goods will produce various phenomena depending on the reactions of various groups in the economy to the pressure. In any event, inflationary pressure, wage-price spirals, and occasionally extreme economic activity may characterize the situation we have entitled "boom."

The onset of the great depression prompted Keynes to provide an apparatus which is adapted to explain variations in employment. Such variations occur primarily during the economic situation we have described as "depression." As the economic situation verges on full employment or boom, various additions to the analysis must be made, if the apparatus is to prove useful. This happens because the variable which the apparatus is designed to explain reaches a constant value, full employment. At this juncture the focus of attention shifts to the level of money prices and wages. In this chapter we will confine our discussion principally to causes of variations in employment. For this reason our discussion will be primarily devoted to the first situation, namely, depression. In a succeeding chapter (Chapter 11) the consequences of the high level of demand leading to boom will be given separate treatment.

Although the level of employment is the focus of attention, the analysis would be incomplete without some discussion of the level of wages. Consequently, we will tie employment theory to the theory of wages, as set forth by Keynes.

Determination of employment: Keynesian theory

UNITS

At the outset it is necessary to decide on units of measurement. Since a study of employment is being undertaken, it is probably not surprising to find that analytical convenience requires the use of labor units. Let us refer to a unit of labor as a "labor unit," rather than a "wage unit" as it was called by Keynes. Employment is expressed directly in labor units; therefore it proves convenient to express income and expenditure (consumption plus investment)

in labor units. If National Income is \$300, expenditure is \$360, and the wage of labor is \$60, National Income in labor units is 5 compared to expenditure of 6 in labor units. In the following discussion the money wage is taken as a constant. Perfect competition is assumed to prevail.

AGGREGATE SUPPLY OF GOODS AND AGGREGATE DEMAND FOR GOODS

According to the Keynesian theory employment is determined by the necessity of equating the aggregate supply of goods with the aggregate demand for goods. In Chapter 4 the thesis was advanced that National Income tends to gravitate to that level at which aggregate supply equals aggregate demand. By a slightly deeper analysis, relating employment to aggregate supply and demand, we can arrive at the conclusion that income *and* employment gravitate to the level at which aggregate supply and demand are equal.

From our preceding analysis we recall that income at factor cost, Y, represents the sum of factor payments. By the same token it is a money measure of the output which can be turned out by this expenditure of money. In short, income at factor cost represents the terms on which businessmen are willing to supply a corresponding volume of output. By reason of the association between the aggregate output and the level of factor payments required to sustain it, this magnitude (Y) may be entitled "aggregate supply."

In the short run the main variable factor of production is labor and the main variable cost, wages. To make it worth while for them to offer a given level of output and the employment required to sustain it, businessmen must take in enough revenue to cover the corresponding factor cost. In general, the level of factor cost, Y, must cover all wage cost and such a return to the fixed factors, land and capital, as is justified by the relation between wages and the prices of goods. The higher the level of prices with given wage rates, the greater is the level of employment which is justified and the greater is the level of factor cost. Briefly, aggregate supply, as measured by necessary factor cost, varies directly with the level of employment. This functional relationship between aggregate

supply and employment is known as the "aggregate supply schedule" or curve.

For analytical reasons it is convenient to express aggregate supply at factor cost in labor units, as noted previously. This is done by dividing the wage into the level of factor cost, Y, giving a magnitude, Y/W, as the measure of output at factor cost expressed in labor units. If output at factor cost is \$420 billion and the weekly wage of labor is \$60, output at factor cost is 7 billion units (weeks). Such an output is generated by some level of employment, say 6 billion labor units. Note that the output at factor cost may exceed employment, implying that more is produced than is required to remunerate labor for its services.

In columns (1) and (2) of Table 30 the relation between labor employed and factor cost is stated. Dividing each level of factor

TABLE 30. The Relation Between Factor Cost in Money and Factor Cost in Labor Units

(1) Labor Employed	(2) Factor Cost of Output in Money Y	(3) Factor Cost of Output in Labor Units $(Y/W)^a$
1	\$60	1
2	150	$2\frac{1}{2}$
3	300	5
4	560	$9\frac{1}{3}$
5	1740	29

[a] W = the wage = \$60, Y is income regarded as factor cost.

cost, Y, as given in column (2), by the wage of \$60, we obtain the factor cost of output in labor units, Y/W. By plotting column (3) against column (1) as in problem 3 (p. 181), we get the aggregate supply curve for goods regarded as a function of employment.

For purposes of the present discussion the existence of schedules (1) and (2) relating employment to factor cost is simply assumed. It seems inherently plausible that a certain level of receipts is required to induce employers to offer a certain corresponding level of employment. For those who want a deeper investigation of the relationship some notes have been included at the end of this chapter (pp. 170–78) which state a basis for the existence of this relationship.

Next, we require the aggregate demand for goods at each level of employment. If we could feel comfortable with the hypothesis that total demand for goods, expressed in money, depended on the level of income, similarly expressed, we could easily secure this desired relation. However, in problems involving the interrelations of employment, output (real income), the price level, and the wage level any elementary hypothesis is soon revealed for what it is. Consequently, we must shift from the simplified approach to demand used hitherto to one which embodies a modification suggested in the chapter on consumption.

Let us assume that consumption "deflated" for price changes depends on the level of income "deflated" in a like manner. The relationship is assumed to be linear in form and to have the same appearance as the short-run consumption functions observed previously. In the graphical representation "deflated" consumption is plotted against "deflated" income.

In the specific model used to construct the diagrams and tables there is but one good produced, which serves both as consumer good and capital good. In making this simplification we are really assuming that the prices of consumer goods and capital goods change in the same direction and by the same percentage. Since there is but one good, the "deflated" demand for consumer goods can be represented simply by the amounts of the good consumers wish to purchase (at corresponding levels of real income).

We also assume that "deflated" investment demand is autonomous. In the present analysis investment demand consists of a decision to purchase a fixed number of units of capital goods. If the price of capital goods doubles, the price of consumer goods also doubles, since both goods consist of units of the same commodity used for different purposes. Both the cost of acquiring the capital good and the returns from the use of the capital good having doubled, no change has occurred in the desirability of acquiring capital.

Consumption in real terms, the number of units of output which consumers wish to acquire, depends on real or "deflated" money income. This real income is the money income deflated for price change. In the present set of tables and diagrams only one good is being used so that real income is simply the total output of the

good in question. Clearly, the real income of society consists simply in the flow of output which becomes available during the year. If the price of consumer goods doubles, and we assume but one good, the price of capital goods also doubles. This implies that all prices double. With the same output or real income money

TABLE 31. A Derivation of the Aggregate Demand Schedule for Goods as a Function of Employment

(1) Labor Employed	(2)[a] Real Income	(3)[b] Real Demand	(4)[c] Money Income	(5)[d] Money Price	(6)[e] Money Demand	(7)[f] Demand in Labor Units
1	12	17.2	$60	$5	$86	1.43
2	20	22	150	7.50	165	2.75
3	25	25	300	12	300	5
4	28	26.8	560	20	536	8.93
5	29	27.4	1740	60	1644	27.4

[a] Real Income denotes units of output.
[b] Real Demand is consumption plus investment in units of output. Consumption = 4 + .6 (real income), implying that people consume 4 units of goods when real income is zero and that MPC out of real income is .6 so that .6 of each additional unit of real income is consumed. Investment is autonomous and equal to 6. Adding, we find that Real Consumption plus Real Investment = Real Demand = 10 + .6 (real income).
[c] Column (2), Table 30. This schedule is derived in the appendix to this chapter.
[d] Money income ÷ real income = (4) ÷ (2).
[e] Real demand × money price = (3) × (5).
[f] Money demand ÷ money wage = (6) ÷ $60.

income will also double. Under these conditions, with double the money income and facing double prices, the consumer ought to buy the same quantity of consumer goods. This is the hypothesis we are using.

Let us now run through the steps necessary to find the aggregate demand for goods as a function of employment. First, we assume that each level of employment gives rise to a corresponding level of real income or output. Of course, the output which results from a given volume of employment depends on the distribution of employment among firms. Once this allocation is settled at every level of employment, and it is assumed to be unique at each level, the corresponding output level will be determined. Leaving aside the details of this allocation problem, we assume in Table 31 a relationship between employment and real income. This relationship is given in columns (1) and (2).

We assume that with every level of real income (output) there is associated a corresponding level of real demand. This real demand is expressed in terms of output which society wishes to purchase at that level of real income. It consists of consumption demand plus investment demand. The relationship between real income and real demand is given in columns (2) and (3). In column (4) we list the money income necessary to induce firms to hire the corresponding quantity of labor. This was listed in column (2) of Table 30 and is derived in the appendix to this chapter.

Money price is the ratio between money income and output (real income). It is derived by dividing the money income column, (4), by the real income (output) column, (2). Money demand is real demand (output demanded), as given in column (3), multiplied by money price. This result is given in column (6). Dividing the money demand given in column (6) by the wage of labor, which equals $60, we arrive at the demand for goods expressed in labor units in column (7). Finally, the relation between the quantity of labor, as given in column (1), and the demand in labor units, as given in column (7), is the desired aggregate demand schedule for goods.

Let us now review informally the nature of the aggregate supply and demand schedules for goods, considered as functions of employment. The aggregate supply schedule represents the schedule of levels of income, expressed in labor units, associated with a corresponding schedule of levels of employment. There is an integral connection between each level of employment and the level of receipts or factor income required to sustain it. The aggregate demand schedule represents the schedule of levels of demand for goods, expressed in labor units, associated with a corresponding schedule of levels of employment. Since each level of employment is associated with a given level of factor income and each level of factor income is associated with a corresponding level of demand, we are in a position to relate employment directly with demand, expressed in labor units.

UNDEREMPLOYMENT EQUILIBRIUM

By combining columns (1) and (7) of Table 31 with column (3) of Table 30, we secure the following table containing the needed

information. At levels of employment smaller than 3, demand for goods exceeds the supply. For example, when the level of employment is 2, the demand for goods is 2.75 compared to a supply of 2.50. As a result of the excess demand, production is increased, more labor is hired, and employment expands until it reaches 3.

TABLE 32. Equilibrium of Aggregate Demand for and Aggregate Supply of Goods with Under-employment Equilibrium

(1) Labor Employed	(2)[a] Aggregate Supply of Goods—Factor Cost in Labor Units	(3)[b] Aggregate Demand for Goods—Expenditure in Labor Units	
1	1	1.43	Demand exceeds supply;
2	2.50	2.75	employment expands.
3	5	5	Equilibrium
4	9.33	8.93	Supply exceeds demand;
5	29	27.40	employment contracts.

[a] Table 30, column (3). [b] Table 31, column (7).

At levels of employment exceeding 3, the supply of goods exceeds the demand. For example, when the level of employment is 4, the supply of goods is 9.33, compared to a demand of 8.93. As a result of the excess supply, production is cut, and labor is laid off until employment declines to 3. Clearly the equilibrium level of employment is 3. At this employment the quantity of goods demanded is exactly equal to the quantity supplied. Such a level of employment is not necessarily one which affords a job to every worker seeking a job. In fact, we explicitly assume the contrary, so that the number seeking work exceeds 3, the quantity associated with equilibrium in the goods market. Suppose that this full employment level is 4. With the existing level of demand for goods conditions do not justify hiring this quantity of labor.

In Figure 32 we depict a situation of this type graphically, using the data of problem 1 of this chapter. The aggregate supply of goods, (Y/W), and the aggregate demand for goods, $(C + I)/W$, expressed in labor units, are plotted against quantities of labor employed.

Employment is determined at E where the aggregate demand for goods generated by the level of employment OA is exactly equal to the supply. If the level of employment were raised to OL, the aggregate supply would become LN, while the aggregate demand would be LM. Since aggregate supply exceeds aggregate demand by MN, production would be cut and employment would fall as indicated by the arrow from L directed toward A.

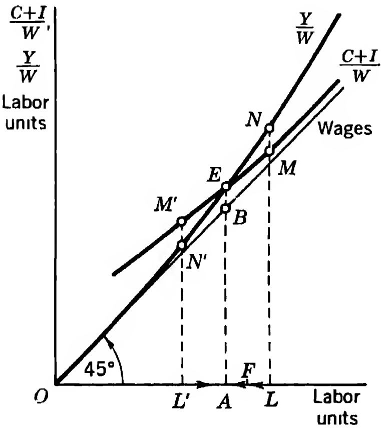


Figure 32. Determination of Employment and Income.

If employment happened to fall to the lower level OL', aggregate demand would be $L'M'$ compared to aggregate supply of $L'N'$. In view of the excess demand of $N'M'$, production would be expanded and employment increased as indicated by the arrow from L' directed toward A. Thus a stable situation exists, with employment tending to the value OA.

WAGE AND NONWAGE INCOME

In Figure 32 we have inserted a 45° line which we label "Wages." Although this line does not play a functional role in the analysis, as in Chapter 4, it indicates the allocation of income between wage and nonwage income. The "Wages" line itself measures wage income in labor units at any level of employment. On the vertical axis wage income in labor units is measured against equal values of employment on the horizontal.

Perhaps a numerical example will indicate the nature of the relation. Suppose 5 laborers are hired at a wage of \$60 apiece, giving a wage bill of \$300. Expressed in labor units this wage income is \$300 ÷ \$60 = 5 labor units. In short, hiring 5 laborers generates the income (5 labor units) earned by 5 laborers. When wage income of 5 is plotted against employment of 5, the resulting point falls on a 45° line passing through the origin.

Equilibrium between production and demand is found at E in Figure 32 where the Y/W and $(C + I)/W$ lines intersect. In turn, the level of income is the vertical distance from E to the horizontal axis, or AE. Wage income is found by going up from the point A, defining equilibrium employment, to the "Wages" curve at B. This gives wage income in labor units as AB. Subtracting wage income of AB from total income of AE, we find nonwage income to be BE.

We have marked out the full employment point at F. As yet this point is undefined, but it will be given a significant meaning later. Note that the equilibrium level of employment OA is less than the full employment level OF. The difference noted is to be accounted for by the fact that the demand for goods does not generate sufficient derived demand for labor.

The employment multiplier

In the theory of income determination the relationship between investment changes and corresponding induced income changes receives detailed attention. In numerical form the relationship is known as "the multiplier." As we might imagine, an explicit theory of employment generates a need for a corresponding "employment multiplier." In numerical form this relationship should reveal the ratio between an increase in employment and the corresponding increase in expenditure on investment. Perhaps a numerical illustration can best state the meaning of this concept.

If investment measured in labor units increases by 1, the number of units increase in employment is the employment multiplier. For example, suppose investment expenditure increases by 1 labor unit, or \$60, since the wage of labor is \$60. If the ordinary multiplier is 4, the resulting increase in money income is \$240, of which perhaps \$180 goes for wages and \$60 for nonwage payments. Since an extra \$180 of wage income is generated, and the wage is \$60, the increase in employment is 3 units of labor. The ratio of increased employment to the increase in investment spending, expressed in labor units, is $3 \div 1 = 3$. This is the employment multiplier.

In the preceding example the employment multiplier is 3 and the ordinary multiplier is 4. To derive the employment multiplier

from the ordinary multiplier it is necessary to know the ratio of additional income to additional wage payments in labor units. In the numerical example this ratio was \$240/\$180 = 4 labor units ÷ 3 labor units = $1\frac{1}{3}$. Let us entitle this ratio "the marginal employment price" of labor. This ratio is equal to or greater than 1 at all times.

In order to justify hiring an additional laborer the employer must take in a sum at least equal to the wage. If a rise in price serves as the justification for hiring an extra laborer, as we assume throughout the analysis, this increased price will generally afford a little additional quasi-rent in addition to the wage cost. To the extent that an increase in quasi-rent (the excess of receipts over variable cost) is afforded by the wage rise, the marginal employment price, MEP, exceeds unity. The value of MEP is measured along the Y/W curve. The value of Y/W represents total factor income in labor units, whereas the level of employment that generates it measures the quantity of labor. Since hiring a laborer gives rise to corresponding wage payments, employment and wage income are synonymous. A movement along the Y/W curve implies an increase in wage income (employment) which necessitates a corresponding rise in all factor incomes. The ratio of the (required) change in all factor incomes to wage income is the MEP.

Suppose both the ordinary multiplier and the MEP are known. In our illustration the multiplier is 4, and the MEP is $1\frac{1}{3}$. If income increases by 4 in labor units, $1\frac{1}{3}$ labor units must be received by employers for every unit of employment offered. Evidently 4 units of extra income will afford only $3 = 4/1\frac{1}{3}$ additional units of employment. Thus the employment multiplier is the ratio of the money multiplier to the MEP, a relation written symbolically as:

$$K_e = K/\text{MEP},$$

where K_e is the employment multiplier and K is the money multiplier.[1]

[1] The algebraic derivation of the formula runs as follows: $Y_w = Y/W, C_w = C/W, I_w = I/W$, where Y, C, and I represent income, consumption, and investment in money, W stands for the wage, and Y_w, C_w, and I_w stand for income, consumption, and investment

GRAPHICAL INTERPRETATION

In Figure 33 equilibrium in the market for goods is found at A where the Y/W and $(C + I)/W$ curves cross. Owing to some autonomous cause the level of investment increases and the $(C + I)/W$ curve shifts upward to the dashed position labeled $(C + I')/W$. Under the changed conditions a new equilibrium is found at D where the unchanged Y/W line intersects the new $(C + I')/W$ line.

By assumption, the increase in investment is autonomous or uniform at any level of income or employment. Consequently, the increase in investment in labor units may be measured by finding the vertical distance between the $(C + I)/W$ line and the $(C + I')/W$ line at any point. For example, the vertical dotted segment AE, extending from A, the old equilibrium point, to E on the new $(C + I')/W$ line measures this increase in investment. Likewise the vertical segment DC, extending downward from the new equilibrium at D to C on the old $(C + I)/W$ line, measures the investment increase. In short, $\Delta I_w = AE = CD$, where $\Delta I_w = \Delta I/W$, the increase in investment, measured in labor units.

in labor units. Also let L stand for employment. Recall that in equilibrium $Y/W = (C + I)/W$, or $Y_w = C_w + I_w$; by the same token $\Delta Y_w = \Delta C_w + \Delta I_w$, the equation holding for changes also.

$$K_e = \frac{\Delta L}{\Delta I_w} = \frac{\Delta L}{\Delta Y_w - \Delta C_w}, \text{ since } \Delta Y_w = \Delta C_w + \Delta I_w,$$

$$= \frac{\Delta L/\Delta L}{\dfrac{\Delta Y_w}{\Delta L} - \dfrac{\Delta C_w}{\Delta L}} = \frac{1}{\dfrac{\Delta Y_w}{\Delta L} - \dfrac{\Delta C_w}{\Delta L}}, \text{ dividing top and bottom by } \Delta L,$$

$$= \frac{1}{\dfrac{\Delta Y_w}{\Delta L} - \dfrac{\Delta C_w}{\Delta Y_w}\cdot\dfrac{\Delta Y_w}{\Delta L}} = \frac{1}{\dfrac{\Delta Y_w}{\Delta L}\left(1 - \dfrac{\Delta C_w}{\Delta Y_w}\right)}, \text{ factoring out } \frac{\Delta Y_w}{\Delta L},$$

$$= \frac{\dfrac{1}{1 - \dfrac{\Delta C_w}{\Delta Y_w}}}{\Delta Y_w/\Delta L} = \frac{\dfrac{1}{1 - \dfrac{\Delta C}{\Delta Y}}}{\Delta Y_w/\Delta L} = \frac{K}{\Delta Y_w/\Delta L} = \frac{K}{\text{MEP}}.$$

In the last line $\dfrac{\Delta C_w}{\Delta Y_w} = \dfrac{\Delta(C/W)}{\Delta(Y/W)} = \dfrac{\Delta C/W}{\Delta Y/W} = \dfrac{\Delta C}{\Delta Y}$, since W is assumed to remain constant throughout, and the W in the numerator and denominator of the ratio cancel. By the same argument $K = \Delta Y/\Delta I = \Delta Y_w/\Delta I_w$.

To find the change in employment extend a horizontal line to the right from A, cutting a vertical line extended down from D in B. Then the increase in employment is the segment AB, measuring the horizontal or employment component of the change in the equilibrium point from A to D. In short, $\Delta L = AB$, where ΔL is the change in employment.

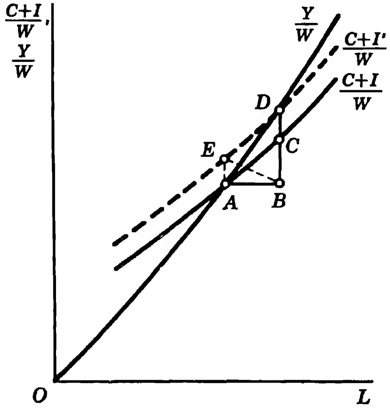


Figure 33. Employment Multiplier in Graphical Form.

Putting these two results together, we find that from the definition, $K_e = \Delta L / \Delta I_w = AB/AE$ = tangent of angle BEA.

We can also interpret the employment multiplier as a ratio of K to MEP. In Figure 33 ΔY_w (the increased income in labor units) is the vertical component of the change in the equilibrium from A to D; that is, $\Delta Y_w = BD$. Using this information, we can write

$$K = \frac{\Delta Y}{\Delta I} = \frac{\Delta Y_w}{\Delta I_w} = \frac{BD}{CD} \text{ (see footnote on page 157).}$$

As explained earlier, the MEP $= \Delta Y_w / \Delta L$ is measured on the $Y/W = Y_w$ curve. From the diagram it can be observed that:

$$\text{MEP} = \frac{\Delta Y_w}{\Delta L} = \frac{BD}{AB}.$$

Taking the ratio of K to MEP, we find that:

$$K_e = K \div \text{MEP} = \frac{BD}{CD} \div \frac{BD}{AB} = \frac{BD}{CD} \times \frac{AB}{BD} = \frac{AB}{CD} = \frac{AB}{AE},$$

since $CD = AE$.

From these manipulations we find that the direct measurement of the definition and the ratio of K to MEP give a common answer.

VARIATION OF EMPLOYMENT MULTIPLIER

With autonomous investment and a constant MPC the money multiplier is a constant. As income and employment increase the money multiplier remains the same. For the employment multiplier the case is different. Recall that the employment multiplier is:

$$K_e = \frac{K}{\text{MEP}},$$

where K is the money multiplier, a constant.[2] In order to find out whether the employment multiplier is a constant it is necessary to consider the behavior of MEP.

Recall that MEP $= \Delta Y_w/\Delta L = BD/AB$, in Figure 33, since BD is the vertical component and AB is the horizontal component of a movement along $Y_w = Y/W$ from A to D. From this it follows that MEP is the ratio of the vertical to the horizontal movement along Y_w, or its slope. By inspecting the Y_w curve we see that it becomes steeper as employment increases. In consequence, the vertical component grows relative to the horizontal component and MEP grows. Hence MEP increases with employment.

In common-sense terms, the reasoning is this: When employment and production are low, costs rise slowly with output. In order to prompt the employment of one more unit of labor the price must rise only slightly. Then a rise in the price sufficient to cover the cost of another few units of output will do little more than cover the cost of labor. At this juncture the firm is induced to increase employment by an increase in receipts only slightly larger than the increase in wage cost. In short, MEP $= \Delta Y_w/\Delta L$ exceeds 1 by a slight margin. As employment expands to a much higher figure and production approaches capacity, costs tend to rise more rapidly with output. In order to provide a little more output the firm must experience a considerable rise in price. Such a higher price will not only cover the cost of the last unit produced, but afford an extra return or quasi-rent on units whose costs are already covered. In short, the MEP rises substantially above 1, as the higher price increases afford quasi-rent in addition to wage cost.

[2] This is subject to one restriction. As income increases, if the economy shifts from a short-run to a long-run consumption function at full employment, MPC rises and with it the value of the multiplier.

Since K is constant under our assumptions, and MEP increases with employment, the ratio, $K_e = K/\text{MEP}$, declines as employment increases. As one might expect, investment expenditure affords less and less leverage effect on employment as employment increases and unemployment diminishes. Indeed, there is less of a body of unemployed on which to exert such a leverage.

LIMITS OF THE EMPLOYMENT MULTIPLIER

Since the employment multiplier, K_e, equals K/MEP, and MEP increases with employment, the value of K_e declines as employment increases. As employment drops to low values, the MEP approaches 1. Under conditions of severe depression employers are willing to hire labor with the expectation that additional gross receipts will do little more than cover the cost of labor. As this situation arises, $K_e = K/\text{MEP}$ approaches $K/1 = K$. In short, K_e increases as employment drops, approaching, but not necessarily reaching, an upper limit equal to K.

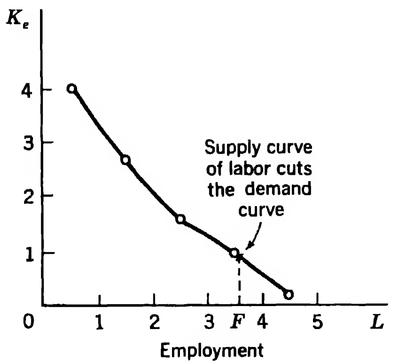


Figure 34. Employment Multiplier as a Function of Employment.

If employment continues to increase, MEP rises, simply because the additional labor is pressing on capacity and the market price of goods must rise sharply to induce the production of more output and the hiring of more labor. If the pressure on capacity is the only force to operate in the situation, K_e will gradually fall, ultimately approaching zero as MEP goes up out of sight. A more likely limit is the arrival of full employment. Let us define *full employment* as the level of employment at which the demand curve for labor cuts the supply curve. When the economy arrives at this point, it is not likely to pass beyond without the appearance of some rather peculiar phenomena. If it cannot, the value of the employment multiplier drops to zero. In Figure 34 a set of values

of K_e are shown. The values plotted are based on a K of 4 and values of MEP drawn from Table 38, column (5).

Wage determination in underemployment equilibrium

According to the pre-Keynesian theory of distribution the wage of labor is determined by the demand for labor together with the supply. Keynesian wage theory rejects this notion, making use of the supply curve and the demand curve of labor in a vastly different way. The key point made by Keynes is that employment does not necessarily equal the value found at the intersection of the supply curve for labor with the demand curve.

In order to facilitate understanding of Keynesian wage analysis we must review briefly the notions lying behind the demand curve for labor.

MARGINAL PRODUCTIVITY AND THE DEMAND CURVE FOR LABOR

According to the marginal productivity theory of wages, labor is paid the value of its marginal product. At the heart of this theory is the notion that a firm seeks to maximize profits. In order to do so, the firm will hire a unit of labor so long as the return added by that laborer equals or exceeds the wage of labor. The return added by the last laborer is equal to the physical product he adds (Marginal Physical Product) multiplied by the price of the product. Restating the rule with obvious abbreviations, we find that the firm hires labor as long as:

$$\text{MPP} \cdot \text{P} \geqslant \text{W}.$$

If labor is assumed to be finely divisible, as it often is, then the inequality is removed and the number of laborers hired is defined by the equation:

$$\text{MPP} \cdot \text{P} = \text{W}.$$

In the present problem, the wage is taken to be constant for society. Evidently, no problem arises of money wage determination. To give the problem meaning, a slight modification is

necessary. Divide both sides of the equation through by P. In its altered form the equation reads:

$$\text{MPP} = \frac{W}{P}.$$

Let us interpret W/P. Suppose the wage of labor is \$60 and the product produced is meals having a price of \$15 (per dozen). Then W/P = \$60/\$15 = 4 (dozen) meals. In short, the expression W/P gives the number of dozen meals which a unit of labor will command. In works on distribution theory this ratio is called the "real wage," since it expresses the power of the money wage to command goods.

Since the money wage is taken to be constant, the price of meals must vary in order that the real wage may change. In its slightly revised form the marginal productivity doctrine states that labor is hired until the product added by the last worker is equal to the real wage. If the real wage falls as a result of a price increase, more workers will be hired until the MPP reaches equality with the lower real wage. If the real wage rises as a result of a decline in the price of the product, workers will be laid off until MPP rises to a position of equality with the real wage. Since the real wage is equal to MPP at any level of employment, the relation between MPP and employment is taken to be the demand schedule for labor, as a function of the real wage.

WAGE DETERMINATION: KEYNESIAN THEORY—VERBAL STATEMENT

Under conditions when the demand for goods is low, employment is determined at that level where the supply of goods is equal to the demand, according to Keynesian theory. At the level of employment determined by this condition a certain value of MPP exists; the last laborer adds a given quantity of output to the total produced at this employment level. In order that the level of employment in question afford maximum profit to the employer it is necessary that this MPP be equal to the real wage, W/P. In short, the real wage is equal to the MPP of labor at the level of employment generated by the need to match the current supply of goods with the demand.

WAGE DETERMINATION: PRE-KEYNESIAN THEORY—VERBAL STATEMENT

According to the theory once generally accepted, the real wage is equal to the value found at the intersection of the demand curve of labor (the MPP curve) with the supply curve of labor. At the level of employment thus determined the real wage could be described as determined by and equal to the MPP. In Keynesian theory the real wage is also equal to the MPP, but at a different level of employment. The difference in the theories lies in the assumed domination of the labor market by the goods market in Keynesian theory. The need to clear the market for goods is thought to override the influence of the supply curve of labor in employment and wage determination. Today the Keynesian views are generally accepted as an account of an *underemployment* situation.

CONTRAST OF THE TWO THEORIES

Generally speaking, in pre-Keynesian days economists were not aware that the demand for goods could be insufficient to justify hiring all the laborers seeking a job. If they had qualms on this score, Say's Law reassured them that the supply of goods always created a like demand. Once Say's Law was questioned, this logical structure also came into question.

Employment determination: classical solution

If a classical economist were drawing the Y/W and $(C + I)/W$ curves, he would have them coincide throughout their lengths. Any addition to the supply, Y/W, would generate an equal demand, $(C + I)/W$, by the same argument used in the text in an earlier chapter. For this reason the level of employment is determined by the supply of and demand for labor. Indeed, this is the only set of circumstances which *can* determine employment, the goods market being consistent with any level of employment. To sum up, it is the labor market which determines employment and output by matching the supply of labor with the demand.

CONTENDING GROUPS AND RELATIVE BARGAINING POWER

Our analysis makes it clear that the employment determined by the equality of demand and supply in the goods market is unique and, with a prevailing condition of depression, less than the level determined by the supply of and demand for labor. Under depression conditions the buyer is highly sought after and must be considered first. Next, the employer, being an intermediary between the productive resources and the buyer, is in a more strategic position than the sellers of those resources. In short, of the three groups operating in the situation the laborer is in the poorest position to make his wishes effective. For this reason the goods market which brings forth an agreement between sellers of goods (employers) and buyers of goods should prevail over any possible agreement between employers of labor and suppliers of labor. Furthermore, the employers' wishes as expressed in the demand curve should be the main force determining the real wage.

Recapitulation and graphical treatment of Keynesian theory

In the upper portion of Figure 35 are pictured the aggregate supply and demand curves for goods as functions of employment. In the lower diagram the demand for labor and the supply of labor, considered as functions of the real wage, are depicted. As we have just explained, the demand curve is identified with an MPP curve representing additional product added by the last worker hired at different levels of employment. In general, the intersection of the demand curve of labor with the supply curve will take place at a level of employment different from the level defined by the intersection of the aggregate demand and supply curves for goods. If this is true, one condition or the other will hold, but not both.

Since buyers of goods and those who cater to them directly (employers) are in a strategic position, it seems reasonable to assume that employment is determined in the goods market. If this is true, employment is determined to be *Oa* in the upper diagram. In this event equilibrium no longer prevails at the intersection of the demand curve of labor with the supply. Once the

level of employment is given from the goods market, the only question remaining is the manner in which the real wage is to be determined. As we have explained, the employer is in a more strategic position than the laborer and will tend to determine the real wage.

Dropping a vertical dashed line downward to the lower diagram from the intersection of the $(C + I)/W$ curve with the Y/W curve, we find the wage to be determined at E on the demand curve for labor. At this wage we find the quantity of labor offered by drawing a horizontal dashed line over to G on the supply curve. Dropping dashed lines to the horizontal axis, we find that employment is OA and the supply of labor offered at the prevailing real wage is OB. The excess of the supply of labor over the demand, AB in the lower diagram, represents "involuntary unemployment." Such a condition crops up when the demand for goods is low, and only then. In a period of strong demand for labor the situation is altered.

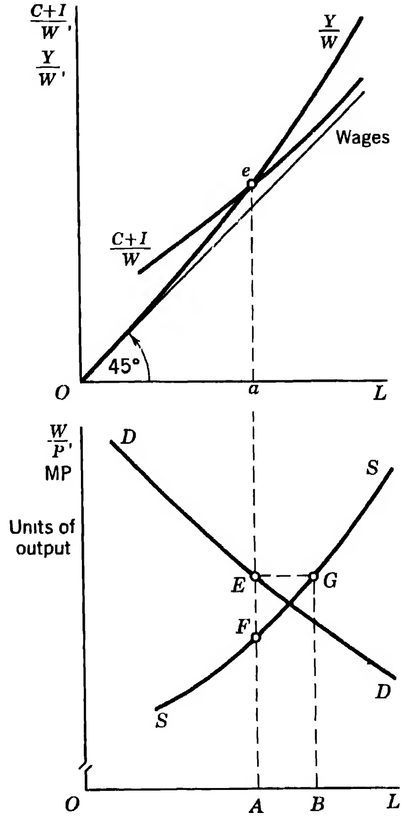


Figure 35. Underemployment Equilibrium.

GRAPHICAL REPRESENTATION OF PRE-KEYNESIAN THEORY

It may clarify the discussion in the text to translate the argument on pre-Keynesian or neoclassical employment and wage theory into a diagram. In Figure 36 the conditions of Say's Law are assumed to prevail. The $(C + I)/W$ line is assumed to coincide

with the Y/W line and the supply of goods is equal to the demand at every level of employment. In this event the goods market is in neutral equilibrium at any level of employment.

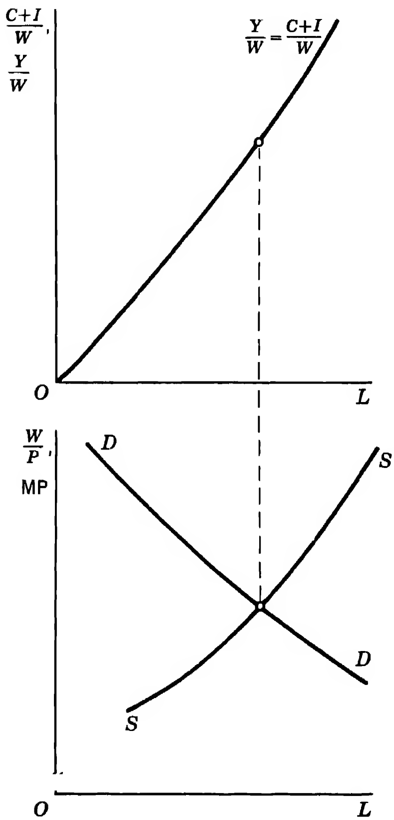


Figure 36. Full Employment Equilibrium with Say's Law.

At a low level of employment, however, the demand price for labor exceeds the supply price. Under these circumstances both employer and laborer gain by moving to a higher level of employment, since a margin of labor productivity over the supply price of labor exists. As employment expands, income increases and with it the supply of goods. Since all money saved is returned to the income stream by investment, all additional income is used to buy goods. Consequently, as employment increases, income and the aggregate supply increase, with demand keeping pace. When the level of employment is reached which affords an equality between the supply price of labor and the demand price, no incentive exists in the labor market for a further expansion of employment. At this level of employment, owing to the operation of Say's Law the demand for goods is equal to the supply. Consequently, combined equilibrium exists in both markets.

SIDELIGHTS

In the preceding analysis we have determined the level of employment, the level of income (in labor units), the real wage,

and the level of excess supply of labor. Let us consider a number of ideas that are bound to rise to the surface of the reader's mind. First, would a cut in the real wage increase employment? If a cut were made in the real wage from AE in Figure 35 to some lower value, employers would be willing to offer more employment. This would be shown forth by a movement down the demand curve for labor. If the additional labor were to be hired, however, the aggregate demand curve, $(C + I)/W$, would fall below the supply curve, Y/W. In this event the level of receipts, $(C + I)/W$, is insufficient to meet the necessary remuneration for the larger level of employment, Y/W. Facing an insufficient aggregate demand, employers would be forced to cut employment to match supply with demand. In short, a cut in the real wage is not an effective means of increasing employment.

In the second place, we assume that in an underemployment situation the bargaining power of the employers is slightly greater than that of employees. Referring to Figure 35, grant that employment is determined at the level Oa in the upper diagram, and OA in the lower. If employers have the upper hand, why do they not beat down the real wage to AF on the supply curve? Since the employees are willing to work for this, why doesn't the superiority in the bargaining power of employers over employees lead to this wage? If employers succeed in doing this, they have become organized into a monopsony which bargains as a single unit with labor. With true competition among employers, bidding must take place until an employer is offering the alternative return in another use which is the MPP. As long as competition (for labor) between employers exists, the real wage will tend to be at a high level in this situation.

FULL EMPLOYMENT AND BEYOND

The situation we have been describing in this chapter is one in which the demand for goods is "low." In such a situation the employment defined by the equality of aggregate demand for and supply of goods is less than the employment defined by the intersection of the supply curve of labor with the demand. The latter value is termed "full employment."

If the demand for capital goods increases, investment rises, and

the $(C + I)/W$ line shifts upward, the level of employment defined by equilibrium in the goods market will increase. If investment continues to rise, the full employment point will be reached in time. At this point simultaneous equilibrium would be reached in both markets. In Figure 37 this situation is pictured. The intersection of the $(C + I)/W$ with the Y/W curve in the upper diagram determines a level of employment OF in the upper diagram. Dropping a vertical dotted line down to the lower diagram from F we find that the line passes through the point of intersection of the demand and supply curves for labor at E. The attainment of this point is associated with one particular level of demand, and its attainment must be regarded as accidental. This is a borderline case.

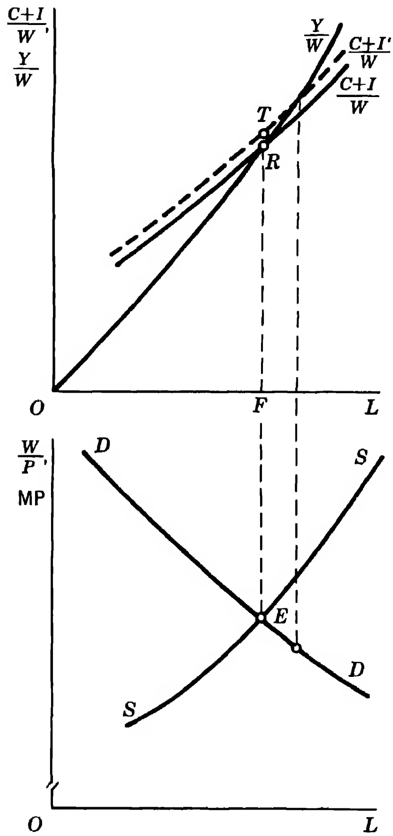


Figure 37. Excess Demand at Full Employment.

Suppose the level of investment increases further. The equilibrium in the goods market will occur at a higher level of employment than in the labor market. To move past the full employment point requires one of two things in the labor market. Note that as employment increases, the demand price falls below the supply price of labor. Either labor is hired by employers at the wage the latter desire, leading to the acceptance by labor of a lower price than they ask, or employers pay a price above their demand price which is the marginal productivity of labor. Under purely static

competitive conditions with perfect knowledge of the market neither of these situations seems likely.

Perhaps we may conclude that in the absence of some weakening of the static competitive setup employment cannot move past the full employment point. If investment demand increases so that the $(C + I)/W$ line intersects the Y/W line at a larger level of employment than is defined in the labor market by equilibrium of supply and demand, a basically new situation arises. The situation is pictured in Figure 37. At full employment, OF, which affords equilibrium in the labor market by definition, an excess demand equal to RT exists in the goods market. Let us briefly discuss this matter of excess demand in the goods market.

In the new situation the scales of bargaining power have shifted. Goods are now scarce and buyers are scrambling for goods in short supply. In such a situation it would appear that the laborer is in the most strategic position. Without the laborer the employer cannot bring forward the desired quantity of goods. As one asking for more goods than are available the buyer is in the weakest position. As an intermediary between the two, the employer is again in a strategic position, but not to the extent that labor is. Without some weakening of perfect competition we cannot expect either employers or laborers to abate their requirements, as expressed in the labor market, to satisfy a strong demand of consumers in the goods market.

According to the theory advanced in Chapter 4, the appearance of an increased demand generated an increased level of production, employment, and National Income. And in the real world it seems all too likely that an excess demand for goods will lead to some important development. It is suggested in a later chapter that this demand leads to a wage–price spiral. In the standard case, however, it would lead to no change in real income, employment, or the real wage.

If we abate perfect competition we may find a situation developing in which employment moves past the full-employment level. Among the requisites of perfect competition is perfect knowledge of prices and other circumstances of the market. If either employer or laborer is slow to recognize price or wage changes, the

development of a wage–price spiral can lead to a movement past the full employment point. Such slowness to recognize price or wage changes is known as "the money illusion."

In any event it seems that the situation becomes inherently dynamic when excess demand for goods develops at full employment. Some outlet will be found for the excess demand and this outlet may be found in a wage–price spiral. Since such a discussion departs widely from the present analysis, it has been dealt with in a separate chapter on "Inflationary Processes."

APPENDIX: A tabular review of the demand for labor

In the text it seemed desirable to sketch the theory of employment in broad outline, stressing general results and underlying assumptions. If the reader feels uneasy concerning the logical derivation of the curves, he should study this appendix. As the discussion in the appendix proceeds, it becomes evident that fine-honed tools brought over from price theory form the basis of employment theory. At first marginal analysis may seem to afford a poor basis for a theory of aggregates. As the discussion unfolds, however, the connection becomes quite evident.

In what follows we wish to describe the derivation of the demand curve for labor, and the aggregate supply curve of goods as a function of employment. These concepts are closely interrelated. Since the supply curve of labor is independent of these, we simply take it for granted without discussion.

BASIC MARGINAL PRODUCTIVITY THEORY

At the basis of the marginal productivity theory is the notion that the firm seeks to maximize profit. In order to achieve this objective, the firm adds units of labor until the last laborer brings in just enough additional revenue to cover the additional cost involved. Under conditions of perfect competition this balancing of added revenue and added cost can be interpreted by an equation. This can be expressed as the equation:

$$\text{wage of labor} = \text{marginal physical product} \cdot \text{price of product.}$$

The *marginal physical product* is the additional physical output added by the last laborer, whereas the marginal physical product times the price

of that product is called the *marginal revenue product.* The marginal revenue product represents, under perfect competition, the value of the product added by the last laborer.

NUMERICAL EXAMPLE

Consider a firm using a fixed amount of land and capital and which employs varying numbers of laborers. Suppose the labor consists of waitresses serving meals at a restaurant, so that the product is "meals."

TABLE 33. Determination of Employment by a Firm

(1) Labor	(2) Total Product (Meals in dozens)	(3) MPP[a]	(4) Price[b] P	(5) (5) = (3) · (4) MRP = MPP · P	(6) Wage W	(7) Real Wage $(W \div P)$
0	0	—	$12	—	$60	5
1	12	12	12	144	60	5
2	20	8	12	96	60	5
3	25	5	12	60	60	5
4	28	3	12	36	60	5
5	29	1	12	12	60	5

[a] Marginal Physical Product consists of the successive changes in the Total Product, consisting of the number of batches of meals produced.
[b] Price of a batch consisting of a dozen meals.

Let the wage of waitresses be $60 while the price of meals is $12 (for a batch of 12 meals). Let the schedule of batches of meals served by increasing numbers of laborers be given in the first two columns of Table 33. By taking the successive differences in the Total Product column, we find the Marginal Physical Product column. Then, multiplying the MPP column by the price of $12, we arrive at the Marginal Revenue Product (MRP) schedule. All these calculations are detailed in Table 33.

As long as the MRP is greater than or equal to the wage, it will pay the firm to hire labor. Thus the first unit of labor brings in an MRP of $144, but costs only $60. Clearly, the addition of this unit nets the firm $84. Next, the second unit affords an MRP of $96, but costs only $60, yielding a net return of $36. The addition of the third unit brings in $60, a return which just serves to cover the wage of the same amount. Evidently, this is the last unit added, since the return on the fourth unit is only $36, a return which fails to cover the cost. In conclusion, the firm stops at the point where MPP $\cdot P = W$.

Table 34. The Aggregate Supply Schedule as a Function of Employment

(1) Labor	(2) MPP	(3) MRP_1 ($P = \$5$)	(4) MRP_2 ($P = \$7\frac{1}{2}$)	(5) MRP_3 ($P = \$12$)	(6) MRP_4 ($P = \$20$)	(7) MRP_5 ($P = \$60$)	(8) Money Wage	(9) Real Wage = Wage/Price
1	12	$60	$90	$144	$240	$720	$60	$60/$5 = 12
2	8	40	60	96	160	480	60	$60/$7½ = 8
3	5	25	37½	60	100	300	60	$60/$12 = 5
4	3	15	22½	36	60	180	60	$60/$20 = 3
5	1	5	7½	12	20	60	60	$60/$60 = 1
Y = Receipts[a]		60	150	300	560	1740		
$Y/W = Y/\$60$		1	2½	5	9⅓	29		

[a] Equal to total revenue actually taken in at the prices listed in the column headings.

THE REAL WAGE

All this is familiar to the student who has had a course in Principles of Economics and is included merely as a refresher. Perhaps somewhat less familiar is the concept of the "real wage," defining the power of the wage to command goods. In our model there is only one good, which greatly simplifies the situation. In this event, the real wage is simply the money wage divided by the price. From the data of the problem we find that the real wage, $\frac{W}{P} = \frac{\$60}{\$12} = 5$. What does the 5 represent? Since the wage is \$60 and the price of a dozen meals is \$12, the real wage (per week) is 5 dozen meals. In short, the real wage measures the number of units of the product that the wage will buy.

Return to our condition of maximum profit, $W = \text{MPP} \cdot P$. Dividing through this equation by P, we get:

$$\frac{W}{P} = \text{MPP}.$$

In this form the equation states that under competitive conditions labor is hired until the real wage equals the Marginal Physical Product of the laborer. In Table 33 the real wage is shown in column (7) as having the constant value 5. Comparing column (7) with column (3) for MPP, we find them to be equal for the third laborer. This confirms our previous results.

VARYING REAL WAGE

In the problem we wish to discuss, the money wage is a constant for society as a whole. However, the money price may vary. If the wage remains constant and the price varies, the real wage, W/P, also varies. What we wish to establish is the way in which employment varies with a change in the real wage.

Let price take on the value \$5. With the help of the MPP schedule given in Table 33, column (3), we can draw up an MRP schedule; this may be designated MRP_1. The labor schedule, MPP schedule, and MRP_1 schedule are reproduced in columns (1), (2), and (3) of Table 34. Comparing the MRP_1 schedule with the money wage schedule given in column (8) (\$60 throughout), we find the point of equilibrium to be at 1 unit of labor. At this point the money wage equals MRP. In column (9) the real wage has been computed for this price and is $\$60 \div \$5 = 12$. This value is recorded in the first row of column (9).

At 1 unit of labor MPP is also 12, as seen in the first row of column (2); the equality of the real wage and MPP confirms that this is the point at which to stop.

If the price of meals rises to $7½, the MRP_2 schedule is realized. In this event the firm finds it profitable to hire two laborers, at which point MRP and the wage are equalized at $60. With the higher price the real wage has fallen to $60/$7.50, or 8 meals; this is the number of meals that the money wage of $60 will buy at $7.50 per meal. Likewise the marginal product of the second worker is 8 meals. Evidently, MPP and the wage have been equalized.

TABLE 35. Demand Schedule for Labor as a Function of Real Wage (Price Variable, Wage Constant)

Labor	Money Wage	Price	Wage/Price	Marginal Physical Product
1	$60	$5	12	12
2	60	7½	8	8
3	60	12	5	5
4	60	20	3	3
5	60	60	1	1

Following these correspondences all the way down the line, it is evident that the firm hires labor to the point where the real wage equals the marginal physical product. From the summary shown in Table 35 we can conclude that the demand curve for labor as a function of the real wage is the Marginal Physical Product schedule of labor. One further question arises. Would the same result be derived if the price remained the same and the wage dropped? The answer is "yes."

In order to show how the problem may be worked out when the wage varies instead of the price, we have included another table. Without following through all the steps in Table 34, we may summarize the relevant information, as follows. The price of the product is assumed to be $20 throughout, while the wage takes on different values. The variation of the wage (in money) causes the real wage to change. In turn this induces the firm to hire more or less labor. Again, the schedule of MPP in relation to the schedule of labor constitutes the demand schedule for labor. This information is summarized in Table 36.

Evidently, it makes no difference whether it is a change in the wage or a change in the price that causes the real wage to vary. In short, it is

the ratio of the wage to the price on which the demand for labor depends.

The lesson to be learned from this is that only a change in the real wage, the ratio of money wage to money price, causes a change in quantity demanded. Thus, if both the money wage schedule and the money price schedule doubled in Tables 35 or 36, the real wage schedule and the demand schedule as a function of real wages would remain the same. This is a significant point to note, since recognition of this condition permits us to separate the problem of employment from inflation (proportional changes in all prices).

TABLE 36. Demand Schedule for Labor (Price Constant, Wage Variable)

Labor	Money Wage	Price	Wage/Price	Marginal Product
1	$240	$20	12	12
2	160	20	8	8
3	100	20	5	5
4	60	20	3	3
5	20	20	1	1

FROM THE FIRM TO THE ECONOMY

Let us take for granted that the demand schedule of a firm for labor as a function of the real wage is the schedule of the MPP at each quantity of employment. In order to secure the demand schedule for all firms, assuming they all produce one product, it is necessary to add the several schedules. By adding for all firms the employment found at a single MPP, we determine the total employment required at that real wage (MPP). When we pair this MPP with the corresponding total employment, we thereby determine one entry in the total demand schedule for labor. Repeating this operation for every MPP, we arrive at a demand schedule for labor by all firms.

In Table 37 the demand schedule for the first firm is depicted by arraying the MPP's in the first column, the corresponding amounts of employment in the second. For Firm B we list the levels of employment at which these MPP's are realized in the third column. Assuming only two firms to exist, we next find total employment at each wage by adding columns two and three, an operation which yields column four. Taking column one, listing the MPP's, with column four, listing total corresponding employment, we have the demand schedule of the two firms

for labor. No problem, save arithmetic, is posed by the consideration of any number of firms.

SUMMARY TO THIS POINT

We have established the existence of a demand schedule for labor in which the quantity of labor demanded depends on the real wage. In our problem the wage is constant, while the price may vary. It is now

TABLE 37. The Aggregate Demand Schedule for Labor

MPP (Real Wage)[a]	Employment		
	Firm A[a]	Firm B	Total, A + B
12	1	2	3
8	2	4	6
5	3	$6\frac{1}{2}$	$9\frac{1}{2}$
3	4	9	13
1	5	12	17

[a] Data drawn from Table 36.

appropriate to see how the same conditions which generate the demand curve for labor generate an aggregate supply curve for goods.

AGGREGATE SUPPLY CURVE FOR GOODS

The aggregate supply curve for goods may be defined as the total receipts, expressed in labor units, which must be received by entrepreneurs or firms to justify a given level of employment (and therefore output). Labor is assumed to be composed of homogeneous units throughout the argument.

In the first place, consider how a curve of this type is derived for the firm. Referring to Table 34, we see that when the money wage is \$60 and the price is \$5, it pays to hire one worker. Total receipts are \$60, and the wage is also \$60, so that the receipts in terms of the labor units commanded by these receipts consist of 1 labor unit. When the price rises to \$7½, the wage being \$60, it pays to hire two laborers. Adding the first two MRP's in the MRP_2 column, we get total revenue in money to be \$90 + \$60 = \$150. Dividing \$150 by the wage, \$60, we get 2½ labor units. When the price rises to \$12, and three units are hired, we add the first three MRP's in the MRP_3 column, giving \$144 + \$96 + \$60 = \$300. When divided by the money wage of \$60, this

gives 5 labor units. In each case, the procedure is to add the MRP's in the *relevant* column down to the number of laborers hired. This gives the total money receipts associated with that employment. The sums to be added are enclosed in a box and are tabulated at the bottom of the relevant column as Y. This sum represents the receipts which businessmen expect to enjoy as a concomitant of hiring this number of laborers. In order to secure Y/W, or expected receipts in labor units, the receipts in money must be divided by the wage to secure expected receipts expressed in labor units.

TABLE 38. Schedule of Y/W or Receipts (in Labor Units) Required to Induce a Firm to Hire the Corresponding Number of Labor Units

(1) Y_w	(2) L	(3) (3) = Change in (1)	(4) (4) = Change in (2)	(5) (5) = (3) ÷ (4) MEP = $\Delta Y_w/\Delta L$
1	1	—	—	—
$2\frac{1}{2}$	2	$1\frac{1}{2}$	1	$1\frac{1}{2}$
5	3	$2\frac{1}{2}$	1	$2\frac{1}{2}$
$9\frac{1}{3}$	4	$4\frac{1}{3}$	1	$4\frac{1}{3}$
29	5	$19\frac{2}{3}$	1	$19\frac{2}{3}$

In order to tie up employment with total receipts note that employment of (say) 3 units is attained only when the price is \$12. Following the arrow from 3 under the Labor column to the right, we arrive at the block of MRP's found when $P = \$12$ in column (5). To find the sum of these we follow the arrow down to the next to last row where the sum of these MRP's are tabulated as \$300. Following the arrow one more step, we arrive at the value 5 for these receipts expressed in labor units as shown in the bottom row. Thus employment of 3 is associated with receipts in labor units equal to 5. A similar correspondence can be traced out for each level of employment and corresponding value of Y/W. The results are tabulated in columns (1) and (2) of Table 38.

MARGINAL EMPLOYMENT PRICE

The marginal employment price, MEP, is the quantity of additional receipts (expressed in labor units) required to induce the hiring of an additional laborer. By taking the differences in Y_w and L and finding the ratio MEP $= \Delta Y_w/\Delta L$, we can derive a table of MEP's. This work

is also recorded in Table 38. Columns (1) and (2), representing $Y_w = Y/W$ and L, give the supply price of output as a function of the quantity of labor.

THE AGGREGATE SUPPLY CURVE: ALL FIRMS CONSIDERED

In the preceding section the aggregate supply schedule has been derived, using as the basis of the analysis a single firm. To extend the analysis to any number of firms producing a single good is not difficult. We first add up the MRP schedules for all firms, using the same technique as the one outlined earlier, except that MRP values instead of MPP values are used. This addition gives the aggregate MRP schedules for the group of firms, one for each price of the final product. With the substitution of the aggregate MRP schedules for the schedules applying to the firm the argument proceeds exactly in the manner already indicated.

USE OF WAGE AS A CONSTANT

In this chapter it is most convenient to take the money sage as a constant throughout the discussion. Variations in the real wage are taken as being accomplished by variations in the price of output. Since labor or employment is the independent variable, symmetry is obtained by expressing the dependent variables in units of labor. Also this permits a comparison of wage and nonwage income to be made in a simple and natural way.

CONSUMPTION, INVESTMENT, TOTAL DEMAND, AND INCOME IN OUTPUT UNITS

In Chapter 11 a study is made of inflationary processes. Such a study makes it necessary to treat the money wage, in addition to the price, as a variable. It is not a good idea to measure variables in terms of units which vary. Consequently, when dealing with problems of inflation it is desirable to find a unit which is more nearly constant than the wage. Output is perhaps the best measuring rod for this case. Under the simplified assumptions being used in this treatment, output has the desired properties. Both the aggregate supply schedule, measured in terms of output, and the aggregate demand schedule have been obtained without reference to money prices or money wages. Consequently, neither of these schedules is affected in any way by changes in these magnitudes. In fact, a change in either wage or price leads to a movement up or down the schedules, but not a movement of the schedules. In the chapter on inflationary processes we take advantage of this property of aggregate supply and demand schedules.

MONEY WAGE CUTS AND EMPLOYMENT (OPTIONAL)

In the Keynesian analysis the *money* wage of labor is taken as a constant. Only by a rise or fall in the price of the product does the real wage vary. As we have already pointed out, a decline in the real wage (occasioned by a rise in the price of the product) will not increase employment in the face of inadequate demand for goods. This assumes that the rate of interest is constant; however a general equilibrium analysis almost certainly leads to the same result. A rise in the price of the product (which reduces the real wage) increases money income and occasions an increased transactions demand for money. In turn, this raises the rate of interest which reduces investment. Therefore, total demand declines and with it the level of employment at which aggregate demand is equal to aggregate supply. This seems to preclude an increase in employment by this method.

Suppose that we consider the effect of a cut in the money wage, accompanied by a cut in the price so that the real wage remains the same. If the wage and prices are cut in proportion, money income changes in the same proportion. Under these circumstances the quantity produced and demanded in real terms remains unchanged. Clearly, no stimulus to employment can arise from this source. However, a reduction in costs and prices all around reduces money income and the transactions demand for money. In turn, this lowers the rate of interest and stimulates investment, thus raising aggregate demand at the given level of employment. Finally, the increased aggregate demand becomes equal to aggregate supply at a greater level of employment.

If the cut in the money wage is great enough, enough of a fall in the interest rate to produce full employment may occur. Such a cut in the money wage is just a method of reducing the pressure on the money supply. As Keynes pointed out, it is much more logical simply to increase the money supply by central bank action.

Several points about money wage cuts may be noted. First, this method of increasing employment is subject to all the limitations of stimulating employment by cheap money noted in Chapter 7. Second, money wage rates are known to be extremely rigid in a downward direction. Such rigidity of the money wage is entirely consistent with a regime of perfect competition, but is certainly aided by trade unions. As a method of increasing employment, taking all institutional factors into consideration, a cut in the money wage seems impractical. Third, as we have noted, the same objectives may be achieved more directly and with less possibility of effective opposition by increasing the supply of money.

PROBLEMS

1. (a) Given the marginal physical product (MPP) schedule shown below, develop a Y/W schedule in the manner of Table 34. (Assume a constant wage, \$480, and let the price take on varying values which are appropriately chosen in view of the equalization of the real wage and MPP.)

L	MPP
1	20
2	18
3	16
4	14.5
5	13

(b) Given the following schedule showing the "real demand" or "deflated $C + I$" as a function of "real income", develop the $(C + I)/W$ schedule as a function of the quantity of labor, L. Use the same wage as in 1(a) (\$480). Recall that "real income" is total product which is the sum of MPP up to that quantity of labor. Refer to Table 31.

uantity of Labor (See Schedule)	*Real Income*	*Real Demand*
	20	37
	38	46
	54	54
	68.5	61.25
	81.5	67.75

(c) Plot the Y/W schedule of 1(a) and the $(C + I)/W$ of 1(*b*) against the quantity of labor. Also draw in a wages line. Find the level of employment, the value of income, the value of wage income and nonwage income from the diagram. Using the wage assumed, convert these values to amounts of money income.

2. In the text we assumed that real demand (expressed in units of output) was a function of real income (also expressed in output). We might change this hypothesis to one used generally in the text, that demand expressed in money is a function of money income. ·onsider how this could work out in the following problem:

Refer to Table 31 for the relation between labor employed, column (1), and money income, column (4). Add to this the following assumed relation between money income and money demand.

Labor Employed [*Table* 30, *Column* (1)]	*Money Income*	$C + I$ *Demand in Money*	Y/W *Supply* (*Factor Cost*) *in Labor Units* [*Table* 30, *Column* (3)]
	$60	156	
	150	210	
	300	300	
	560	456	
	1740	1164	

(a) Convert the money demand into labor units, corresponding to (but different from) column (7) of Table 31. Use wage of $60.
(b) Plot the new data for "demand in labor units" and "supply in labor units" against labor units. Draw in a "wages" line.
(c) Estimate employment total income, wage income, and nonwage income, in labor units, from the diagram.

3. Plot Table 32 (p. 153). Draw in a wages line. Find employment, total income, wage income, and nonwage income, all in labor units.

CHAPTER 7

Economic policy

Out of the great economic problems of each era come great intellectual attempts to define and solve them. Among the greatest of such efforts was that of Adam Smith who wrote the *Wealth of Nations* in an effort to provide a philosophical basis for early capitalism. On a lesser scale the profound economic depression of the 1930's sounded a sharp challenge to economists. On the one hand a need existed for a clear description and explanation of the situation. On the other, an economic policy which could remedy the situation was required. As an economist who combined practical with theoretical talents John Maynard Keynes found this double assignment to be a worthy task. In the previous material we have outlined his explanation of the phenomena then observed. In a later section of this chapter we shall discuss remedies which he suggested.

Theory as the basis of policy

Every scheme of economic policy is undergirded with a corresponding theory. Without the theory the system of policy would lose its basis, and without the policy the theory would lose its impact. Thus Adam Smith's theory explained how a price system set in motion by self-interest would stimulate the production and distribution of the national income. Since Smith's theoretical analysis implied that a competitive price system was economically efficient, the policy recommendations which emerged defined conditions necessary to maintain such a system. In this way policy flowed from theory.

Lest we be too naïve, it will pay to take note of the fact that every theory flows from certain assumptions. Clearly, then, the policy which emerges from a system of thought depends on the basic assumptions of the theory. Subconsciously, an economist may form policy conclusions which condition his choice of assumptions for the theory. Were the assumptions to be altered the

theoretical and policy conclusions would change. For this reason it is very important to examine the underlying assumptions of the theory.

To illustrate, the appearance of monopoly to a marked degree in the price system reduces its efficiency, and this, in turn, weakens the case for such a system. In short, an economic policy designed to maintain a system of monopolies has no sound basis in theory. Since monopoly is now widely prevalent in our economic system, Smith's system of policy cannot be accepted without reservation today.

SEPARATION OF THEORY AND POLICY

In the previous section it was argued that theory and policy are closely connected. So true is this that the two often are confounded. In a closely woven statement of theory and policy the reader may be offended by the policy recommendations and reject the entire discussion on this account. Yet the theory may be perfectly sound. To avoid this reaction on the part of the reader we have tried to keep these aspects separate. In this way the distinctive features of the policies suggested may be made to stand out in clear relief.

Let us note that problems in economics may be divided into three categories: (1) science, (2) policy or art, (3) ethics. Science may be defined as a set of organized rules about the behavior of things under specified conditions. In economic life extreme variability of the conditions prevails. As a result, it is extremely important that the conditions assumed bear a close resemblance to those which are believed to exist. For example, a good theory of price must deal with conditions of monopoly or monopolistic competition to make the theory capable of current application. In line with this interpretation science may be defined more narrowly as a set of organized rules about "what is." Such a definition would exclude the consideration of situations which are not in line with current economic conditions.

Art (policy) may be defined as that set of rules which will suffice to bring about a desired result. Here the objective is specified and the conditions are sought which will bring about the result in question. In representational art the objective is specified as the creation of a realistic image of the object. Certain means such as

the rules of perspective, light and shade, color and so forth are employed to create a certain impression in the mind of the viewer. Whatever means is adopted the attempt is deemed successful if it conjures up the desired image.

In practicing the art of economics it is first necessary to set a goal for the operation of the economy. The investigator then seeks to define those conditions which must prevail in the system for the desired goal to be realized. Often the necessary rules define the character of our basic economic institutions. Adam Smith desired a system which could produce, exchange, and distribute the national income in an effective manner. As he saw it, the necessary conditions were comprised in the operation of free, competitive capitalism. At the present time a commonly accepted goal of economic policy is the maintenance of full employment without inflation. Here it is assumed that full employment may coexist with an inflationary situation which should be checked. To achieve this goal various actions binding on the monetary and fiscal authorities have been proposed. In the latter part of this chapter the means for obtaining this objective will be given study.

Ethics may be defined as the study of the "ideal ends of human action" (Webster). In an ethical system certain goals are ruled out, while others are given the stamp of approval. In a way corresponding to the ethics of individual behavior there are standards against which the economic system may be measured. Such a group of ethical standards or ideals forms the basis for every system of economic policy. In the preceding paragraph we named as an ideal the attainment of full employment without inflation. Another goal is the attainment of the greatest rate of economic growth compatible with a satisfactory level of consumption and a stable price level. Perhaps these two ideal ends are the ones most closely related to the questions of policy relevant in this text.

As a rule economists steer clear of ethical discussions. Possibly his attitude arises because economics is a social science and its oals are therefore social in character. In turn, the formation of ocial goals must arise from a consensus among those who determine the course of events. Since the economist cannot determine uch a consensus, he is likely to be wary of explicit discussion of such matters. In what follows this question will be touched on briefly.

Economic policy and the Keynesian system

In a period marked by its high level of economic activity, the greatest unsolved economic problem of the hour lies in an obstinate inflation. With the need for sustained economic growth to compete with the socialist countries has come an increased temptation to slide into an inflationary situation.

Note that a broad difference exists between the underemployment problem of the 1930's and the inflation-growth problem of today. Underemployment is a condition, while inflation and growth are processes; the first problem is static, the second dynamic. As we have developed our subject to this point, the analytical tools provided have been essentially static. Let us continue and extend our study of underemployment, since it lies within the limits of our present capabilities.

UNDEREMPLOYMENT EQUILIBRIUM

Let us recall some of the analysis we have used to analyze income and employment levels. In Figure 38 we find the economy to be in a state of underemployment equilibrium at income OA, defined by the equality of total demand $(C + I)$ and total supply (Y). A movement of the economy to full employment income OF will cause an excess supply TR to develop. As a result production and income will fall until the income level OA is reached. In other words, a stable equilibrium of the economy is reached at a level of income less than that which corresponds to full employment.

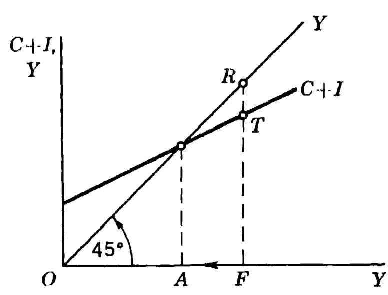


Figure 38. Equilibrium with Underemployment Income.

If the monetary factor is introduced, the conclusions are the same. In Figure 39 the "natural rate" which equates I and S is equal to the "money rate" which equates the demand for and supply of money at income OA. Since

income OA is less than full employment income OF, the economy is in underemployment equilibrium. Moreover, the equilibrium is stable. At full employment income OF the money rate is Fh, while the natural rate is Fg, so that the money rate exceeds the natural rate by gh. In this situation it pays income recipients to direct the flow of saving into paying off bank debts or adding to bank balances instead of investing in capital goods. Since these actions drain money from the income stream and reduce the level of investment, they bring about a reduction in the income level. In fine, income tends to decline from the full employment level OF to the underemployment level OA.

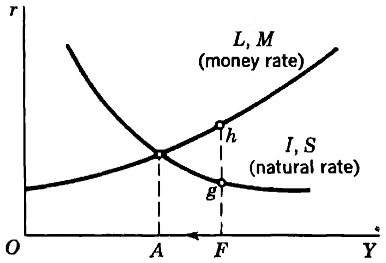


Figure 39. Money Rate, Natural Rate, and Underemployment.

In this analysis we find the key to the problem of unemployment as Keynes saw it. As Figure 38 clearly illustrates, a situation may develop in which aggregate demand $(C + I)$ falls short of that level which will absorb full employment output (Y). Such analysis leads naturally to policy measures based on the manipulation of aggregate demand. Evidently, the policies suggested will have two basic properties: (1) emphasis on demand; (2) emphasis on the aggregate.

SOCIALISM AS A REMEDY FOR UNEMPLOYMENT

In contrast with the implied policy system of Keynesian economics is the alternative of socialism. Under a socialistic system a detailed blueprint for the operation of the entire economy would be required. Such a plan would touch on every individual and firm in the economy. In fact, detailed regulation of the individual and the firm is implied. Rationing of consumer goods, allocation of resources, control of individual prices are all matters for concern under socialism. In Keynesian economics the assumption is made that the price system is an adequate allocative mechanism. Given the level of national income, this system is supposed to provide for its production, exchange, and distribution. In fact, it is presumed

that the price system can perform these functions as well as or better than a socialist system. Under Keynesian policy the vital issues relate to the behavior and control of aggregates, not parts.

Traditional policy and the banking system

From the time of Adam Smith traditional or classical economics proceeded on the assumption that private enterprise was the foundation of the economic system. Pursuant to this belief, government was assigned the role of referee in the great game of competition. As long as the rules of play were observed, the game proceeded without government interference. True, government was supposed to carry out certain functions such as providing for defense, internal improvements and the like, but these were minor in scope. Since the economy was supposed to operate at full employment owing to the operation of Say's Law, the level of economic activity was not taken to be a matter for public concern. In fact, variability in the price level was commonly regarded as the main unsolved economic problem. Since the price level was thought to depend on the money supply, its regulation devolved on the banking system. For this reason the banking system represented the traditional means of influencing economic behavior.

A RESTATEMENT OF EQUILIBRIUM

What can the monetary and banking system do to relieve underemployment? Let us formulate the problem in Keynesian terms and give the obvious solution. In the graphical analysis we will give shapes to the several curves which are accepted by Keynesian economists as the probable ones. First, let us define the equilibrium at full employment income, a situation which we will assume to prevail at the start. In the process of defining the equilibrium we will generalize slightly the analysis of saving.

At full employment income a certain demand schedule for money is defined. This is called the liquidity preference schedule and gives the relationship between the quantity of money which the public wants to hold and the rate of interest. Taken with the supply schedule of money this determines the interest rate in the money market. In our analysis we have called this the *money rate*.

In the capital market where saving and investment become equal the rate of interest prevailing at the point of equality of these variables is entitled the *natural rate*. On the investment side the natural rate represents the earnings on the last dollar invested, expressed in percentage form, or the marginal efficiency of capital. On the saving side we have assumed so far that the saving at a given income level is independent of the rate of interest. Let us alter this assumption, introducing here a bit of traditional analysis.

According to the theory of interest advanced by Irving Fisher[1] saving is influenced by a preference for present goods or money over goods or money in the future. When a loan is made, the lender gives up a sum of money, say $100, for a promise to return a like sum (at least) in the future. If the lender is *indifferent* between $100 now and $105 a year from now, we say that his (marginal) time preference is $5 or, as a percentage, $5/$100 equals 5 percent. In short, the borrower must throw in $5 interest along with the original $100 to leave the lender as well off as before.

According to Fisher's theory people increase their lending (saving out of income) until marginal time preference (MTP) is equal to the rate of interest. It is assumed that marginal time preference increases with each increase in the quantity lent. Suppose the interest rate is 10 percent. If the MTP on the first $100 is 5 percent, the interest rate more than offsets MTP and the individual finds it worth while to lend. If the second $100 lent is associated with MTP of 8 percent, it is still worth while to save and lend. On the third $100, if the MTP is 10 percent, the interest rate of 10 percent just offsets time preference, and the quantity saved and lent is fixed at $300. In equilibrium the MTP of every individual is equated to the rate of interest; the common MTP of every saver may be called simply the MTP of society.

We may now draw up a schedule of MTP for society at every level of savings. This relationship of MTP to the corresponding level of saving is the traditional supply curve of saving, as a function of the rate of interest. Saving is taken to be functionally dependent on the rate of interest and MTP equals the interest rate at every saving level. In the diagrams to be drawn we will label this relationship the S curve. At high rates of interest the S curve

[1] *The Theory of Interest*, New York, MacMillan, 1930.

is represented as bending backward, owing to what is known in price theory as the "income effect."

A graphical representation of full employment equilibrium is given in Figure 40. Let Y_f stand for full employment income. Since the demand for money, *L*, saving, *S*, and investment, *I*, are all assumed to be influenced by the income level, we insert the

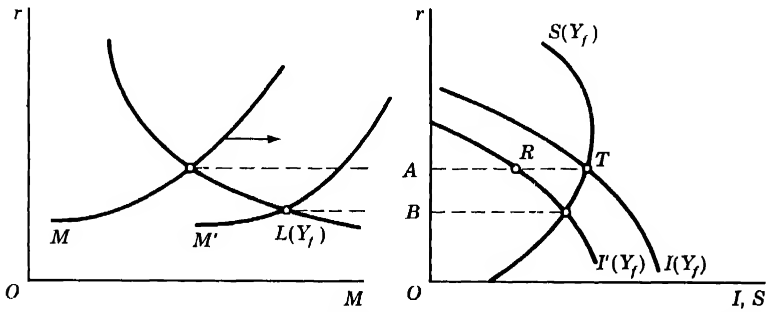


Figure 40. Maintenance of Full Employment by Monetary Manipulation.

notation, Y_f, in parentheses by these letters. This indicates that each of these curves assumes a position determined by the fact that full employment prevails. The *I* curve depicts the schedule of the MEC, whereas the *S* curve depicts the MTP schedule at different levels of saving. Note that the *S* curve turns back on itself at a fairly high rate.

A DECLINE IN THE *I* (MEC) SCHEDULE: TRADITIONAL POLICY

Suppose now that anticipations change and that businessmen expect a lower schedule of returns from investing money. This leads, in turn, to a downward shift in the *I* (MEC) curve. At the rate of interest determined in the money market, *OA*, a margin of saving not filled by investment, equal in size to *RT*, tends to develop. This margin may be entitled a "deflationary gap"; it marks an excess of *S* over *I* or of aggregate supply over aggregate demand. In the absence of some outside intervention the level of National Income will fall.

Suppose the monetary authorities now act to increase the supply of money, moving the M schedule to the right. This action will satisfy liquidity preference to a greater degree than before and reduce the interest rate. This lowering of the interest rate will stimulate investment and check saving without a change in the income level. If the supply of money is increased sufficiently, investment and saving will be equalized at the lower interest rate, OB. In turn, this implies equality of the money and natural rates with no change in the income level. This is illustrated in Figure 40.

In the left-hand diagram the money rate is determined by the intersection of the L and M curves; in the right-hand one the natural rate is determined by the intersection of the I and S curves. When equilibrium prevails, the money rate on the left equals the natural rate on the right. In Figure 40 equilibrium exists when the money and natural rates have reached the common level OB. As long as this condition prevails at full employment no problem arises.

This situation can also be interpreted by the general equilibrium method. Assume that the natural and money rates are initially equal at full employment income. Suppose that the MEC or I schedule now falls. Such an event will lower the natural rate. At the full employment income level the natural rate will have fallen to a level below the money rate. Of course, this implies that money flows from the capital (or goods) market into the money market. When the funds arrive in the money market, they are used to extinguish bank loans or to increase idle balances. As the supply of money is reduced and as more of it becomes idle, the level of income falls. In consequence, income declines below the full employment level. An alternative argument which runs strictly in terms of saving and investment and the multiplier may also be given, but the conclusions are the same.

To forestall these developments the Federal Reserve may take steps to increase the money supply. As this is accomplished the money rate schedule falls. If it is lowered sufficiently, it may cut the natural rate curve (I,S curve) at the original full employment income level. In this way the income level is maintained by

adjusting the money rate to the lower natural rate now prevailing at full employment.

OBSERVATIONS ON THE USE OF TRADITIONAL MONETARY REMEDIES

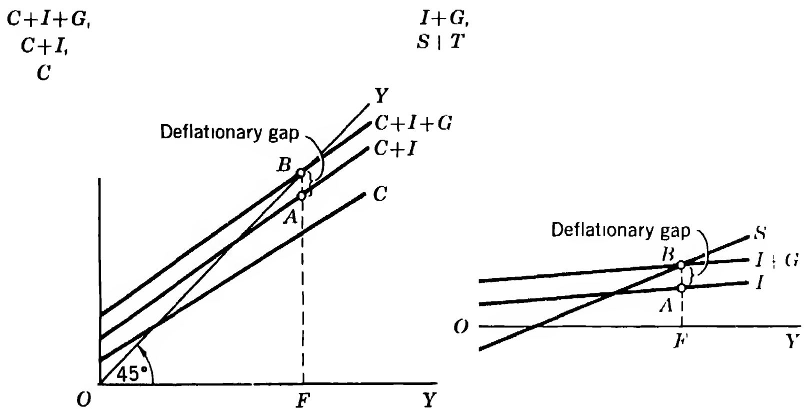
Note that there is nothing automatic about the monetary remedy suggested. As the MEC, or I, curve falls, no unseen force calls into play an increase in the M schedule. In fact, such an increase calls for a deliberate decision on the part of the monetary authorities. Accordingly, a policy of cheap money in depression constitutes a form of planning by the banking authorities to stabilize demand and income.

Let us note also a difficulty in applying the method. In order to bring about an increase in the M schedule in the United States the Federal Reserve authorities would apply their three basic tools. First, they would lower the rediscount rate (or rate on advances) at which member banks may borrow from the Federal Reserve Banks. Second, they may order Government Bonds purchased on the open market. Third, they may lower reserve requirements. The last two actions are designed to increase excess reserves of member banks. In turn, the presence of a given quantity of excess reserves is designed to stimulate greater lending on the part of member banks. However, it is by no means certain just how many additional dollars the member banks will wish to lend on the basis of one additional dollar of reserves. This implies that the Federal Reserve authorities cannot calculate very closely the effect on the M curve of a given change in the three basic determinants.

Filling the gap: a Keynesian approach to policy

The solution we have outlined above is a logical extension of orthodox thinking about the problem of stabilizing income. At the same time it is consistent with the analytical scheme set forth in this text. Yet it does not flow naturally out of the most basic National Income analysis. In fact, the most basic element in the theory is the analysis of income determination by aggregate supply and demand or saving and investment. Consider the problem of income stabilization from this point of view.

In Figure 41 we represent income determination by the total-spending and saving–investment methods, respectively. On the left the C line is drawn up on the assumption that a given tax schedule prevails. At full employment income, $OF = FB$, income or aggregate supply exceeds private demand, $C + I$, by the amount AB; this margin may be called the "deflationary gap." If the



(A) (B)

Figure 41. Filling the Deflationary Gap at Full Employment.

government spends the sum AB in purchasing output, the aggregate demand will be raised to FB which exactly clears from the market aggregate supply of the same amount.

In the right-hand diagram the same result appears with a varying interpretation. At full employment income OF the drain of money from the "income pool" in saving and taxes is FB; the corresponding inflow from private investment is only FA. In the absence of a further inflow more is going to be drained from the income pool than will be returned to it; the excess is measured by distance AB which is the deflationary gap. Consequently, income will fall under these conditions. Governmental expenditure in the amount AB will bring up the inflow to amount FB which will exactly offset the outflow and stabilize the level of income at OF.

DIRECTNESS OF KEYNESIAN APPROACH TO UNDEREMPLOYMENT

The Keynesian tools of analysis point to deficient demand for goods as the cause of underemployment. By lowering the rate of interest it may be possible to increase investment and thereby make up the difference between deficient and full employment demand. However, this policy will fail if interest rate manipulation is ineffective in stimulating investment to the needed extent. On the other hand, if the government fills the gap directly with additional spending, full employment will almost certainly be attained. In fact, the economic policy suggested by the Keynesian analysis is a manipulation of government spending to secure full employment. Such an approach has the force and intellectual appeal carried by a direct attack on the source of a problem.

IS THE KEYNESIAN POLICY SOCIALISTIC?

Under the policy proposed government could purchase that part of full employment output not sold to the private sector. Clearly, the buyer is going to influence the kinds of goods produced so that output purchased by government will have a distinct character. One would expect the items to consist mostly in direct services rendered by individuals, bought by government, and designed to be of use to the public. Among possible examples we may list defense, public health, education, police protection, and like activities. In part, the items bought would consist of goods provided by private industry but having a public use. Among these items are military hardware, highways, and the like. When government buys the unsold part of full-employment output, it will acquire goods and services having a public usefulness. We may call this use of output by society "collective consumption," a process quite different from private consumption.

On the face of it this policy involves the social use of a part of full-employment output. To this extent it is clearly "socialistic." Even as we recognize this, we should take note of two other points. First, the private sector of the economy could muster up no demand for this part of output. By the same token, use of this output for public purposes does not thereby deprive the private sector

of output which it would otherwise have used. Consequently, collective consumption is designed to supplement but not to compete with private demand.

Second, the policy contemplated does not require government production of the margin of output purchased. Insofar as the items purchased consist of material goods it is to be assumed that these would be privately produced. If the government expanded the services it rendered to the public, it would act as employer and thus might be regarded as increasing its share of production. For example, an increase in postal service might be regarded as an increase in government production to satisfy greater collective consumption. In summary, collective consumption does not imply corresponding collective production.

GOVERNMENT SPENDING AND THE INTEREST RATE

In the discussion of filling the gap with government spending we ignored the interest rate. As government spending expands, however, National Income rises through the multiplier. As a result the demand for money goes up and the interest rate rises. If the resulting tight money were to be ignored, investment might fall, and the effect of increased spending would be partly undone. To make sense of this policy we add the stipulation that the monetary authorities expand the money supply as the level of income rises. For maximum simplicity, we assume that the money supply is increased sufficiently to hold the rate of interest constant. As a result of this assumption, the simplest Keynesian policy constitutes a combined fiscal-monetary attack on unemployment. It proceeds by: (1) filling the deflationary gap at full employment with government spending; (2) stabilizing the interest rate, or avoiding the forcing up of interest rates as income is expanded.

THE MULTIPLIER APPROACH

An approach essentially similar to filling the gap is involved in a straightforward application of the government multiplier. Assume that the MPC out of national income is .6 and that the tax structure is kept unchanged. Suppose that the actual income level is 300, while full employment income is 400. In this case $K = 1/(1 - .6) = 2.5$. In order to expand income by 100 it is

necessary to spend an extra 40. This will expand income by 100 and restore full employment.

If the rate of interest is 4 percent at an income level of 300, an increase to 400 will increase the transactions demand for money. In turn, this will raise the rate of interest which will lower investment, an undesired phenomenon. To ward off this contingency we may assume that the quantity of money is increased sufficiently to satisfy the increased transactions demand and stabilize the interest rate.

The multiplier analysis assumes that the MPC is constant over the income range in question. If this is untrue, an error will be introduced into the analysis. When using gap analysis, the investigator uses a C and $C + I$ line which he finds to be realistic. These may or may not be straight lines, characterized by the assumption of a constant MPC. Consequently, a study of the gap does not necessarily involve the pitfall of assuming a constant MPC.

OTHER FISCAL MEASURES: INCOME TAX CUTS

Perhaps the chief objection to increased government expenditure is that it tends to collectivize consumption. By means of tax cuts disposable income may be enlarged with a view to increasing expenditure on consumption. In turn, this change would help to close the gap. For simplicity, consider the effect of a decrease in taxes with the following conditions: (1) the marginal propensity to tax (MPT) is unchanged; (2) the distribution of private income after taxes is unchanged at any given level. To fulfill this condition the tax burden must be fixed so that the distribution of income after taxes is equivalent to that formerly reached from an increase in national income to the required point.

We can trace out the process graphically, with the aid of Figure 42. For simplicity, assume a tax rate of 20 percent and plot a private income line Y_p against National Income, Y. Then Y_p rises at .8 the rate of the Y line. Now let a tax cut be made (or a bounty given) so that at any level of National Income the tax is \$1 less. (The absolute numerical magnitude of the tax change does not affect the solution in a significant way.) In turn, this raises the Y_p line by \$1, shown as FG in the diagram. Before the tax change the private income at G would have been found at H on the old Y_p

line. Assuming the distribution of private income to be the same in the two cases, consumption will rise from R to T, as if caused by a movement from F to H on the old Y_p curve. To represent the rise in consumption we extend a dotted line backward from T to a point U directly above R. The movement RU represents the upward shift in the C line caused by a tax cut.

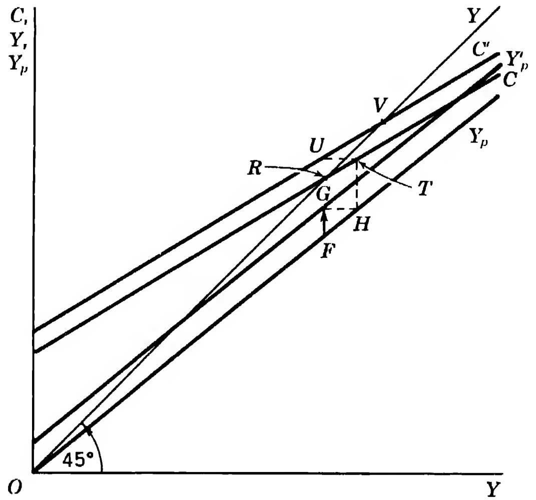


Figure 42. Effect of Tax Cut on Consumption Line.

What is the value of this rise in the C line? Under the assumptions made the initial increase in C is the tax cut, the initial increase in private income, times the MPC out of private income, MPC_p. When the decrease in tax is 1, the initial increase in C is simply $1 \times \text{MPC}_p = \text{MPC}_p$.

Assume for simplicity that I and G are zero. Equilibrium is attained where all goods produced, Y, are taken from the market by consumption demand, C. The original intersection is at point R. As the curve shifts upward from R to U an increase in income takes place, marked by the intersection of the Y line with C' at V.

Consider now the over-all effect of the tax cut on National Income. A tax cut of 1 causes an upward shift in the C line equal to MPC_p, RU in the diagram. Such an excess causes a rise in

National Income by the multiplier. Evidently, the total increase in National Income is $MPC_p \cdot K$, where $K = 1/(1 - MPC)$. Note that MPC is the fraction of an additional dollar consumed out of National Income. It is connected to MPC_p by the formula, discussed in Chapter 4 on income determination, namely:

$$MPC = MPC_p(1 - MPT), \tag{7-1}$$

where MPT is the marginal propensity to tax, in this case .2. To summarize, the total increase in the level of National Income may be expressed by the formula:

$$K^t = \frac{MPC_p}{1 - MPC_p(1 - MPT)} = \frac{MPC_p}{1 - MPC} = MPC_p \cdot K \tag{7-2}$$

K^t being a symbol for the tax-cut multiplier.

In the United States K^t is nearly as large as K, if the tax cut is made on personal incomes alone. We found the short-run MPC out of Disposable Personal Income to be about .86 and the long-run MPC to be about .92. Roughly, we could say that the tax-cut multiplier would amount to about nine-tenths of the government spending multiplier. Of course, if a tax change were to be designed to alter the distribution of Disposable Personal Income as well as its amount, a further effect would be experienced.

To avoid misunderstanding let us note that private income is not the same as Disposable Personal Income. The term "private income" is used to mean National Income less corporation income taxes and personal taxes. Owing to the drain of retained corporate earnings and certain other elements MPC_p is probably less than MPC out of Disposable Personal Income. If the tax cut is made to impinge directly on Disposable Personal Income, as in a reduction of personal tax schedules, then MPC_p is identical with the MPC out of Disposable Personal Income for this case.

For practical purposes a cut in taxes on individual incomes is essentially as effective in the United States as an increase in government spending. Only the fact that avoidance of taxes is possible makes this formula uncertain. Probably a tax cut would be more favorable to those who do not earn high incomes and do not succeed in avoiding taxes than those in the higher brackets. For this reason a tax cut would tend to redistribute income in favor of those with higher MPC's and would further raise the C line.

In order to raise the economy to full employment where a deflationary gap exists it is necessary to lower taxes so that the C line rises enough to fill the gap. To be more precise, the rise in the C line will push the $C + I + G$ line upward to the point where it cuts the Y line at full employment output. This is much the same approach as one which works through increased government spending.

Planning for full employment: the tools applied

With the aid of multiplier formulas we can answer certain elementary questions about fiscal policy. In fact, with the expression for the multiplier given by equation (4–3) (p. 84) we can answer one question immediately. Let us determine that question. By definition, we have:

$$\frac{\Delta Y}{\Delta G} = K,$$

where K is given by (4–3) and is a constant. Cross-multiplying by ΔG, we get $\Delta Y = K\ \Delta G$. Of course, this tells us that the rise in the level of income is the increase in government outlay, ΔG, multiplied by a factor K. However, the appropriate question at this point is somewhat different in character and may be put as follows.

With a given multiplier, K, what is the increase in government spending, ΔG, which will provide a desired increase in income?

In this problem ΔY is given, K is a constant and ΔG is an unknown. To answer this question we rewrite the formula as:

$$\Delta G = \Delta Y/K. \tag{7–3}$$

Suppose ΔY is 90 and represents a desired increase in the level of income to the full employment level. The multiplier K is given, say with the value 3. This being given, we know that $\Delta G = 90/3 = 30$. In short, the quantity of government spending required is seen to be 30.

The question stated above is the simplest of all those which may be asked about fiscal policy. In order to answer this question it is necessary to know the value of K. In turn, this requires that the

tax structure be known, since formula (7–1) reveals that it is necessary to have MPT before the value of K can be determined. Accordingly, the question is susceptible to being answered only after a decision about the tax structure has been made. If the tax structure is not to be altered, K is given and the rule may be applied directly. If the tax structure is yet to be settled, it is impossible to answer our first question immediately. In this case it is necessary to settle on the tax structure.

Evidently, a question prior to the first must be decided which has to do with defraying the cost entailed by the new government spending. This question may be put as follows. *What fraction, f, of the additional expense is to be met by taxation, and what by borrowing?* This is a basic question which must be decided at the outset. It must be settled on the basis of fundamental considerations of economics and politics. Given the value of f, it is possible to determine the increment of government spending ΔG which will provide the desired rise in the level of national income. Let us investigate the mechanics of this relationship.

First, note a very simple relationship between f, MPT and K. By definition we have:

$$f = \frac{\Delta T}{\Delta G} = \frac{\Delta T}{\Delta Y}\frac{\Delta Y}{\Delta G} = \text{MPT} \cdot K. \qquad (7\text{–}4)$$

Here K is given by equation (4–3) (p. 84), with the value of MPC shown in (7–1). Equation (7–4) alone is not useful, since one equation can serve to define but one variable. By substituting the value of K into the equation we can obtain the value of MPT in terms of f and MPC_p. Since MPC_p is taken to be a constant and f is a quantity whose value is to be determined from the outside, we have one unknown, MPT, defined in terms of two given quantities. This relationship is given by the following equation:

$$\text{MPT} = \frac{f(1 - \text{MPC}_p)}{1 - f(\text{MPC}_p)}\,. \qquad (7\text{–}5)$$

Actually, we are not interested in MPT for its own sake, but only in ΔG, the increase in government spending required to raise the level of income the desired amount. Consequently, we take the value of MPT, substitute it in the value for K, and substitute the

value of K into the expression for ΔG, giving us an expression for ΔG in terms of ΔY, MPC_p, and f

$$\Delta G = \Delta Y\left\{1 - MPC_p\left[1 - \frac{f(1 - MPC_p)}{1 - fMPC_p}\right]\right\} \qquad (7\text{–}6)$$

The expression in the outside brackets is $1/K$; it is the reciprocal of the multiplier.

By giving various values to f we can find corresponding values of ΔG which will afford a given rise in the level of income, ΔY. By considering the following table we can verify the sense of this relationship. If the Treasury and Congress decide that none of the

TABLE 39

ΔG	ΔY	K	MPC_p	MPT	
25	100	4	.75	0	0
$33\frac{1}{3}$	100	3	.75	$\frac{1}{9}$	$\frac{1}{3}$
50	100	2	.75	$\frac{1}{3}$	$\frac{2}{3}$
$62\frac{1}{2}$	100	1.6	.75	$\frac{1}{2}$	.8
100	100	1	.75	1	1

additional expense is to be raised by taxation, MPT is obviously going to be zero. Then we make an application of the simple multiplier formula using the value of MPC_p, since we do not have to worry about taxes. With an MPC_p of $\frac{3}{4}$ the multiplier turns out to be 4. Since income is to be raised by 100, this entails additional spending of 25.

If the Treasury secures support for a policy which covers all the additional spending with taxes, f equals 1. From formula (7–5) we see that MPT must be 1. In turn, from the formula for the multiplier K is also equal to 1. This can also be seen by noting that the expression in outside brackets in equation (7–6) which is $1/K$ is equal to 1. We can arrive at the same result by a verbal argument. Suppose the government spends an additional dollar on goods. This generates an additional dollar of income in the affected industries. As long as the additional money is being spent the additional production (income) will be forthcoming. However, the government decides to tax all *additional* income at the rate of 100 percent.

Consequently, the extra dollar is taken from the individual as taxes, leaving his personal income the same as before the extra government spending and income. As a result, he will spend no additional money on consumption. Consequently, there is no further spending induced by the initial amount provided by the government. Evidently, the multiplier will be exactly equal to 1. In this situation there is no leverage, and the government must spend an extra sum exactly equal to the desired rise in the income level. Thus ΔG and ΔY have the common value 100 in the table.

The intermediate values of f give rise to other values of ΔG which will afford the desired rise in the level of National Income. When we plot ΔG against f we get the curve shown in Figure 43. From this figure we can find the amount of additional spending, ΔG, required to raise National Income a given amount, ΔY. This is the essence of the problem of fiscal policy in simple terms. Note that, by definition, $f \cdot \Delta G = \Delta T$, where ΔT is the increment of taxes. It is, accordingly, easy to find the combinations of ΔG and ΔT which will afford a given rise in the level of income. This relationship could be plotted instead of the one shown, but the basic information is the same.

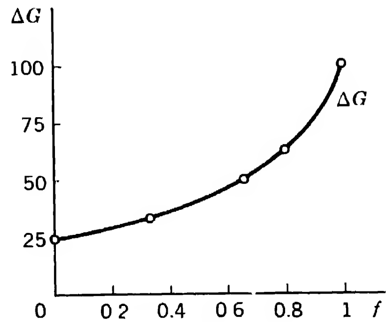


Figure 43. Required Government Spending [illegible] Proportion of Income Taxed.

This discussion illustrates clearly the distinction between economics as science and as art. As a matter of science we ask: "With given values of MPC and MPT what is the effect on income of an increase of 1 in government spending?" The answer is the government multiplier, K. In fiscal policy a different sort of question arises, to wit: "What amount of government spending will be required to raise income by a given amount, given the fraction of the additional spending to be raised through taxation?"

In the case of economic policy we first determine the objective. Then we manipulate the various means necessary to reach the

objective. Here the objective is a given rise in National Income, whereas the means consists of additional spending, some part of which is covered by taxation. Hence the decision as to the fraction of spending to be raised by taxation is the first one. By a manipulation of the multiplier analysis we make the required spending an outcome of the specified income increase, f and MPC_p.

In effect, the economic policy approach reverses the direction of the causal relation traced out by the multiplier. There the line of causality goes from an arbitrary increase in government spending to a resulting rise in National Income. On the contrary, the economic policy approach makes the direction run from a desired increase in income to the increase in government spending which will induce this desired change. The main purpose of this discussion is to make clear that it is necessary to reverse directions to operate in the realm of economic policy.

Interest rate manipulation and government spending compared

In our review of the subject we found that an increase in the money supply and a reduction in interest rates will sometimes restore a depressed economy to full employment. Likewise, we found that fiscal remedies such as increased government spending or reduced taxes will serve the same purpose. Which of these two approaches is to be preferred?

We have already stated that the basic cause of underemployment is deficient demand for goods. In turn, the most direct attack on the problem is increased government expenditure on output; reduced personal income taxes serve as a somewhat less direct attack on the problem. Since the monetary remedy operates through the interest rate, it constitutes an indirect and somewhat uncertain approach.

On the other side of the matter, fiscal remedies are both more expensive and more difficult to initiate. They involve the taxpayer's pocketbook and may be hard to control. Furthermore, the governmental expenditure approach involves increasing collective consumption, though without encroaching on private consumption. These points might be regarded by some as sufficiently

weighty to offset the advantages of the more direct fiscal approach. However, the case against exclusive reliance on monetary measures is based partly on a further point. Monetary measures alone may not bring an economy out of a deep slump. Let us consider this point.

MONETARY REMEDIES IN DEEP DEPRESSIONS

In the following discussion the shapes of the curves to be pictured and the institutional considerations introduced reflect mostly the Keynesian viewpoint. Nevertheless, they seem realistic and are probably the most generally acceptable ones.

Suppose that a sharp decline of anticipations takes place, causing a marked downward shift in the *I*, or MEC, curve. Such an event sets up a deflationary gap between *I* and *S* which will lead to a decline in the level of income unless action is taken. At this point the Board of Governors takes steps to loosen credit by lowering the rediscount rate and freeing reserves. This will make it possible to satisfy liquidity preference more fully and will lead to a lower rate of interest. In turn, this will increase the amount invested and (possibly) reduce saving a little. However, these actions may not suffice to close the gap between *I* and *S* for several reasons.

First, consider the relations of investment and saving to the rate of interest. Saving is thought nowadays to bear a rather weak relation to the rate of interest. If the rate of interest were lowered, there is little reason to believe that saving would be strongly affected. To a lesser extent this is thought to be true of investment in the short run; the MEC curve is believed to be fairly inelastic with respect to changes in the rate of interest. Consequently, after investment had declined sharply, owing to a change in anticipations, it is unlikely that the decision to invest could be completely restored by a decline in the rate of interest. To sum up, the rate of interest has insufficient influence on saving and investment to restore equality once a large gap develops. This is the belief of those who follow Keynes' line of thought.

A second consideration lies in limits to the rate of interest. In the chapter dealing with interest it was pointed out that banks, like all firms, have a need to cover variable cost. If the price (rate of interest) drops to a low enough point, it will no longer be possible

for the firm to cover out-of-pocket costs. While additional considerations of liquidity preference or risk enter into the minimum rate, it seems likely that some such rate exists. Even a flood of excess reserves created by Federal Reserve action will not overcome the condition in question. Hence it seems likely that some minimum to the rate of interest, probably positive, will exist.

Such a minimum would restrict the ability of the monetary authorities to cheapen money and thereby to eliminate the deflationary gap. While institutional changes could be introduced by the government to alter this situation, it would involve a reconstitution of the entire banking and monetary system as we know it. Such a radical move is unnecessary, because fiscal tools are also available in case of a deep depression.

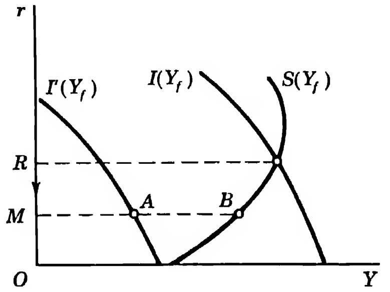


Figure 44. Failure of Monetary Remedies for Depression.

A graphical picture of the situation discussed is given in Figure 44. As the I curve shifts downward and to the left, a gap between I and S develops. By monetary action the rate of interest may be pushed downward from OR toward the minimum OM. However, at the institutional minimum OM a deflationary gap AB between I and S exists. As a result, income falls below the full employment level and the control scheme fails. This indicates the need for some alternative action to remedy the situation. Obviously, one such alternative consists in the use of appropriate fiscal measures.

INDEPENDENT TREASURY ACTION

When the Treasury seeks to fill the gap between full employment production (income) and demand, it must find the financing to make this possible. As it does so one of two things will happen: (1) the Federal Reserve authorities will go along with the policy and create enough new money to satisfy the need and hold the interest rate constant; (2) the Federal Reserve authorities will not

take such action, and the increased need of the Treasury and others at the higher income level prevailing will force up the rate of interest. As a general rule the two institutions are likely to take a common view of the unemployment problem and act in a concerted manner to ease the situation. However, it is possible that the Treasury will find itself alone in seeking a remedy, leading to a different sort of problem.

In verbal terms the analysis of the second case is rather simple. Suppose the economy is in underemployment equilibrium. If the economy moves all the way to full employment income, the demand for money will increase and thereby raise the rate of interest. In turn, a rise to the full employment "money rate" will lower the sum businessmen wish to invest. Clearly, this implies that the deflationary gap is larger than it was at the lower interest rate. Failing the cooperation of the Federal Reserve, the Treasury will be faced with an even larger gap which it must seek to fill with government spending.

What forces up the rate of interest initially is the demand by government for additional cash to finance the extra expenditures. As governmental expenditures get under way, the level of income rises, and the transactions demand for money is pushed up. This occasions a further rise in the interest rate. As a result of the higher rate investment is somewhat reduced and government expenditures must be increased to offset this.

In Figure 45 the graphical interpretation of the situation is shown. In the original situation the $S + T$ and $I + G$ curves cross to provide an equilibrium income level OR, less than full employment income OF. At income OR the money rate is 4 percent, as seen by extending the dashed line down to the L, M line and over to the interest axis. At full employment income, however, the rate is 6 percent, as seen by dropping a line down from F to the L, M line and over to the interest axis. Evidently, the Treasury is going to have to cope with this rise in the interest rate.

As the interest rate is raised from 4 percent to 6 percent the level of investment falls, G remaining constant, and the $I + G$ line shifts downward to the lower position indicated by the dashed line. At full employment income OF the gap between $I + G$ and $S + T$ is AB. This deflationary gap must be filled, if full employment is to

prevail. The government then increases spending by ΔG to fill the gap and assure full employment.

All this is essentially obvious. However, it is well to point out clearly that Treasury and Federal Reserve policies are interrelated.

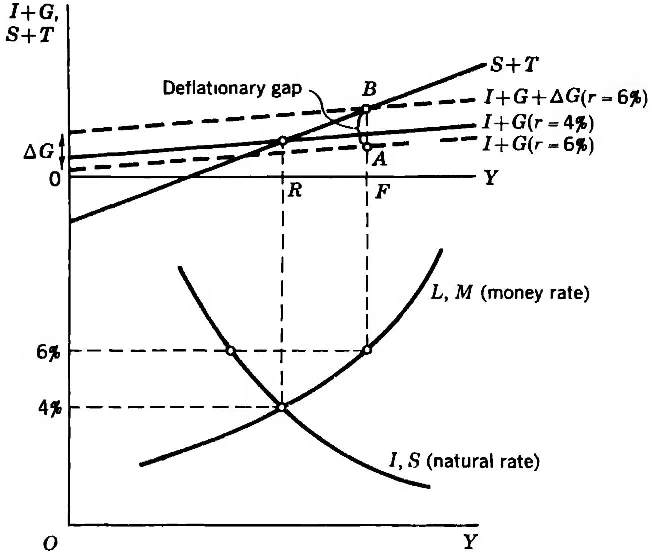


Figure 45. Filling Deflationary Gap without Help from Banks.

An independent Treasury policy is harder to carry through than one in which cooperation with the banks is obtained. The same remark holds true for independent action by the Federal Reserve.

Interrelations between fiscal and monetary policy

From the fact that the government spending program depends on monetary factors we see that fiscal and monetary policies are interrelated. Let us develop this relationship in a slightly more formal way. In the first place, the greater the money supply, the lower is the rate of interest and the lower is "full employment government spending." (The latter is the spending necessary to fill the gap at full employment income.) Reasoning

in this way we might relate the money supply inversely to the level of government spending. Let us note why this is unlikely to be helpful.

As we have already noted, the Federal Reserve has the power to vary excess reserves of member banks by changing reserve requirements and engaging in open market operations. If the member banks expanded loans and deposits in some predictable fashion as a result, the outcome of the process could be predicted. However, we cannot be sure that banks are going to be willing to create some definite quantity of money as a result of having given excess reserves. In short, it is not certain what quantity of money banks are willing to create as a final result of the actions taken. For this reason the quantity of money does not make a good independent variable for policy purposes.

Consider the rate of interest as an alternative. First, note the theoretical relationship of the interest rate to National Income. If the interest rate is raised, the quantity of investment will be reduced, thus increasing the spending which government must undertake to maintain full employment income. This relationship is short and direct.

Is the rate of interest (the money rate) subject to manipulation by the Federal Reserve authorities? In normal circumstances the answer is "yes." If the Federal Reserve wishes to lower the rate, it may lower the rate on rediscounts and advances. This acts as a signal to the banks that any needed action will be taken to bring about the desired rate. To back up its action the Federal Reserve can cut reserve requirements and buy government bonds, actions which serve to increase excess reserves. If these actions are pushed far enough, the banks can usually be pressed into lowering their rates to a point slightly above the rediscount rate.

In general, the Federal Reserve is more effective in raising than in lowering interest rates. It is doubtful whether the rate can be pushed below some positive minimum, at any given time, by enlarging excess reserves of member banks. However, the Federal Reserve can be very effective in raising rates (or causing a refusal of credit) by reversing the aforementioned actions. Although some limitation on the power to control the rate is present, the Federal Reserve

does possess this power within a fairly wide range. In consequence, the rate of interest can be used directly as a variable of policy.

Let us consider the combination of interest rate and spending policies which will ensure full employment income. First, in Figure 46(A) draw up the investment curve, labeled I (MEC) which is found at full employment income and varying interest rates. Next, assuming a given structure of tax rates, draw up a

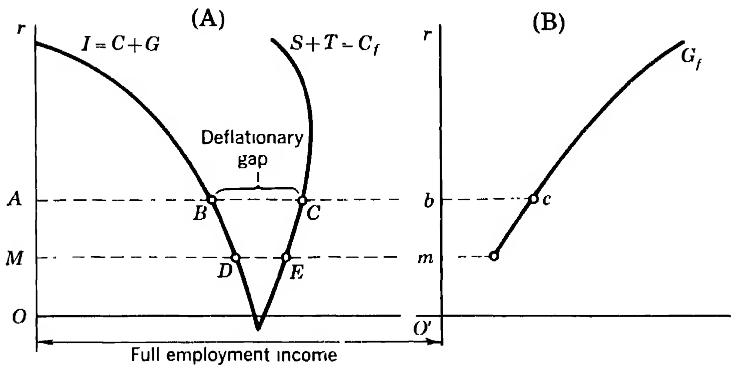


Figure 46. Full Employment—Government Spending Curve.

saving plus taxes curve, labeled $S + T$, which is found at full employment income and varying interest rates. Then mark off the institutional minimum at M below which the interest rate cannot be pressed by Federal Reserve action. Let OO' represent full employment income.

These curves are shown in the left-hand diagram. At an interest rate OA investment at full employment income is AB, while saving and taxes amount to AC, leaving a deflationary gap BC. To sustain full employment income with no change in other variables the government must spend a total of BC. In Figure 46(B) we lay this distance off as $bc = BC$. Point c represents the government spending necessary to sustain full employment when the interest rate is OA in the left-hand diagram. By plotting the deflationary gap at each interest rate we arrive at a curve which we may label G_f for "government spending required to sustain full employment

income." This curve gives the desired relationship between government spending and the interest rate.

Several comments are in order. First, the curve does not extend below the institutional minimum on the interest rate. Since the rate cannot be lowered below this figure, it is pointless to consider lower values. Second, note that the deflationary gap is large in comparison to investment and that the I and $S + T$ curves intersect only at a negative rate. If full employment is to be sustained, some government spending is required. Last, consider the slope of the curve. If this curve is steep, it indicates that a slight decrease in government spending can only be offset by a sharp reduction in the interest rate. If the G_f curve is rather flat, a reduction in government spending can be offset by a very slight reduction in the rate of interest. Clearly, the slope of the G_f curve is highly important in making a choice of policy.

According to one's beliefs about the slope of the G_f curve, one will have corresponding views on policy. Most followers of Keynes believe the curve to have a rather steep slope. If this be true, maintenance of full employment demands a level of government spending falling within a narrow range. By the same token this belief leads one to lay heavy stress on the importance of choosing the right amount of government spending. In short, the Keynesian puts his faith mainly in government spending as the factor which assures full employment. If the G_f curve is believed to be rather flat, it is logical to rely more on the interest rate. Only by a proper choice of the interest rate can full employment be assured in this case, whereas government spending can be varied within wide limits.

FISCAL POLICY AND TAXES

In the above argument taxes are held constant. With a given tax structure, the tax take at full employment income will be fixed. Government spending was taken to be the variable element. This treatment may be altered by holding government spending constant and varying taxes. Without going into details we may sketch the necessary technique.

First, in Figure 47(A) draw in a saving curve at various rates of interest. Next, assuming government spending on output to be

constant, draw up an $I + G$ curve at various rates of interest.[2] At interest rate OA, $I + G$ exceeds S by BC, implying the presence of an inflationary gap. Lay off $bc = BC$ in Figure 47(B). Point c designates the taxes necessary to avoid inflation (or deflation) at the given interest rate. Tracing out similar points, we arrive at the full employment tax curve, T_f. Let OO' represent full employment income.

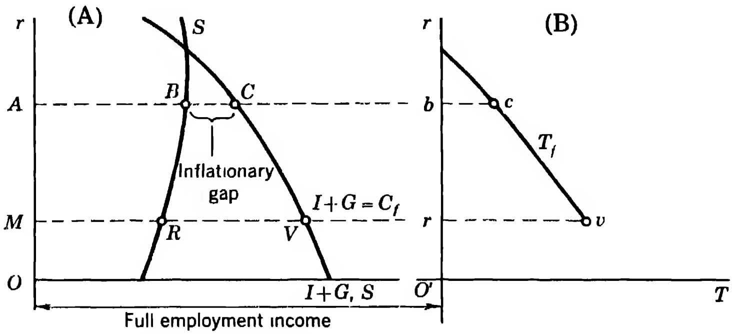


Figure 47. Full Employment Taxing Curve.

Let us note that raising the rate of interest above some fixed level may arouse opposition from the Treasury. With a very large national debt a rise in the interest rate will cause a large increase in the service on the debt. In turn, this would present to the Treasury the unpleasant alternatives of: (1) asking for increased appropriations; (2) cutting down on existing governmental services or purchases. By political pressure or the printing press the Treasury may be able to set an upper limit to the interest rate. In turn, this will set an upper limit to the T_f and G_f curves. Also it will restrict further the use of monetary measures to stabilize income at the full employment level.

THE CHARACTER OF THE ECONOMY: INVESTMENT VERSUS GOVERNMENT SERVICES

In Figures 46 and 47 let the distance OO' equal full employment income. Since $Y = C + S + T$, the distance from the $S + T$ curve

[2] Interest charges paid by the government on the national debt increase with the rate of interest. In turn, the payment of these sums constitutes an income transfer

in Figure 46 to the second vertical axis erected from O' equals C. (Let us denote full employment income and consumption out of this income by the letters Y_f and C_f, respectively.) In the diagram we have $C_f = Y_f - (S + T)$; at interest rate OA the value of C_f is Cb, while at the institutional minimum it is Em.

As we have drawn the $S + T$ curve, it is not very responsive to changes in the interest rate. As a first approximation it is normal in Keynesian economics to assume that this curve is a vertical straight line. This means that the fraction of income consumed will not be altered much by variations in government spending compensated by corresponding changes in the rate of interest. What is changed more significantly, we assume, is the relationship between government spending and investment.

At high interest rates investment is relatively low. To provide full employment with given taxes it is necessary to fill the gap with more government spending. Obviously, then, a high interest rate–large government spending combination gives an economy with low investment and a great deal of governmental services. On the other hand, a low interest rate-small government spending combination gives an economy with a good deal of investment and a minimum of government services. Moreover, the fraction of income consumed is not likely to be affected in a significant way.

THE CHARACTER OF THE ECONOMY: INVESTMENT VERSUS CONSUMPTION

In Figure 47 also the distance OO' represents full employment income. Since $C + I + G = Y$, $C_f = Y_f - (I + G)$. In turn, C_f is measured by the distance Cb from the $I + G = C_f$ curve to the vertical axis of the right-hand figure at interest rate OA. Consequently, we have labeled the $I + G$ curve C_f, the values being measured to the left from the vertical axis above O'.

At low interest rates investment is rather large so that $I + G$ exceeds S by a substantial amount. The inflationary gap can only be filled by larger taxes. In turn, the larger fraction of income devoted to I and G implies less C. Keep in mind that G is taken as

from the taxpayers as a body to bondholders. We assume here that this transfer is not such as to change the level of saving. However, this can be allowed for in the shape of the S curve and need not affect the conclusions.

constant (except for variations in transfer payments necessitated by changing interest rates). Consequently, it is I which increases at low interest rates to offset the decline in C, subject to the exception noted.

A higher interest rate which reduces I and $I + G$ narrows the inflationary gap, permits a reduction in T and an expansion of C. These results can be read from the figure. Consequently, a low interest rate–high tax combination gives an economy with high investment and low consumption, whereas a high interest rate–low tax combination gives an economy with low investment and high consumption.

It goes without saying that an economy with high investment will tend to be progressive. In the last case under analysis it was seen that this investment displaces consumption. Obviously, a society may choose how progressive it wants to be, within limits. But progressiveness and high consumption compete with one another to some extent.

CHAPTER 8

Economic policy—more advanced techniques

Spending and taxing

At one time it was believed that only that government spending which resulted in a deficit could raise the income level. As a result of further thinking economists began to realise that additional spending, with corresponding taxes, could also raise income. In our analysis of the multiplier we gave one approach to this subject. Let us now consider the alternative taxing and spending policies which will yield full employment with the aid of a different technique. In so doing we will seek to lay out a complete map of the taxing and spending situation, following a figure developed by John G. Gurley.[1]

Let us entitle all income received by individuals *gross private income*. That part of this sum earned as a result of current productive activity is called *National Income* while the unearned part is entitled *transfer payments*. The term National Income is used in the same sense here as elsewhere in the text. The excess of gross private income over taxes is called *private income*.

In Figure 48 lay off from point O a horizontal distance OE to represent full employment National Income. From O lay off to the left the distance $O'O$ to represent transfer payments. From E lay off to the left the distance AE to represent (income) taxes. Then $O'A$ (that is, $O'E - AE$), the excess of gross private income ($O'E$) over taxes (AE), is private income. Complete the square on the base $O'E$ and draw a 45° line connecting the lower left and upper right corners.

Next draw a total private spending line, $C + I$, as a function of private income. Next lay off distance EN equals OE along the farther vertical side of the square and complete the smaller square $OENM$. The length of any side of this square gives full

[1] Deficits, Surpluses, and National Income," *Southern Economic Journal*, XXI, No. 1, July 1954, pp. 15–24.

employment National Income. Note also that $O'O = NO'' = KH$ which measures transfers.

With private income $O'A$ the level of private spending is AD. Since $OE = EN = AK$, the part of total output (National Income) not taken off the market by private spending is DK. If this income is to be sustained, government spending on output must equal DK

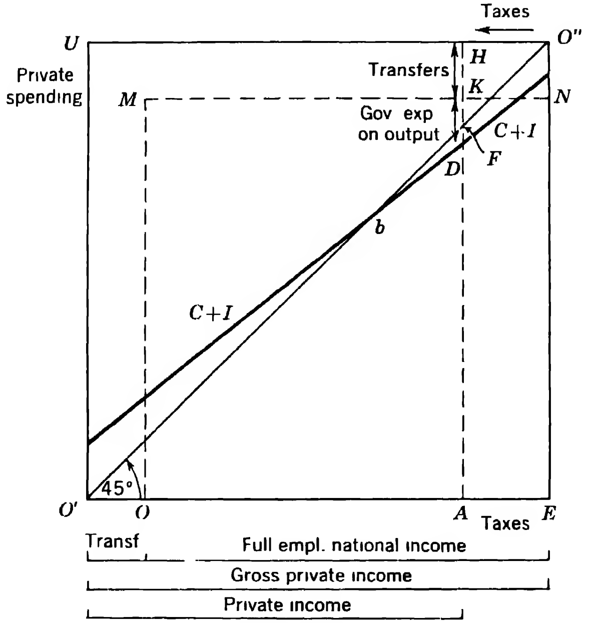


Figure 48. Full Employment Income Box.

to take off output of this amount. Note also that $AE = O''H = HF$ measures the level of taxes. Also, since DK equals government spending on output and HK equals transfers, HD represents total bugetary expenditures. The excess of budget expenditures over taxes, $HD - HF$, is the budgetary deficit and is measured by FD.

THE FIGURE LOOKED AT FROM THE GOVERNMENT VIEWPOINT

Note that point A is arbitrary and that National Income may be divided in any proportion between private income and taxes,

provided the right spending and taxing policy is followed. What we need to do is get at this matter from the viewpoint of fiscal policy. To get this viewpoint rotate the box 180° so that O'' comes down into the lower left corner, as in Figure 49.

The B line defines the budget-tax combinations necessary to sustain full employment and is the same thing as the $C + I$ line, except that the B line is oriented from O'' instead of O or O'. The T line

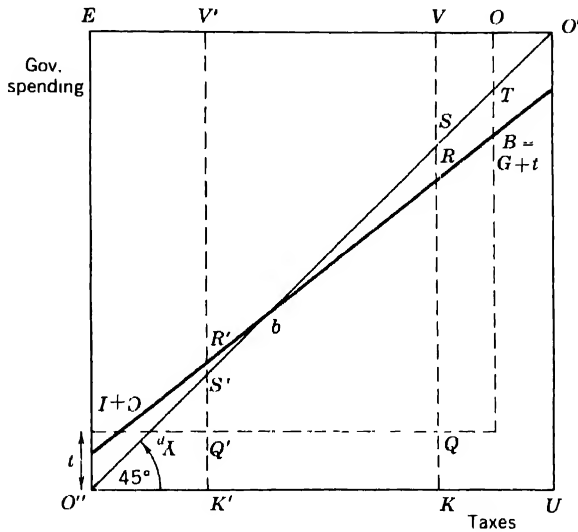


Figure 49. Government Spending in Relation to Taxes.

is simply the 45° line which serves to convert a given horizontal distance to a like vertical distance, thus permitting taxes to be measured with reference to the vertical axis. This permits taxes to be compared with budgetary spending as registered in the line B. Comparing the T and B lines, we find that the vertical difference between them, distance RS, represents the surplus with the budget-tax combination shown at R.

PROPERTIES OF THE FIGURE

Consider the properties embodied in Figure 49. First, note the construction of the budget line. It consists by assumption of a

constant amount of transfers, t, and a variable amount of government purchases of output, G, necessary to fill the gap between full employment output and private spending. G increases with taxes and the corresponding fall in private income. In fact, the slope of the budget line is the MPX out of private income—MPC if investment is unaffected by a change in private income. The geometrical proof is based on the fact that the $C + I$ line (the B line upside down) makes equal angles with either the tax or private income axis. The common sense of this relation may be explained as follows.

Assume, for simplicity, that investment does not vary with private income. An increase in private income will be distributed partly to individuals and partly to corporations. Of the latter amount a part will be distributed to stockholders and part will be retained by the corporation. Of the part finally received by persons in Disposable Personal Income a part will be consumed based on the MPC out of such income. In any event, a fraction of the increment of private income will be consumed, and we may label this MPC_p. Suppose this fraction is .8. If taxes are reduced by 1 and private income increased by 1, consumption will increase by .8. In turn, this permits government spending to be reduced .8 without lowering total spending. Reversing the direction, an increase of 1 in taxes will lower consumption by $\text{MPC}_p = .8$, requiring an additional .8 in government spending to keep total spending constant. If $\Delta G = .8$ denotes the increase in government spending necessary with an increase in taxes $\Delta T = 1$, then $\Delta G/\Delta T = .8$.

As we have just shown, $\text{MPC}_p = \Delta G/\Delta T$. Let us entitle the last ratio the marginal spending requirement out of additional taxes and abbreviate it as MSR. Since the MSR is less than 1, the government surplus tends to increase or the deficit decrease as taxes rise. If $\text{MPC}_p = \text{MSR} = .8$, an increase of 1 in taxes, accompanied by an .8 increase in government spending, will increase the surplus by .2 without changing national income. As the tax level increases, a movement along the B line brings the government a larger surplus. Point b depicts an exactly balanced budget, where the T and B lines cross. However, such equality does *not* represent an equilibrium in the economic sense or a point

toward which the Treasury should move from economic considerations.

FISCAL POLICY AND THE RELATIVE SHARES OF OUTPUT

By choosing alternative combinations of taxes and government spending found on the B line the Treasury can change the mix of private and public output. Recall that in Figure 49 the budget expenditure line B measures budget expenditures from $O''U$, but private spending, $C + I$, from $O'V'$. At the high budget-tax level at R on the B line in Figure 49, taxes equal KS, whereas budget spending is KR, providing a surplus of RS. A much more important consideration is the relative share of output taken by the two parties, government and the private sector. National income is $QV = Q'V'$, the width of the inner square, excluding double counting from transfers. The government share of output is measured by its expenditure on output, QR, KQ of its total budget spending being devoted to transfer payments. The private sector's share is RV, as measured by private expenditure on output, $C + I$. Comparing QR with RV, we see that the government share is quite large—so large that it would characterize only a capitalistic economy in wartime.

At the low budget-tax level at R', taxes equal $K'S'$, while spending is $K'R'$, which implies a deficit of $R'S'$. Now the government share in output is only $Q'R'$, while that of the private economy is $R'V'$. Evidently the government share in output is greatly reduced by a policy of low taxing and spending.

SUMMARY OF FISCAL POLICY ISSUES

There are two extreme policies which may be followed by the Treasury. First, it may run a surplus with heavy taxing and spending, and increase in government's share of output at the expense of that part going into private hands. Second, the government may run a deficit with light taxing and spending, reducing the government's share of output to the advantage of the share going into private hands. All this is pretty elementary, but confusion often arises as to the exact assumptions made about the handling of variables and constants. In consequence, it was deemed desirable to spell out the analysis in detail.

A combined fiscal–monetary analysis

We can now join together the taxing, spending, and interest variables in Figure 50, modifying the figure just used. A reduction in the rate of interest will shift the $C + I$ line away from the private

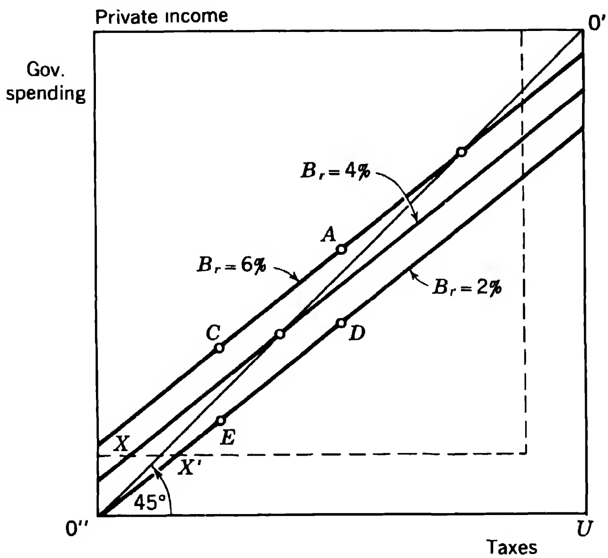


Figure 50. Combined Fiscal-Monetary Analysis.

income axis (top of box) toward the tax axis, $O''U$. This means increased private spending, $C + I$, at every level of private income. Since the $C + I$ line is the budget or B line, referred to the tax axis, the B line shifts downward. This implies that less government spending is required to sustain full employment income, when private spending increases.

We may now draw up a set of B lines (which is also a set of $C + I$ lines) for different interest rates. With the aid of this set of lines we will be able to consider alternative policies combining the considerations of interest rate, budget spending, and taxation. For simplicity we make three assumptions: (1) interest rates do not affect saving; (2) a change in the interest rate produces a uniform

effect on investment at any level of private income; (3) a change in the interest rate causes a negligible change in transfers (interest on the public debt).

In Figure 50 the B lines are labeled according to the interest rates which generate them. Let us assume that budget spending must be sufficient to pay transfers. Accordingly, X and X are end-points of the $B_r = 4$ percent and $B_r = 2$ percent lines. Also consider the fact that a movement along a budget line B is a fiscal adjustment, and involves no change in the interest rate. A movement from one curve to the next in a vertical direction denotes a change in the interest rate and is of monetary origin. Such a vertical movement implies both a rise in the interest rate and an increase in government spending. Note that when one of the three variables—taxes, budget spending, and interest rate—changes, at least one of the others must be altered.

Perhaps the most important question which can be discussed is the mix between consumption, investment, and government shares of output as determined by the relative expenditures. Alternatively, we could discuss the mix between private and government shares of output. Of some interest, also, is the price tag attached to government output in the form of government deficit or surplus.

RELATIVE SHARES OF OUTPUT AND OTHER MATTERS

Consider points A, C, D, and E in Figure 50. Each point depicts a fiscal-monetary policy which affords full employment but with differing relative shares. Point A, for example, is marked by high taxation, high interest rates, and high budget spending. As a result of the high taxes consumption is low, and low investment follows from the high rate of interest. Government's share in output is large, whereas the private share, consisting of consumption and investment, is correspondingly small. Owing to low investment, the private business economy will be progressing slowly. A deficit is present because of the large government spending required to compensate for low consumption and investment.

At point C the interest rate is at the same high level, but taxation has been greatly reduced. As a result consumption has been expanded greatly along with the rise in private income. Since the MSR = MPC is less than 1, spending was reduced by less than taxes. In consequence, a large deficit develops. Consumption is at

a high level; investment is low; the private share of output is moderate; the government share is also moderate.

At point E the interest rate is very low and taxes are also quite low. By virtue of the low interest rate investment is quite high. Consumption is also quite high, owing to the low tax level. Since government spending can be quite small, owing to large private spending, the government experiences a surplus in spite of low taxes. Evidently, the government share of output is going to be small and the private share large.

The conditions prevailing at D may be worked out by the student. He may also work out the complications caused by the effect of a change in the interest rate on transfers.

WEAKNESS OF MONETARY POLICY IN DEPRESSION

Earlier, we pointed out that monetary policy is subject to a weakness in periods of deep depression. In the first place, there are limits below which the banks cannot lower the interest rate and cover costs. For this reason the existence of banks, organized as privately owned, profit-seeking institutions requires that a lower limit be set to the interest rate. Ordinarily, this lower limit will not be the only barrier to full employment. However, a sharp falling off of anticipations, leading to a decline in the investment (MEC) schedule, will focus attention on this matter.

As the Keynesian sees it, investment is likely to be somewhat insensitive to a cut in the interest rate. Saving is regarded as likely to be even less sensitive. The insensitivity of investment to interest rate changes is likely to be especially marked in a depression situation, when expectations are pessimistic. With inherently limited power to equate investment and saving in a depression the interest rate suffers as a tool of policy from the further limitation of the "institutional minimum rate."

Limitations on fiscal policy in boom

When a period of boom is reached, the $C + I$ line is raised and the B line required to sustain full employment is lowered to the same extent. If the Treasury simply holds taxes constant under these conditions, the reduction in required budget spending would tend

to increase the surplus or reduce the deficit. In Figure 51 with the original fiscal situation at A, an upward shift of the $C + I$ line relative to the private income axis lowers the B line relative to the tax axis. In turn, this permits the deficit AR to be replaced by the surplus RD. As an alternative, the Treasury might hold government spending constant and increase taxes rather steeply, as shown by the movement from A to E. Again, a surplus would result with income being held constant at the full employment level.

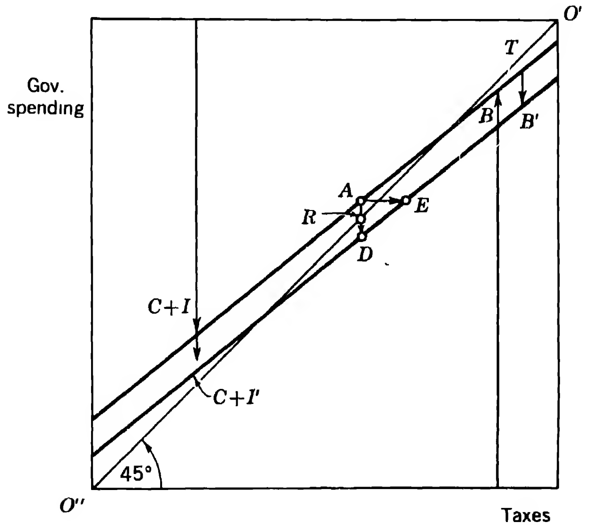


Figure 51. Government Offsets to Increased Private Investment.

Consider the first alternative of reduced spending with the same taxes. If the tax structure is not modified, and the income level remains unchanged, the specification of constant taxes follows automatically. Clearly, the key point is whether government spending can be reduced to the same extent as private spending rises. This is not at all an easy path to follow, for several reasons. In the first place, there is an internal logic and impetus for the growth of all government services. As a rule they represent very worthy aims which compare favorably with those satisfied by the private use of funds. Defense, veterans' benefits, education, highways, public

health—can one gainsay the need for these? Furthermore, groups whose welfare may be affected by a cut in spending are likely to let themselves be heard at such a juncture.

In addition to these considerations the psychology of prosperity is contagious. When private investment and consumption are expanding, a generally confident frame of mind is prevalent in society. At such times a bold scheme for expanded government activity is more likely to win approval than it would be in depression. Moreover, when private businessmen and consumers are expanding private output and consumption, they feel in a weak moral position to resist an expansion of government services, much less to roll back the amount. If private activity can expand, why shouldn't government share in the growth? It is a difficult psychology to combat. More could be said, but we are verging here on the theme of economic growth.

Difficult as it is to cut back government spending in this situation, the alternative of increasing taxes appears even less likely to gain approval. As we pointed out earlier in this text, the main source of fluctuations in spending lies in expenditure on capital goods. If taxes are increased to compensate for a rise in investment, the level of consumptions is being forced down to offset the impact of the first event. This is stiff medicine. Consumers are being made to give up their claim on output whenever businessmen wish to invest more. As you may well imagine, the taxpayer will not take this sort of treatment calmly.

We can see the makings of an impasse in this situation. It is far easier to *talk* about cutting budgetary spending or raising taxes that it is to do it. Indeed, the very appearance of a surplus could be embarassing to the Treasury under certain conditions. The event in question could produce calls for a tax out which would undo the policy needed at such a stage. All this amounts to saying that the power to carry out these fiscal measures may not, in view of the political considerations, be exercised.

AUTONOMOUS CHANGES IN TRANSFER PAYMENTS

Let us consider briefly the effects of an autonomous increase in transfer payments on the budget situation. Assume that the additional transfers are allocated in about the usual proportions

to the several classes or types, such as direct aid payments to the indigent and interest payments on government bonds. In this event MPC_p should have its normal value; we here assume it to be .8. Let the increase in transfers be 10 billion. If taxes are changed, it would appear offhand that the deficit would increase by this amount. On further consideration, however, we note that the increase in private income of 10 billion will generate increased private spending of 8 billion. Government spending on output should now drop by the same amount at this level of taxes to maintain an exact position of full employment. The outcome of the combined actions will be an increase in the deficit of only $2 = 10 - 8 = 10(1 - .8) = \Delta t(1 - MPC_p)$. The last expression represents the upward shift in the *B* line of the diagram used in this section.

In this result two main features are noteworthy. First, a proportionately small deficit results from the extra spending, provided MPC_p is close to 1. The increase in the deficit comes about because part of the transfers paid out are drained off in saving before generating any additional demand for output. In this case only 8 billion is consumed of the extra 10 billion in transfer payments, meaning that government spending on output can be cut 8 billion, 2 billion being required to offset the drain of like amount into saving. Second, 8 billion dollars worth of output is transferred from government use to the consumption requirements of the recipients of increased transfers.

In a situation discussed below a change in transfers is *induced* by Federal Reserve Action. Here the analysis is more complicated.

INTERRELATIONS OF MONETARY AND FISCAL POLICY IN A BOOM

During an upswing of business verging on a boom the investment demand (MEC) curve exhibits an upward shift. In forming an appropriate policy the Federal Reserve will have to take into account a number of considerations. Obviously, a rise in the interest rate is appropriate in order to check investment, when the full employment level is attained. Yet the raising of the interest rate at full employment, the capital stock being approximately constant, involves an increase in the income flowing to capital.

Some part of the extra income will become disposable income to consumers, but the fraction will probably be low, since corporations hold a substantial part of all capital assets. Whatever income is made available will result in extra consumption and cause some upward shift in the consumption line.

If the Federal Reserve is doing its job well, it will take into account this indirect effect of increasing the interest rate. Assume that the Federal Reserve raises the interest rate enough to keep private spending from rising. In turn, this action will stabilize money income and ward off inflation. As it carries out its program the Federal Reserve precipitates an economic change of some importance.

If money income is stabilized at full employment by raising interest rates, a transfer problem results. Most generally, this action involves a transfer of income from noncapitalists to capitalists. In short, wages and/or rent are squeezed in order to pay more interest out of a fixed money income. Under the circumstances, however, this outcome is neither surprising nor wholly undesirable. During a boom, society has both the opportunity and the wish to progress by means of a rapid accumulation of capital. At such a juncture the most needed factor of production is capital, and it is natural that its return be raised, even at the expense of other factors. Undoubtedly, a question of values in the distribution of income is involved. However, if the value of accumulating capital is conceded, then the necessity of a higher interest rate follows readily.

A more important consequence of higher interest rates from the present viewpoint is the resulting fiscal problem. Suppose the original budget is 80 billion, the national debt is 300 billion, and the interest rate is 3 percent, so that the service on the debt is 9 billion. Let the budget be divided into 71 billion spent on output and 9 billion on transfers which are made up by the interest charge.

When the interest rate is raised, say to $6\frac{1}{3}$ percent, the service on the debt rises to 19 billion, an increase of 10 billion. We assumed above that the Federal Reserve calculates the adjustment in the interest rate with a view to stabilizing total expenditure on output. If it assumes government expenditure on output to be constant

and stabilizes private spending, then the outlay of 71 billion on output by government is still required to sustain full employment. With the rise in transfers (interest payments) to 19 billion the budget will go up to 90 billion, and an increase of 10 billion in the deficit will occur. It is clear that this is not a happy result from the viewpoint of the Treasury.

Suppose that the Federal Reserve with this possibility in mind consults with the Treasury before taking action. Both agree on the necessity of increased interest rates to check investment. In this case, however, the Federal Reserve aims at allowing an increase in private spending. Suppose the interest rate is raised only to 5 percent, a result which permits an increase in investment amounting to (say) 6 billions. In this event the Treasury can cut government spending on output by a like amount. This will suffice to pay for the 6 billion rise in the interest charges from 9 to 15 billion. Under these conditions the budget remains unchanged at 80 billion with no change in the deficit. However, a cut of 6 billion in government expenditure on output is involved. This implies a reduction either in direct government services or in government purchases of private output. In either event a major reappraisal of the budget is required.

In summary, a rise in the interest rate is going to pose problems for the Treasury. If total private spending is stabilized, the budget deficit is going to have to be substantially increased. The political repercussions of this may be left to the reader's imagination. If private spending is allowed to increase, so that government spending on output may be reduced, the size of total spending and the budget deficit may be held constant. However, the beneficiaries of government spending will be charged to a marked extent. "Free" government services must be contracted. Moreover, suppliers of output for government use are in a poorer position relative to those who produce for private purchase. Of course, the government bondholder, like all recipients of interest, rides high. So a considerable redistribution of government benefits ensues from the second course of action. Again, considerable dissatisfaction may result from this policy. Evidently, a restrictive monetary policy carried through by the Federal Reserve will precipitate significant budgetary problems.

THE DIFFERENTIAL IMPACT OF MONETARY POLICY

During prosperity it is good monetary policy to raise the rate of interest. Such a policy is designed to make it hard for borrowers generally to obtain funds. In fact, however, the impact is not evenly distributed throughout the economy. As the figures in Chapter 2 indicate, corporations are rather large savers. As such, they have two characteristics. First, they can tap the flow of income directly through the significant income share which flows into corporate profits. Moreover, the use of part of this share in saving and investing is not likely to be affected greatly by a high external rate of interest. Second, they can and do accumulate significant precautionary balances, not highly related to interest. When the investment opportunity arises, corporations can draw on these surplus balances. To sum up, large corporations are insulated to a considerable extent from changes in the rate of interest.

Relatively small, unincorporated enterprises are in a different position. Unable to accumulate large cash resources, and without large business profits to draw upon, such firms rely to a considerable extent on bank credit. When interest rates are raised, they bear the impact directly. Furthermore, in case of an application for a loan in times of tight money, the small businessman is much more likely to meet with a refusal than his corporate counterpart. For this reason a boosting of interest rates raises broad social issues ranging well beyond the conventional economic ones.

Fiscal policy in depression

To provide stimulation to the economy in a depression the Treasury may raise spending on output or lower taxes. Cutting taxes on personal incomes is likely to have a multiplier effect nearly as large as increasing government spending. Cutting all income taxes, including corporate, would have a smaller leverage, because part of the taxes remitted to corporations would be retained by them and thus drained from the income stream. The most powerful fiscal tool is direct spending by government on public works or other projects. Let us consider two of the difficulties involved in implementing such an approach.

First, the timing of public works is a considerable problem. Most of the projects which receive public approval require considerable time to set in motion. For instance, increased spending on highways cannot be effected overnight. A planning period is necessary before spending can begin. Yet the need may manifest itself in a rather sudden and unexpected way that precludes advance planning. Thus there is a lag in the spending. By the time spending begins, the recovery phase may be under way and the program may accelerate it to an unwanted extent. A similar lag analysis applies to the boom phase.

A second difficulty lies in the impact of the spending program. For purposes of restoring morale the initial impact of public spending in depression should favor the areas of greatest unemployment. Yet this is not necessarily feasible, since the affected area may lie outside the field of government as a producer or consumer. If the automobile industry is suffering the greatest unemployment, a public works program will not relieve its workers directly. Only when secondary spending commences will these workers come to be employed. Meanwhile government spending in the generally approved directions will draw marked criticism, because it misses the mark. Such criticism is hard to answer.

FISCAL-MONETARY TOOLS INADEQUATE

A criticism of current control schemes is that the tools are actually inadequate in the light of changing economic conditions. For example, all commercial banks in the United States had total assets of around $79 billion at the end of 1941, while life insurance companies and saving and loan associations, combined, had total assets of about $39 billion. Clearly, commercial banks had about twice the assets of these savings institutions at that time. In March 1959 commercial banks had assets of about $229 billion, compared to a figure of about $166 billion for the two savings institutions.[1] Currently, then, banks have only about one-third more assets. Clearly, savings institutions are destined to become as large as or larger than commercial banks, unless this trend is reversed.

Of course, banks and savings institutions have widely different functions. Yet the latter are coming to play an increasingly

[1] *Federal Reserve Bulletin,* June 1959, pp. 607, 617.

important role in the working of the economy. This suggests that these institutions might be subjected to regulation of the general kind now exercised over the banks. Let us note that savings institutions constitute a part of the mechanism whereby saving is channeled into investment. If this flow is to be regulated, the fact would betoken more or less direct control of the saving-investment process by the government. As such, it would be a step in the socialization of this important process. If these and other financial institutions continue to outgrow banks, the risks of such regulation will have to be carefully weighed.

Built-in versus discretionary stabilizers

In the analysis hitherto given, it was assumed that every decision on fiscal and monetary policy was completely discretionary. Actually, a good deal of the fiscal decision-making is automatic. Let us consider briefly the scope and basis of these built-in stabilizers.

Built-in stabilizers are designed to set in motion forces which automatically offset inflationary and deflationary forces. Of course, the objective is the maintenance of full employment without inflation. Discretionary controls are designed to offset changes in economic activity which cannot readily be foreseen. Until economic science develops further, most controls will have to be of this character.

In the area of Federal fiscal policy the planning of taxes and spending is such as to build into the system a certain degree of stability. Thus, the economy tends to be more stable with than without the Treasury's fiscal apparatus. Consider the character of government spending. By law the appropriations are fixed for the period of a year. Regardless of fluctuations in the level of economic activity, the appropriations will tend to be spent. In turn, this expenditure tends to maintain the level of income and lend an element of stability to the economy.

It hardly seems necessary to observe that investment and consumption do not exhibit a high degree of stability. If expectations of future profits become poor, investment will decline sharply. Likewise, consumption will fall off with any decline in National Income. Consequently, both these vary according to the phase of

the cycle or other conditions. Evidently, the sharing in total demand (or output) of an element which does not change tends to add an element of stability. During the period 1929–1958 the share of government spending on output has risen from a little over 8 percent to about 21 percent of the total. Such a change tends to stabilize income.

Consider the effects of the tax structure on the situation. Broadly speaking, the existence of an income tax system lowers the MPC out of National Income. For this reason a given change in the level of National Income will produce a smaller change in consumption after the tax has been imposed. This assumes that the MPC out of private income remains the same. Assuming a uniform tax rate on marginal income, MPT, the MPC after tax is expressed by the formula: $MPC = MPC_p(1 - MPT)$. The larger is MPT, the smaller is MPC. And as MPC becomes smaller, the C line becomes flatter. Finally, this implies that the consumption element of total spending has become more stable.

Another way of expressing this idea is that the multiplier drops in value as the tax rate rises. In turn, this means that variations in the level of investment produce a smaller effect on National Income. Using the formula, $K = 1/[1 - MPC_p(1 - MPT)]$, we can construct a table of values to illustrate. We assume that MPC_p

TABLE 40. The Multiplier with Different Values of MPT

K	MPC	MPT	MPC_p
5	.8	.0	.8
3.57	.72	.1	.8
2.5	.6	.25	.8
1.67	.4	.5	.8

is .8; this is less than MPC out of Disposable Personal Income which is about .9. This is done to take account of the fact that a part of corporate income is retained, rather than being passed out in the form of dividends. However, the figure .8 does not purport to be realistic. By varying the marginal propensity to tax we can secure a corresponding schedule of values of the multiplier. We also include the values of $MPC = MPC_p(1 - MPT)$.

If the marginal propensity to tax is zero with an MPC_p of .8 the multiplier is 5. An increase in the tax rate to .25 lowers the multiplier to 2.5. Such a change means that an increase in the level of investment by 1 will produce a change of only 2.5 in the income level instead of 5. This indicates a marked alteration in the stability of the income level. Obviously, the higher the marginal tax rate, the lower is the multiplier and the greater is the stability.

On the other side of the ledger is the fact that the discretionary power of the government is diminished. In seeking to raise income a given amount by spending extra sums the government will now have a smaller leverage. If the government wished to raise income by 5 with an MPT of 0 and a multiplier of 5, an increase of only 1 in spending would be necessary. With an MPT of .25 and a multiplier of 2.5 an increase of 2 is necessary. Although the tax system increases stability, it reduces the power of the Treasury to control the income level. In short, the system becomes insulated to a degree against change in the spending level by the government itself. Only by alteration of the tax structure can this tendency be offset.

As we observe these phenomena in prosperity and depression the outward signs show up most clearly in the budget surplus or deficit. A decline in the income level reduces the tax receipts of the government, while the expenditures are constant. In turn, this leads to a budgetary deficit. Such a deficit implies that the government is putting more into the income stream than it is withdrawing. This excess is inflationary in effect.

Let us note that at any given equilibrium level, a deficit in the public budget implies a surplus in all private accounts. By this we mean that private income exceeds private spending ($C + I$) to this extent. This may be seen in the standard box figure. Thus the deficit on government account is $R\,S'$, while the surplus on private account is also $R'S'$ at budget-tax point R' in Figure 49. This excess of private income over spending is sometimes called "hoarding," since it implies an attempt (not necessarily successful) to add to bank balances. If the public sector has a surplus, the private sector will run a deficit, i.e., spend more than it receives in income.

When private spending increases and income rises, tax receipts will increase and with fixed expenditures, the government will tend

to have a surplus. In turn, this will match an excess of private expenditure over income, that is, a private deficit. So a high National Income is often marked by a private deficit and a Treasury surplus, whereas a low income is often marked by a private surplus and a Treasury deficit.

PROGRESSIVE TAXATION AND INFLATION

In an economy with progressive taxes the tax system automatically increases the tax take as income rises. Beyond the full employment level per capita incomes in money will certainly tend to rise, and the rate of progression will increase. In turn, the tax bite will constitute a larger proportional share of the income, a fact which will increase the Treasury surplus. Evidently, this development implies a resistance to any inflationary tendency or deficit in the private economy. All well and good. However, the individual pays a price for this stabilizing tendency.

Suppose real income (output) remains the same, while prices and wages double and the tax structure is not altered. Assume that population is constant. Clearly, National Income in money and per capita income in money will double, while real per capita income remains the same. With double the per capita money income a higher tax rate would apply and the per capita tax take would absorb a larger share of income. Evidently, real per capita income after taxes would be smaller.

Obviously, the tax effect of inflation with a progressive tax system is to reduce per capita disposable real income. By the same token this reduces consumption in real terms and checks inflation. If the cause of inflation was excessive private consumption, no complaint is in order. If the cause lay in excessive private investment, the individual may feel with some truth that his living standard is being depressed because of an excessive desire for adding to the stock of capital. This is a possibility that ought not to be left entirely to chance. The means for controlling investment rather than higher taxation, might be invoked.

THE ROLE OF AUTOMATIC STABILIZERS

Can the place of automatic stabilizers be widened? We might set up a scheme like this: Whenever the level of National Income

fell by some given percentage, additional Federal expenditures would be started. The greater the fall in the level of income, the greater would be the increased spending. For increases in income above the full employment level expenditures would be cut in a corresponding manner. We could also work tax cuts and increases into the scheme.

It is doubtful that such a scheme would be workable. The economic system is so complex that it is hard to gauge the dimensions of a recession until it is occurring. No formula based on National Income, employment, or other indicators can serve as an adequate guide to action. Before formulating a fiscal policy it is necessary to have in hand all the relevant facts at the time.

A generalized graphical multiplier analysis (optional)

In the preceding pages we have concentrated on the combinations of spending, taxing, and interest rate which result in full employment. In addition, we have considered changes in the income level through the operation of the multiplier. This analysis was mainly algebraic. Let us now integrate the basic graphical analysis of this chapter with the multiplier theory. Throughout the discussion the rate of interest is assumed to be constant.

Consider the familiar box rotated 180° so that taxes are measured along the horizontal axis and budgetary spending is measured along the vertical axis. Let the National Income, as measured along the private income axis, equal OE. If in Figure 52 it is proposed to increase National Income by governmental action from OE to OE', the dimensions of the box will be correspondingly enlarged. This can be represented by a movement of the government origin from O'' to O'''. Under the new conditions we have the same B and T curves, but referred to a new origin, O'''. For convenience, let us show the B and T lines from the old set of axes $O''E$ and $O''A$. By working this change out, together with its implications, we can find such shifts in the curves as may be required to attain the several desired levels of income.

When the B and T curves are referred to the new origin, every point on these curves must move in the same way. To move from O''' to O'' requires a horizontal move $O'''W = E'E$, equal to the

increased income and an equal vertical movement WO''. These movements are equal, because the angles at O''' and O'' are both 45 degrees and the sides opposite these angles in triangle $O'''WO''$ are equal. Similarly, point R is carried into U and R' into U'. Since the horizontal and vertical movements of points O''', R, and R' are identical, $O'''W = RS = R'S'$ and $WO'' = SU = S'U'$.

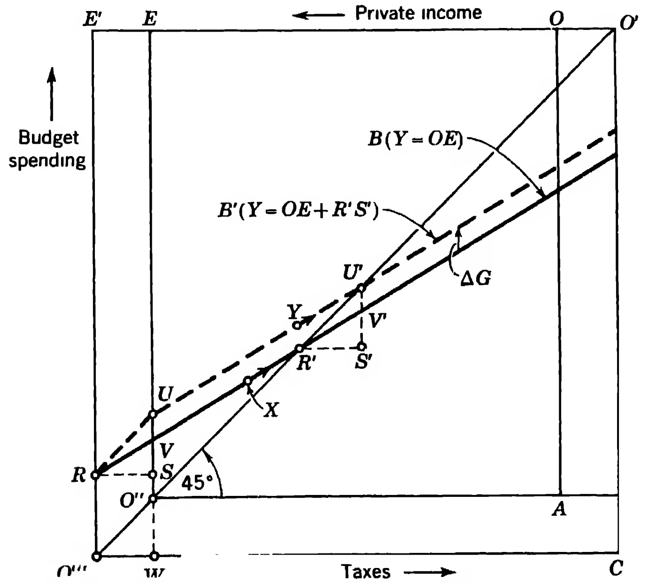


Figure 52. Generalized Graphical Multiplier Analysis.

As just explained, $O'''W = WO''$, and this implies that $SU = RS$, and $S'U' = R'S'$. Obviously, triangles $O'''WO''$, RSU, and $R'S'U'$ are isoceles right triangles with two equal sides, meaning that they are congruent. Also angle VRS equals angle $V'R'S'$; the corresponding triangles are also congruent, since all the angles are equal and one corresponding side of each is the same ($RS = R'S'$). As a result, $SV = S'V'$. Since $SU = S'U'$, we get by subtraction $VU = V'U'$. This implies that the new B line, B', is parallel to the old.

Let us interpret these results. At any point on budget line B

income is constant. On the higher budget line, B', income is higher by the amount $EE' = O'''W = R'S' = S'U'$. In short, B represents a higher budget line associated with a higher level of income. As we found in the last paragraph the new budget line is parallel to the old.

Implicit in these remarks is a method of constructing and interpreting these curves. Suppose an initial point X is found on an initial budget line B, as shown in Figure 52. Point X defines a level of taxing and spending which gives rise to income OE. The Treasury decides to alter the combination of taxing and spending to the one defined by point Y. We construct a new budget line by drawing a line B' through Y parallel to B. We can find the rise in income by moving from X to R', the equivalent balanced budget point on B, and from Y to U', the equivalent balanced budget point on B'. We proved above that the income rise equals $R'S' = S'U'$.

In the method outlined above the change in taxing and spending is represented by a change from one point to another in the tax-spend plane. The effect of such a change on the level of income has been shown. It now remains to relate the change in income to the change in taxes or spending on the figure in some convenient manner. In short, we should like to have a graphical representation of the government multiplier. In what follows we will outline a method for finding the value of the government spending multiplier.

In Figure 53 let the initial and new tax-spend policies be represented by points f and c, respectively. With a given B line[3] we construct B' as before, and note equivalent balanced budget points A and C. The rise in income is:

$$\Delta Y = AB = BC. \tag{8-1}$$

The increase in the tax level is the horizontal distance fd, while the increase in government spending is dc. Draw a line through d

[3] Given the initial point f, all we need to do to construct the B line is to pass a line through that point with a slope equal to MSR − MPC. If MPC_p = .6, we lay off a distance 10, say, in a horizontal direction, and add a distance 6 in a vertical direction. This defines a new point on the B line, call it f'. Passing a line through f and f' gives the B line.

parallel to B, intersecting CB in D. Then we have:

$$\Delta G = DC = dc, \tag{8–2}$$

since these are the opposite sides of a parallelogram. Since the increase in income is AB and the increase in tax is:

$$\Delta T = fd = FD = GB, \tag{8–3}$$

private income increases by the difference

$$\Delta Y_n = AG = AB - GB. \tag{8–4}$$

Finally, since the slope of B = MSR = MPC_p, the increase in

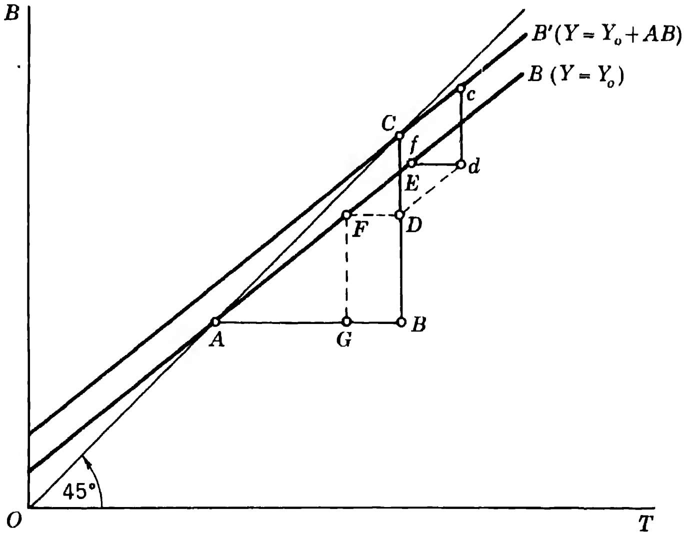


Figure 53. Finding the Value of the Multiplier Graphically.

private income of AG leads to a rise of $GF = BD$ in consumption. This was necessarily true, because in equilibrium,

$$\Delta C = \Delta Y - \Delta G - \Delta I = \Delta Y - \Delta G = BC - DC = BD = GF, \tag{8–5}$$

ΔI being zero in this case.

It is now a matter of straightforward manipulation to secure the multiplier formula from the diagram, using the results above.

$$K = \frac{\Delta Y}{\Delta G} = \frac{BC}{dc} = \frac{BC}{DC}, \text{ by (8–1) and (8–2),}$$

$$= \frac{BC}{BC - BD} = \frac{1}{1 - \frac{BD}{BC}} = \frac{1}{1 - \frac{GF}{BC}} = \frac{1}{1 - \frac{GF}{AG}\frac{AG}{AB}}$$

$$= \frac{1}{1 - \frac{GF}{AG}\frac{AB - GB}{AB}} = \frac{1}{1 - \frac{GF}{AG}\left(1 - \frac{GB}{AB}\right)}$$

$$= \frac{1}{1 - \frac{\Delta C}{\Delta Y_P}\left(1 - \frac{\Delta T}{\Delta Y}\right)}, \text{ by (8–1), (8–2), (8–4), and (8–5)}$$

$$= \frac{1}{1 - \text{MPC}_p(1 - \text{MPT})}.$$

By means of the graphical methods shown we can both illustrate easily by a comparison of lengths the value of the multiplier and derive the general multiplier formula itself. Note that the derivation is entirely independent of any particular assumptions about the relation of the changes in spending and taxes. We could also show graphically the government tax-cut multiplier, but will not pause to do so here.

PART II

NATIONAL INCOME FROM THE DYNAMIC VIEWPOINT

In this section the static concepts of National Income economics are modified and applied to the study of economic change. Three main problems present themselves and receive analysis, namely, economic fluctuations, inflationary spirals, and growth. To provide insight into these three problems is the theme of this section of the text; no common explanation is advanced.

CHAPTER 9

Simple economic dynamics of national income

When Keynes set out his version of National Income theory the Western world faced a problem of economic stagnation. As a result, no doubt, the tools of analysis he presented had a distinctly static character. Although dealing with the effect of variations in the level of investment on income and employment, this apparatus did not specifically involve the element of time. As a result of the work of many economists, it is possible to progress from the static building blocks into the study of change through time. Today, with public attention focused on problems of inflation, growth, and those interruptions caused by fluctuations it is necessary to concentrate on the dynamic side of National Income. For this reason the remaining chapters are concerned with the elements of economic change.

Statics and dynamics

In seeking an explanation of economic phenomena we may imagine an equilibrium of the various forces involved. Such an equilibrium represents a state of balance between the economic magnitudes involved. If the economic system is viewed as a sort of mechanism which requires balance, we may assume that such a balance is attained. Once the balance is assumed we can work backward from such a steady state to find the necessary values of the various economic magnitudes.

In the study of National Income we regard a balance between saving and investment as necessary to equilibrium. Next, we may find the conditions which such a state of equilibrium imposes on the variables involved. In short, the condition of balance between saving and investment determines the value of National Income.

By a slight extension of this method we can investigate the effect of certain changes on the equilibrium position. If one of the "constants" in the situation changes, the old state of balance is disturbed. Sooner or later a new equilibrium will emerge from the altered situation. In the new equilibrium another set of values emerges for the variables involved. By investigating the response of various economic magnitudes to the changed situation we can find out something about the character of change in the system.

Such an approach to the study of economics is known as "comparative statics," because it compares one state of equilibrium with another. When studying the multiplier, we investigated the effect of a change in the level of investment on income as the system moved from one equilibrium to another. In this and other such problems the element of time plays no essential role.

As the preceding discussion indicates, the study of equilibrium is designed to reveal how things *are*. In many cases, however, the point of particular interest is how things *change*. To some extent the study of comparative statics leads to an understanding of change, but this approach shackles change to the concept of static equilibrium. Further progress in this direction requires us to free the analysis from the bonds of static equilibrium and investigate explicitly the timing of events. In broad terms the explanation of change involves the statement of economic relationships in a form involving time in some essential way, that is, economic dynamics.

Between the fields of comparative statics and dynamics lie certain cases which combine the characteristics of both categories. In one such case (let us call it "structural dynamics") some attention is paid both to the relative timing of events and to the results of such timing on the values of the variables in time. However, the main emphasis is laid on a pattern of timing so loose that the results follow although the timing varies. In this case the assumptions as to timing are so loose that the changes being explained are made to depend more on economic structure than on timing.

Each of the three methods of analyzing change has its advantages. The method of comparative statics is simple, and may well be used when the changes in economic magnitudes, but not their timing, are important. When the timing of events assumes importance, it is necessary to resort to a dynamic analysis. If a

description of events requires only a rough account of timing, it is advantageous to use "structural dynamics." When a relatively precise account of the timing of events is required, the use of straight dynamics is mandatory. However, the difficulties in such an analysis range, according to the problem, from moderate to insuperable. We will start with a reasonably simple problem in straight dynamics.

Simple dynamics of the multiplier

ECONOMIC FLUCTUATIONS AND THE MULTIPLIER

From our study of National Income it is reasonably clear that investment is more changeable than consumption. Whereas consumption depends in a fairly predictable way on income, investment is capable of autonomous change. Any such change of investment produces a leverage effect on income through the multiplier. Economists are now convinced that alterations of investment operating through the multiplier are a principal source of the income changes to be observed in the economic system. As a first step in the study of dynamics it will be appropriate to consider the effect of investment changes on income *through time*. To do this it is necessary to consider explicitly the timing of events. This gives rise to an elementary problem in dynamics.

TIME LAG AND EQUILIBRIUM

According to static equilibrium theory, income tends to move to the point at which income and expenditure are equal, provided the two variables are also related in a manner that affords stability. While static analysis ignores the time required to adjust income to an increase in expenditure, a time interval doubtless occurs between the initial appearance of additional expenditure and the ultimate rise in income which results. In the argument that follows we take it for granted that investment is the part of expenditure which changes independently of income.

When new investment demand makes itself felt, businessmen will want to increase production. However, various steps have to be taken. Orders must be sent in to manufacturers when the extra demand is manifested. Moreover, the manufacturer must make

plans to add to his labor force and raw material flow. This cannot be done instantaneously; a time interval occurs before a new and higher level of income will have been established.

To summarize, the appearance of extra demand occasioned by a rise in autonomous investment, sets in motion a rise in the level of income. However, extra production and corresponding income can be generated only after the lapse of a time interval. Additional demand in one period generates corresponding income in the following period; that is, an excess of demand over supply in one period generates a corresponding rise in income in the following period. Now let us formulate the hypothesis in symbols.

Let the sum of consumption and investment be entitled expenditure and denoted by the letter E. In the static theory the condition of equilibrium was stated symbolically as:

$$Y = C + I.$$

In the revised notation this is stated as:

$$Y = E.$$

In the present problem it is necessary to date the variables Y and E, an operation which may be effected by the use of a subscript t. Letting the subscript t take on the integral values 0, 1, 2, . . . which designate corresponding time periods, the hypothesis of the preceding paragraph can be written:

$$E_t - Y_t = Y_{t+1} - Y_t = \Delta Y_t.$$

Clearly, this implies that:

$$Y_{t+1} = E_t. \tag{9–1}$$

With this interpretation of the lag of income behind demand, we can proceed to the study of the simplest dynamic case.

EFFECTS OF A SINGLE DOSE OF INVESTMENT ON INCOME

Suppose the level of investment rises for *one period* and then lapses back to its original level. Let us assume that the marginal propensity to consume is .6. In period 0 investment rises by \$1, causing an excess demand of \$1 to arise in the same period. In period 1 income rises by \$1, generating additional consumption of \$.6. At this point investment has lapsed back to its original figure,

while consumption is \$.6 above normal. Accordingly, income of \$.6 above normal will be generated in period 2, a fact which occasions consumption of .6 of this value or $.6(\$.6) = \$(.6)^2 = \$.36$. In turn, this generates additional income of $\$(.6)^2$ in period 3, and so forth.

Following out this sequence we can arrange the material in tabular form as in Table 41. Here the arrows make clear that the expenditure in one period determines income in the next which

TABLE 41. The Effect of a \$1 Autonomous Change in Investment on Income, Expenditure, and Consumption (in Dollars)

Period	0	1	2	3		$n-1$	n
Excess of variable over base period value							
E_t	1	.6	$(.6)^2$	$(.6)^3$	$\cdots$	$(.6)^{n-1}$	$(.6)^n$
Y_t	0	1	.6	$(.6)^2$	$\cdots$	$(.6)^{n-2}$	$(.6)^{n-1}$
C_t	0	.6	$(.6)^2$	$(.6)^3$	$\cdots$	$(.6)^{n}{}_{-1}$	$(.6)^n$
I_t	1	0	0	0	$\cdots$	0	0

fixes the consumption of that period. In turn, this determines expenditure which translates itself into income in the following period.

The reader may wonder what happens to saving and whether it is equal to investment. Of course, the excess of saving in a given period over its base period value is the difference, $Y - C$. In period 1, for example, income rises by \$1 and consumption by only \$.6; the difference of \$.4 is saved. Can this \$.4 saved in period 1 be used for investment in the same period? If so, investment would equal \$.4, instead of the zero value shown. Quite clearly, the answer is "no." By our initial assumption investment is an autonomous variable, independent of income. Saving is assumed to depend on income and can therefore differ from investment. In such a case any excess saving can best be used to repay bank loans. This point has already been explained in Chapter 4 in the discussion of saving and investment.

The basic purpose of the present discussion is to show the path of income through time following a single, unrepeated dose of investment. In period 1 income is \$1 above normal; in period 2, \$.6 above normal; in period 3, $\$(.6)^2$ above normal; in period n,

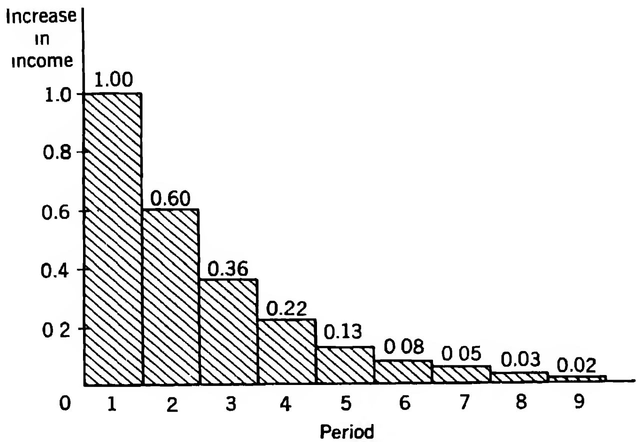


Figure 54. Effect of One Dose of Investment on Income.

$\$(.6)^{n-1}$ above normal. Clearly, income in any period n is $\$(.6)^{n-1}$ above normal. Since .6 stands for MPC, we can express this result symbolically by the expression:

$$Y_n - Y_o = (\text{MPC})^{n-1} = (\Delta C/\Delta Y)^{n-1}. \qquad (9\text{–}2)$$

In the numerical example analyzed and the formula above, the change in I is assumed to be \$1. If I rises by ΔI where this expression indicates a change greater or less than \$1, the level of income will rise by ΔI times the quantity indicated.

The expression (9–2) gives the value of the deviation of income from the base period in period n. Graphically, the situation appears as shown in Figure 54. As the number of periods increases, the income deviation decreases and gradually approaches zero. After the lapse of a given number of time periods, say 4, income has deviated from normal by a total equal to the sum of the first four blocks. This raises the question: "What is the total deviation of income from equilibrium after n periods?" In turn, this amounts

to an attempt to find the sum of the first n blocks in a systematic way.

Our first objective will be to find the sum of the blocks generated in the first n periods following the disturbance in period zero. This can be expressed by the following sum:

$$K_n = 1 + (\Delta C/\Delta Y) + (\Delta C/\Delta Y)^2 + \cdots + (\Delta C/\Delta Y)^{n-1},$$

where $(\Delta C/\Delta Y) = \text{MPC}$. Such a sum is known as the "truncated multiplier." As we shall see, the number of blocks ultimately generated is indefinitely large. Therefore our procedure in limiting the number of blocks to n implies that the series is "truncated" or cut off at an arbitrary point.

In the case where n is 4 and $(\Delta C/\Delta Y) = .6$, the need is for a formula to find:

$$K_4 = 1 + .6 + (.6)^2 + (.6)^3$$

Clearly, the exponent of the last term is one less than the number of terms in the series. Let us try, first of all, a straightforward algebraic calculation of the sum.

By definition:

$$K_n = 1 + (\Delta C/\Delta Y) + (\Delta C/\Delta Y)^2 + \cdots + (\Delta C/\Delta Y)^{n-2} + (\Delta C/\Delta Y)^{n-1}.$$

Multiplying K_n by $(\Delta C/\Delta Y)$, we find:

$$K_n(\Delta C/\Delta Y) = (\Delta C/\Delta Y) + (\Delta C/\Delta Y)^2 + \cdots + (\Delta C/\Delta Y)^{n-1} + (\Delta C/\Delta Y)^n.$$

In the last equation the term immediately prior to $(\Delta C/\Delta Y)^{n-1}$ is $(\Delta C/\Delta Y)^{n-2}$. Keeping this in mind, and subtracting the second equation from the first, we find that all but the first term of the first equation and the last term of the second vanish, giving:

$$K_n - K_n(\Delta C/\Delta Y) = 1 - (\Delta C/\Delta Y)^n.$$

On the left-hand side factor out K_n, giving:

$$K_n(1 - (\Delta C/\Delta Y)) = 1 - (\Delta C/\Delta Y)^n.$$

Dividing both sides by $1 - (\Delta C/\Delta Y)$, we get the desired answer:

$$K_n = \frac{1 - (\Delta C/\Delta Y)^n}{1 - (\Delta C/\Delta Y)}. \tag{9–3}$$

In the following discussion we will omit the \$ sign. In the case where the number of periods n equals 4 and $(\Delta C/\Delta Y) = .6$ the equation implies that:

$$K_4 = 1 + .6 + (.6)^2 + (.6)^3 = 1 + .6 + .36 + .216$$
$$= \frac{1 - (.6)^4}{1 - .6} = \frac{1 - .1296}{1 - .6} = .8704/.4 = 2.176.$$

By straight addition of the four terms in the series this is seen to be the correct answer.

Our problem is not yet concluded, because the level of income is still above normal after the lapse of four periods; at this point it is $(.6)^3$ or .216. Evidently, there is a residue of income passing on and generating further income no matter where the series stops. To use an analogy, the situation resembles the process of cutting slices from a piece of cheese. We start with a piece of cheese whose size is equal to 1. In the next period .6 of the cheese is cut and handed on, .4 being set aside. The .4 set aside may be likened to saving, while the .6 passed on may be likened to the income generated by additional consumption of .6. At the next round .6 of the reduced cheese of size .6 is passed on, while .4 is set aside. In short, .6 of .6 or $(.6)^2$ is passed on, while the remainder of the .6 is set aside. No matter how many times the cheese has .4 of its size removed there is always some cheese to be passed on.

Owing to the persistence of the series through successive periods a problem arises of calculating the total effect of the initial investment of 1 on income. In Figure 54 the successive income increments are represented as blocks; these blocks diminish rather rapidly in size, but never reach zero. This situation suggests the question: "What is the sum of *all the blocks generated* by investment of 1? What is the sum of all the increments of income generated by a single dose of investment?" This question amounts to asking the value of the series,

$$K = \text{limit } K_n = 1 + (\Delta C/\Delta Y) + (\Delta C/\Delta Y)^2 + \cdots + (\Delta C/\Delta Y)^j \cdots,$$
$$n = 1, 2, \cdots \infty$$

where the notation ∞ indicates an integer value for n larger than any preassigned integer and j indicates any integer whatever.

Since we know the value of K_n, our problem is to discover the value of the expression:

$$K = \lim_{n=1,2,\cdots\infty} K_n = \lim_{n=1,2,\cdots\infty} \frac{1-(\Delta C/\Delta Y)^n}{1-(\Delta C/\Delta Y)}$$
$$= 1 + (\Delta C/\Delta Y) + (\Delta C/\Delta Y)^2 + \cdots \text{ indefinitely.}$$

Consider the value of the expression $(\Delta C/\Delta Y)^n$, as n becomes large. For illustrative purposes take $(\Delta C/\Delta Y)$ to be .6. As n takes on the values 1, 2, 3, $(\Delta C/\Delta Y)^n$ takes on the declining values .6, $(.6)^2$ or .36, and $(.6)^3$ or .216. By the time n equals 10 this term is $(.6)^{10} =$ .006 approximately. As n takes on a very large value (much larger than 10), the term $(\Delta C/\Delta Y)^n$ becomes so small that it can be neglected. Neglecting this term in the preceding equation gives the desired answer:

$$K = \lim_{n=1,2,\cdots\infty} K_n = \frac{1}{1-(\Delta C/\Delta Y)}, \tag{9-4}$$

which is nothing more nor less than the static multiplier. The sum K_n is entitled the "truncated multiplier," because the terms beyond the nth are omitted.

At this point let us review the results we have attained, giving some numerical examples and otherwise trying to interpret the results. Consider the following table which presents the results from the time of the injection of 1 in investment in period zero, $(\Delta C/\Delta Y)$ being .6.

The second column indicates the deviation of income in any period from the base period value. Since more and more of the 1 invested "leaks out" of the income stream into saving, a constantly diminishing amount is handed on in successive rounds. Quite obviously, income tends to fall back to the level prevailing before the single dose of 1 was invested in period zero. In short, the deviation of income from the original equilibrium value of period zero tends toward nothing.

The third column in Table 42 expresses the cumulative rise in income compared with the base period. As long as current income exceeds base period income the cumulative increase in income shown in column (3) will continue to rise. In fact, the cumulative

sum shown in column (3) increases by exactly the last block or income deviation shown in column (2). For example, K_3 is 1.96 compared to 1.6 for K_2, the difference of .36 being accounted for

TABLE 42. Values of Income Blocks and the Truncated Multiplier in Successive Periods

(1) n Period	(2) $= (Y_n - Y_o) = (\Delta C/\Delta Y)^n$ Deviation of Income from Base Period Value: Value of Successive Income Blocks	(3) $= \Sigma(Y_n - Y_o)$ Cumulative Sum of Blocks (Equals K_n)[a]
1	1.	1.
2	.6	1.6
3	$.36 = (.6)^2$	1.96
4	$.216 = (.6)^3$	2.176
5	$.130 = (.6)^4$	2.306
6	$.078 = (.6)^5$	2.384
7	$.047 = (.6)^6$	2.431
8	$.028 = (.6)^7$	2.459
9	$.017 = (.6)^8$	2.476
10	$.010 = (.6)^9$	2.486
11	$.006 = (.6)^{10}$	2.492

[a] Defined in formula (9–3)

by the .36 block shown in column (2) for period 3. In order to show the cumulative effect of investment on income up to a certain time it is necessary to add the successive income blocks of column (2). Column (3) exhibits this calculation.

According to our previous argument the cumulative sum of the increments in income in the several periods can be measured by formula (9–3). Let us check the use of the formula by comparing the results it yields with the table. First let us find K_3, which is at once the sum of the first three income blocks and the cumulative increase in income effected by period three. According to the formula,

$$K_3 = \frac{1 - (.6)^3}{1 - .6} = \frac{1 - .216}{.4} = .784/.4 = 1.96.$$

Clearly, this is the same as the sum of the first three blocks in column (2) or $1 + .6 + .36$; by the same token it is equal to the cumulative sum, K_3, opposite period three in the last column.

As formula (9–4) indicates, the sum of all the blocks tends toward a limit equal in value to the multiplier. In this case $\Delta C/\Delta Y = .6$, giving rise to $K = 1/(1 - \Delta C/\Delta Y) = 1/(1 - .6) = 1/.4 = 2.5$. If the argument is correct, K_n as given in the last column should approach the value 2.5 as the number of periods becomes large. By the time the eleventh period is reached K_{11} has reached 2.492. Obviously, this differs by only 8 parts in 2500 from the alleged final limit. Consequently, the example tends to support the contention that K_n approaches an upper limit equal to the multiplier.

Multiplier and super multiplier

Under certain conditions it is plausible to assume that a rise in the level of income will induce a rise in investment. Essentially the same method of analysis is used when such induced investment occurs. However, the induced investment must be added to the induced consumption. In the case above with a marginal propensity to invest of .2, Table 41 may be revised to take the form shown below. In the formulas it is merely necessary to substitute the marginal propensity to expenditure, MPX $= \Delta C/\Delta Y + \Delta I/\Delta Y = .6 + .2 = .8$, in place of the marginal propensity to consume.

TABLE 43. Effect of a $1 Autonomous Change in Investment on Income, Expenditure, and Consumption

Period	0	1	2		n
Excess of variable over base period value					
$E_t = I_t + C_t$	1	.8	$(.8)^2$	....	$(.8)^n$
Y_t	0	1.	.8	....	$(.8)^{n-1}$
C_t	0	.6	.6(.8)	....	$.6(.8)^{n-1}$
I_t	1	.2	.2(.8)	....	$.2(.8)^{n-1}$

In Table 43 note that E in any period is the sum of I and C. Note how the addition of these gives .8, the marginal propensity to expenditure, raised to some power. For example, in period 2 the sum of C (equals .6(.8)) and I (equals .2(.8)) gives

$$C + I = .6(.8) + .2(.8) = (.6 + .2)(.8) = (.8)(.8) = (.8)^2.$$

In reality, we could ignore the C and I rows, remembering simply that MPX is .8, and applying the idea that expenditure in any period is .8 times the income of the same period.

REPEATED EQUAL DOSES OF INVESTMENT—INVESTMENT AUTONOMOUS

Suppose that investment increases by 1 from a base period level of zero and continues at this increased level indefinitely. In this case an opportunity is afforded for income to build up to a new and higher level. Such a higher income level would be required to raise saving by the same amount as the autonomous increase in investment.

To show the effect of the investment change most clearly, it is advisable to confine our attention to income changes. As before, let investment expenditure rise \$1 in period zero and let this higher level be maintained indefinitely. Each dose of investment generates a dwindling sequence of increments of income. When the investment doses continue, the consequent increments of income begin to pyramid. Eventually, the level of income builds up to a new and higher level. The following table illustrates the process, with the aid of arrows.

TABLE 44. Effect of a Constant Dose of Investment Equal to \$1 on the Level of Income

Rise in Income	1	1.6	1.96	· · ·	$\frac{1-(.6)^n}{1-.6}$	· · ·	$\frac{1}{1-.6} = 2.5$
							· · ·
					$(.6)^{n-1}$		$(.6)^{n-1}$
				· · · · · ·	· · · · · ·	· · · · · ·	· · · · · ·
			$(.6)^2$	· · ·	$(.6)^2$		$(.6)^2$
		.6	.6	· · ·	.6		.6
	1	1	1	· · ·	1		1
Income Generated by Doses, Period	1	2	3	· · ·	n	· · · · · · · ·	∞

CONCLUSIONS FROM TABLE 44

When the successive doses of investment are equal through time, the rise in income builds up by the same sequence of incomes as that generated by a single dose of investment. In the table the diagonal sequence starting with 1 and extending across and upward to $(.6)^{n-1}$ has the same sum as the vertical sequence in the column above period n. What this amounts to saying is that the increase in income per period after n periods have elapsed is the same as the cumulative increase in income from a single dose of investment after n periods. Since we have already calculated the latter amount, the expression for the rise in income as compared with the base period is:

$$Y_n - Y_o = \frac{1 - (\Delta C/\Delta Y)^n}{1 - (\Delta C)/\Delta Y}. \tag{9–3a}$$

As the doses continue indefinitely, we get the following final result:

$$Y_n - Y_o = \frac{1}{1 - (\Delta C/\Delta Y)}, \tag{9–4a}$$

as n becomes large. This is nothing but the ordinary multiplier formula. It indicates that income ultimately rises by the amount of the multiplier, if the dose of investment is continued indefinitely.

CUMULATIVE MULTIPLIER

In formulas (9–3a) and (9–4a) we find the growth in income per period as a consequence of an increment of investment which is maintained for n periods, or indefinitely. In some connections it might be interesting to know the cumulative increase of incomes generated by a sustained increment of investment. If investment rises by one unit per period, and is sustained at this level for n periods, income rises above the base level in each period. What is the sum of the increases for the first n periods? In Table 44 we would like to know the sum of .1, 1.6, 1.96 . . . $(1 - (.6)^n)/(1 - .6)$.

In a later chapter this value is calculated.

Writing

$$K = \frac{1}{1 - \dfrac{\Delta C}{\Delta Y}}, \quad K_t = \frac{1 - \left(\dfrac{\Delta C}{\Delta Y}\right)^t}{1 - \dfrac{\Delta C}{\Delta Y}},$$

we find the cumulative sum of the increased incomes to be

$$Y_t \text{ cum.} = K \cdot (t + 1) - K \cdot K_{t+1}, \tag{9-5}$$

where Y_t cum. is the cumulative increase in income at period t. To find the cumulative sum after a large number of periods have elapsed and K_{t+1} approaches K, we set K_{t+1} equal to K in (9–5), giving:

$$Y_t \text{ cum.} = K \cdot (t + 1) - K^2 \tag{9-6}$$

Let $t = 3$, and MPC $= .6$, as in Table 44. By formula (9–5),

$$\begin{aligned} Y_3 \text{ cum.} &= \frac{1}{1 - .6} \cdot 4 - \frac{1}{1 - .6} \cdot \frac{(1 - (.6)^4)}{(1 - .6)} \\ &= 2.5\,(4) - 2.5(2.176) = 10 - 5.44 = 4.56. \end{aligned}$$

By simple addition of the first three top items in Table 44 we get:

$$Y_3 \text{ cum.} = 1 + 1.6 + 1.96 = 4.56.$$

Evidently, the formula is valid for this instance. A proof is given in Chapter 13.

DYNAMIC STABILITY OF INCOME

In the section on determination of income under static economic conditions a rule for stability of the income–expenditure equilibrium was suggested. If time is not a factor, stability is achieved when expenditure is more stable than income. Graphically, this means that the $C + I$ line is flatter than the Y line. Analytically, it means that the marginal propensity to expenditure is less than 1. When investment is autonomous (as we usually assume), the rule is that the marginal propensity to consume is less than 1. Suppose a one-period time lag of income behind expenditure is introduced, giving $Y_{t+1} = E_t$. Also assume that production is not adjusted to

compensate for variations in inventory. In this case what is the condition for stability?

In a stable situation any divergence of expenditure from income will tend to be corrected by a change in income. Such a change will correct the disparity of income and expenditure. Suppose the divergence is caused by an arbitrary increase in expenditure at a former equilibrium. In Figure 55 expenditure increases by $EF = 1$,

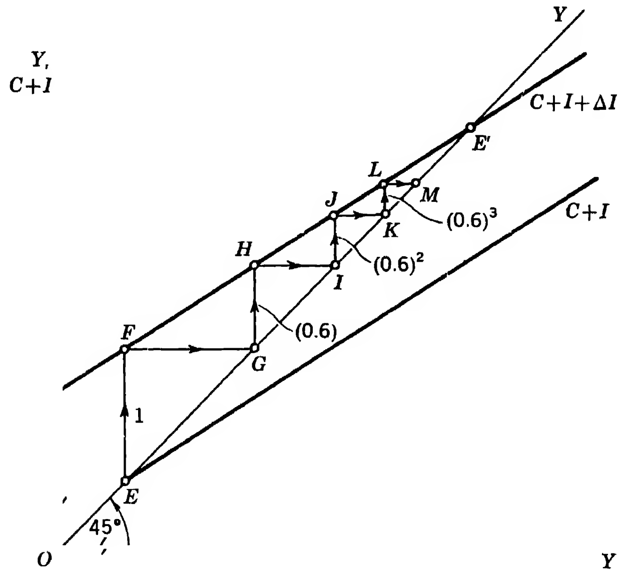


Figure 55. Income Build-up Shown on Equilibrium Diagram.

causing a divergence between expenditure and income of this amount. As Figure 55 shows, a stairstep movement would take place, initiated by the excess demand of EF. Evidently, the equilibrium is stable. As the steps are squeezed between the convergent Y and $C + I + \Delta I$ lines, they grow smaller in size. As we might suppose, the stability condition is that the $C + I + \Delta I$ line, the new $C + I$ line, be flatter than the Y line. In analytical terms the MPX or the marginal propensity to expenditure must be less than 1. If induced investment is zero, the MPC must be less than 1.

Consider briefly the meaning of this condition[1]. Let us remember that MPX has the numerical value of the additional quantity of

[1] Note the three following relations:

(a) $E_t = Y_{t+1}$, by assumption.

(b) $\text{MPX} = \dfrac{\Delta E}{\Delta Y} = \dfrac{E_{t+1} - E_t}{Y_{t+1} - Y_t}$, by definition.

(c) $E_{t+1} - E_t = \dfrac{E_{t+1} - E_t}{Y_{t+1} - Y_t} \cdot (Y_{t+1} - Y_t) \quad \text{MPX} \cdot (Y_{t+1} - Y_t)$ from (b).

Write down the set of equations derived from (c) by setting $t = 0, 1, \ldots n - 1$, and substituting from (a). This gives:

$$(1)\ E_1 - E_0 = \text{MPX}(Y_1 - Y_0) = \text{MPX}(E_0 - Y_0)$$
$$(2)\ E_2 - E_1 = \text{MPX}(Y_2 - Y_1) = \text{MPX}(E_1 - E_0)$$
$$(n-1)\ E_{n-1} - E_{n-2} = \text{MPX}(Y_{n-1} - Y_{n-2}) = \text{MPX}(E_{n-2} - E_{n-3})$$
$$(n)\ E_n - E_{n-1} = \text{MPX}(Y_n - Y_{n-1}) = \text{MPX}(E_{n-1} - E_{n-2}) = E_n - E_{n-1} = E_n - Y_n.$$

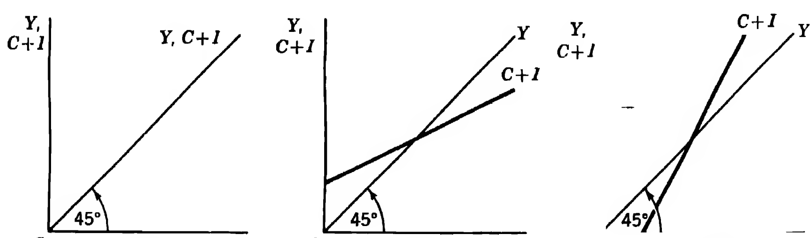
Start with the value of $E_n - Y_n$ in (n) and proceed with successive substitution of values back to (1), as follows:

$$\begin{aligned} E_n - Y_n &= E_n - E_{n-1} = \text{MPX}(E_{n-1} - E_{n-2}) = \text{MPX}\,[\text{MPX}(E_{n-2} - E_{n-3})] \\ &= \text{MPX}^2\,(E_{n-2} - E_{n-3}) = \ldots = \text{MPX}^{n-2}\,[\text{MPX}(E_1 - E_0)] \\ &= \text{MPX}^{n-1}(E_1 - E_0) \\ &= \text{MPX}^{n-1}\,[\text{MPX}(E_0 - Y_0)] = \text{MPX}^n(E_0 - Y_0). \end{aligned}$$

In brief, the result is:

(d) $E_n - Y_n = \text{MPX}^n(E_0 - Y_0)$.

If $\text{MPX} = 1$ and the equilibrium is neutral, $E_n - Y_n = E_0 - Y_0$. The disequilibrium neither grows nor diminishes. If $\text{MPX} < 1$, $(\text{MPX})^n$ diminishes as n increases and ultimately tends to zero. Then $(\text{MPX})^n(E_0 - Y_0)$ tends to zero, and E_n tends to Y_n. If $\text{MPX} > 1$, $(\text{MPX})^n$ grows with n, and $E_n - Y_n$ increases with n. These situations are illustrated in the figures below.



Neutral Stable Unstable

expenditure on output generated by \$1 of additional income. If income increases by \$1, consumption increases by \$.60 and investment by \$.30, MPX is .9. In short, \$1 of additional income generates additional expenditure on output of \$.90. Such a condition leads to a stable equilibrium of income and expenditure.

Suppose the level of expenditure is arbitrarily raised by \$1.00 at the equilibrium point. If the equilibrium is a stable one, income and expenditure will eventually tend to equality, after income has expanded for a large number of periods. In this case an excess expenditure of \$1 generates a rise in income of \$1. In turn, this causes expenditure to rise by \$.90. At this juncture the excess demand is only \$.90. In turn, this excess demand generates additional expenditure equal to .9 of \$.90 or \$.81. As income expands, the divergence between expenditure and income diminishes and the two approach equality.

If MPX equals 1, the initial equilibrium is a neutral one. In this case the $C + I$ or E line coincides with the Y line and there is an equilibrium at every income level. If $C + I$ is increased by 1 uniformly, the $C + I$ line rises 1 above the Y line. As E increases by 1, additional income of 1 is generated. Since MPX is 1, additional expenditure of 1 is generated. In turn, this implies an excess demand of 1, and so forth. The gap between $C + I$, or E, and Y remains constant at 1, as income increases by 1 per period. In fact, a new equilibrium is never attained, income increasing 1 in each period.

If MPX exceeds 1, the equilibrium is unstable. When $C + I$ is increased by 1, a steady divergence of $C + I$ from Y takes place. If MPX $= 1.2$, an increase of \$1 in expenditure generates an extra \$1 of income. In turn, this generates an additional \$1.20 of expenditure. Evidently, the excess expenditure grows in succeeding periods as time elapses and income grows.

GRAPHICAL INTERPRETATION OF TABLE 44

By plotting income against the period in which it is realized we get the graphical interpretation of Table 44 shown in Figure 56. The arrangement of blocks in the figure corresponds exactly to Table 44. As the bottom set of blocks we have selected those

whose age is one period and which are equal in value to $1. These blocks of common size are generated by corresponding quantities of investment in the preceding periods. In Table 44 these income values are shown in the first row. Next comes the set of blocks

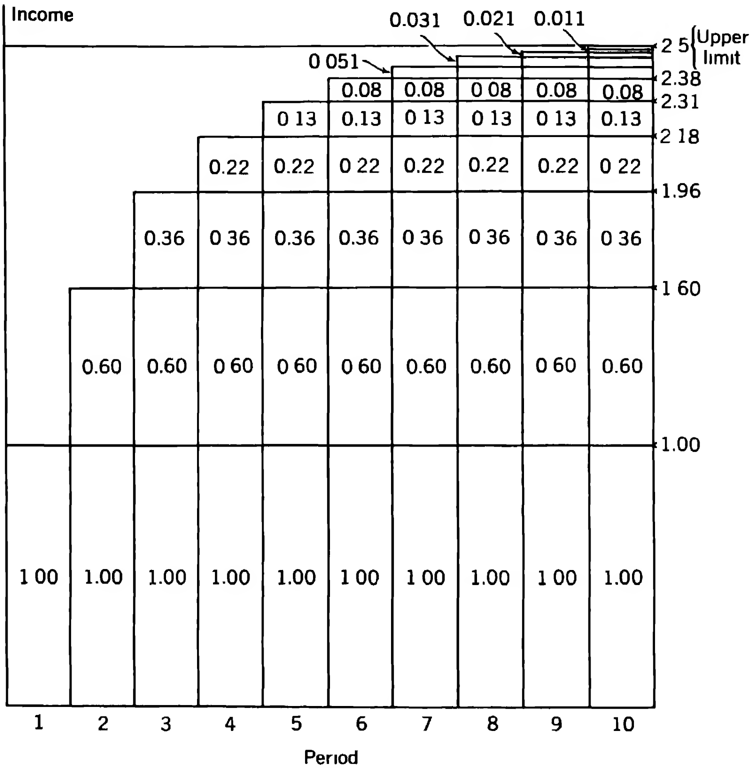


Figure 56. Effect on Income of Repeated Investment Doses.

representing consumption induced at the next round by the preceding income blocks of $1 and equal in value to $.6. Again, this set of blocks corresponds to the second row of Table 44. In fact, the first ten rows of Table 44 are represented in Figure 56 by corresponding rows of blocks. As the diagram indicates, income builds up rapidly and then more slowly toward a limit of 2.5, approached but never reached.

CONTINUOUS CHANGE IN INCOME

When investment increases autonomously in equilibrium, it causes thereby an excess of expenditure over income. Let us assume that income increases at such a rate at any moment that it will erase in precisely one year any difference between income and expenditure. If E exceeds Y by 100 at a given time, income begins to rise at the rate of 100 per year. Clearly, as income rises and the difference is lessened, say to 90, income will then increase at the rate of 90 per year. Such a buildup of income takes place smoothly, instead of jerkily, as in the case of discrete time periods with discrete changes in variables. The continuous case appears to be more realistic, but is much harder to expound.

GRAPHICAL INTERPRETATION BY THE STATIC EQUILIBRIUM FIGURE

In the case of the discrete multiplier the idea contained in the block figure is conveyed by the income–expenditure graph. In Figure 55 the initial equilibrium is given at E where $Y = C + I$. Below this point the scales are broken. Investment increases by EF or 1, raising expenditure by the same amount. In the following period income increases to the same extent, $EF = FG = 1$. When income increases by $FG = 1$, expenditure rises by $GH = .6$. In turn, this generates additional income of $GH = HI = .6$. By following the jumps EF, GH, IJ, KL, and so on, which amount to 1, .6, $(.6)^2$, $(.6)^3$, we find the successive increases in expenditure.

Since every jump in expenditure generates an equal increment in income, it is convenient to think of the jumps in expenditure as the increases in income themselves. We get a ready view of the approach to the new equilibrium through successively shorter and shorter jumps of income and expenditure.

PROBLEMS

1. Assume the MPC to be .8. Let investment increase by 1 in period zero. Construct a table showing the variation of income from the initial level in periods one through ten, assuming the "dose" of investment not to be repeated.

2. Construct a block figure for problem 1.
3. Assume that a "dose" of investment, equal in amount to 1, is made in period one and is repeated in each succeeding period. Construct a triangular table showing the various income blocks and the total variation of income from the initial level in periods one through ten.
4. Draw a block figure for problem 3.
5. We may entitle the ratio, $F_n = K_n/K$, the *fulfillment ratio* at period n. It represents the fraction of the full multiplier K realized at period n, and may be expressed by the equation, $F_n = 1 - (\Delta C/\Delta Y)^n$. In general, the function varies directly with n and inversely with $\Delta C/\Delta Y$.
   a. Work up a table with three columns: n, F_n when $\Delta C/\Delta Y$ has the value .6, and F_n when $\Delta C/\Delta Y$ has the value .8.
   b. Let us call the smallest n for which $F_n \geqq \frac{1}{2}$ *the half fulfillment time.* Find the half fulfillment times, expressed in terms of n, for the cases in which the MPCs are .6 and .8, respectively. Use the above table.
   c. If the time period in use is three months, what are the half fulfillment times expressed in months?
   d. Why might a low half fulfillment time lead to a lack of appreciation on the part of politicians of the practical value of additional government spending?
6. What is F_n when MPC is zero? Draw a block figure showing the effect on income of an unrepeated dose of investment manifested in period zero.
7. Draw a block figure showing the effect on income of an unrepeated dose of investment when MPC = 1. Summarize your conclusions from this figure as to the value of the multiplier K. What is the value of the truncated multiplier K_n?
8. What result do you secure by substituting $\Delta C/\Delta Y$ in the truncated dynamic multiplier formula?

   Mathematicians recognize the result as an indeterminate form. We may treat $\Delta C/\Delta Y$, as a variable, differentiate the numerator and denominator of the equation for K_n with respect to $\Delta C/\Delta Y$, and give $\Delta C/\Delta Y$ the value 1 in the resulting ratio. Compare the result with what you found in the initial question. What is F_n in this case?
9. Let $\Delta C/\Delta Y = 1.1$. Construct a table like Table 42 and a block figure like Figure 54, assuming a single investment of 1.
10. Check the result of adding the four first elements in the table against the "truncated multiplier" formula (9–3). Are the results the same?

11. If $\Delta C/\Delta Y = 1.1$ is formula (9–4) valid?
Suggestions. (a) Substitute in the formula, noting the sign, compare with the tendency in the block figure in problem 9, remembering that formula (9–4) is supposed to measure the *sum* of *all* the blocks that are ever generated (or the limit to which that sum tends). (b) Compare with the third column of the table in problem 9. Does this sum appear to be reaching a limit? (c) Note the value of $(\Delta C/\Delta Y)^n$ in (9–3) as n becomes large. What does this imply about the value of (9–3) as n becomes large? Note that both numerator and denominator are negative.

12. Note that if $\Delta C/\Delta Y$ is greater than 1, we may write $\Delta C/\Delta Y = 1 + v$, where v is a positive number, which represents the percentage of overspending of consumers out of an additional dollar of income. Thus if $\Delta C/\Delta Y = 1.2 = 1 + .2$, consumers who get an additional dollar, spend \$1.2, and overspend by \$.2. In this notation we may rewrite (9–3) as:

$$Y_n - Y_o = \frac{1 - (\Delta C/\Delta Y)^n}{1 - (\Delta C/\Delta Y)} = \frac{1 - (1 + v)^n}{1 - (1 + v)} = \frac{(1 + v)^n - 1}{v}.$$

But this is a formula used in mathematics of finance. It is known as the amount of an annuity of \$1 per year invested for n years at compound interest with a rate equal to v.

a. What happens to the deviation of income from its base value, according to the equation just above? Explain.

b. Assume $\Delta C/\Delta Y = 1.06$ and $v = .06$. Look up the value of $Y_n - Y_o$ for $n = 48$ by consulting a text on mathematics of finance with the table for $S_{\overline{n}|v} = \dfrac{(1 + v)^n - 1}{n}$.

CHAPTER 10

Business cycles and income analysis

Nature of economic cycles

By common observation and statistical investigation economists have discovered that prices, production, income, and employment are subject to periodical fluctuations. During one span of time prices, production, income, and employment may be low, but such periods of depression are followed by rising values of these variables. Subsequently they reach a peak or plateau, after which their values decrease and ultimately reach a trough at which all the values are low. For the sake of simplicity we shall confine the discussion to fluctuations in income.

Perhaps the most striking thing about these fluctuations is their persistent recurrence in wavelike form. Despite variations in the time interval between successive peaks or successive troughs a rough average for these intervals can be struck. This average period is usually estimated at something like eight and a half years. A basic economic problem, then, consists in accounting for the existence of roughly wavelike alternations in economic activity with a period of about eight and a half years. Although other cycles have been identified, this one appears to present the greatest practical economic problem.

Acceleration principle

In the past few decades economists have begun to develop a concept that has been known for some time. This concept, entitled the *acceleration principle*, helps to explain variations in investment. In turn, these variations in investment, working through the multiplier, act on income. According to this principle net investment depends on the increase in the consumption of the present period over that of the preceding period. If consumption exceeds its

former value, new capital goods will be demanded in order to produce the extra consumer goods demanded.

GROSS AND NET INVESTMENT

To lay the groundwork for future discussion it is necessary to make a distinction between gross and net investment. Gross investment comprehends all production of capital goods including: (1) replacement capital employed to take the place of capital goods used up in the current period; (2) new capital used to add to the stock of productive equipment. In its simplest versions the acceleration principle deals only with new capital production or net investment. By the same token the income concept appropriate to such versions is National Income, which excludes the element of depreciation for worn out equipment. Since the replacement of capital is capable of variation with time, the simplest version of the acceleration principle neglects a possibly significant cause of cyclical variations.

SIMPLE EXAMPLE ILLUSTRATING THE ACCELERATION PRINCIPLE

Suppose the economy employs \$2 of capital for every additional \$1 of consumption. The ratio between the additional investment and the additional consumption, $\Delta I/\Delta C = g$, may be entitled *the acceleration coefficient*. Suppose that prior to the changes to be considered replacement demand was evened out perfectly over time. In this event the only immediate changes occurring in gross investment will take place in net investment. For simplicity, let us ignore momentarily the repercussions of changed investment on income, and, through changed income, on consumption.

Suppose that the consumption of period two exceeds that of period one by \$10. In order to adjust to the additional consumption businessmen will seek to acquire \$20 worth of capital goods. This amount is just sufficient to turn out the additional goods. Suppose consumption repeats this increase of \$10 in period three, rising to a level \$20 over period one. Again, \$20 in new capital is needed to provide additional consumer goods. So long as consumption *rises* at a steady rate of \$10 per period, investment demand will be constant at \$20.

Suppose that in period four consumption rises by only \$5. Clearly, only \$10 of additional capital will be required. Investment will necessarily fall from \$20 to \$10, although consumption is still increasing. The reason lies in the fact that consumption is increasing less rapidly than previously. In summary, the acceleration principle states that net investment declines when consumption increases at a decreasing rate.

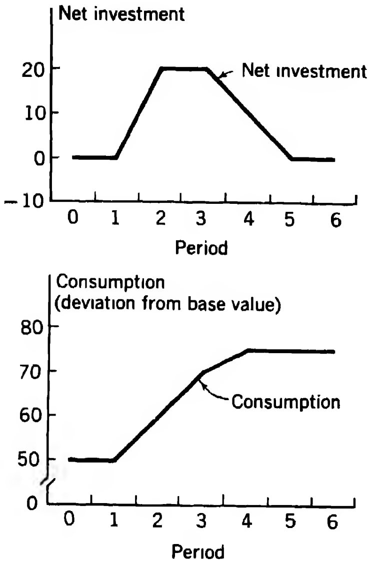


Figure 57. Consumption Change and Net Investment Level: The Acceleration Principle.

It may be helpful to see the principle exhibited graphically. Following the same assumptions used above, consumption occurs at a steady rate prior to period zero and investment is for replacement only. As consumption rises in Figure 57 at the rate of \$10 per period, investment jumps by \$20. When consumption rises more slowly, at the rate of \$5 per year, investment falls to only \$10 above the replacement level. Finally, when consumption levels off at \$25 above its initial value, gross investment lapses to a replacement level, net investment to zero. In the future, increased replacement demand will necessitate an increase in gross investment as capital wears out.

Acceleration principle and cycles[1]

From Figure 57 it is evident that an irregular line of increase in consumption can cause ups and downs in investment. If the principle developed is to be useful, something must be done to justify

[1] P. A. Samuelson, "Interaction Between the Multiplier Analysis and the Principle of Acceleration," *Review of Economic Statistics,* May 1939, pp. 75-85.

the pattern of growth in consumption. In the preceding discussion the consumption line represented an autonomous pattern of growth. In actuality, since the level of investment affects consumption through the multiplier effect on income, a "feedback" from investment to consumption occurs. In short, consumption and investment are related through the multiplier, as well as the acceleration principle. In order to clarify the situation it is desirable to employ an example.

BASIC DATA AND INITIAL CONDITIONS

In the example that follows the marginal propensity to consume is .5 and the acceleration coefficient 2. Investment demand is taken to be twice the difference between current and preceding consumption. Each period's expenditure "becomes" income in the following period.

To determine successive values of income it is necessary to know the values of income in two initial periods. Such given values of income are known as "initial conditions," and their values affect the size of each subsequent income level. Let us assign the arbitrary values 4 and 5 to income in periods zero and one, respectively, and see how this starts off the model.

In period zero the income of 4 determines consumption of 2, while in period one the income of 5 determines consumption of 2.5. Since income is known to be 5 in period one, the expenditure in period zero which generates this income must be 5. In turn, since C is 2 and $C + I$ is 5, I must be 3 in period zero. Between periods zero and one consumption rises from 2 to 2.5, an increase of .5, leading to investment of twice this, or 1, in period one. Adding C and I together for period one we get $C + I = 2.5 + 1 = 3.5$. Since expenditure is 3.5 in period one, income in period two is 3.5. For the first time a value of income has been derived from the conditions of the problem. Continuing in this way, we determine the successive values of the several variables.

Inspection of Table 45 reveals quite clearly that income exhibits a tendency to a fluctuate in a cyclical manner. For the student who is unacquainted with the properties of difference or differential equations it is natural to want to know *why* income fluctuates. Before answering this question it is worth while to throw in a side

comment on the propriety of this question. In modern scientific analysis there is a tendency to describe a situation by formal models such as the scheme we are now studying. If the model provides the desired behavior, many scientists will be content to conclude the

TABLE 45. Interaction of Accelerator and Multiplier: Regular Recurring Cycles (Figures Given as Deviations from Original Equilibrium Values)

Period	Y[a]	C[b]	ΔC[c]	I[d]	$C + I$[e]
0	4[f]	2	—	—	5
1	5[f]	2.5	.5	1.	3.5
2	3.5	1.750	−.75	−1.500	.25
3	.25	.125	−1.625	−3.250	−3.125
4	−3.125	−1.56250	−1.6875	−3.375	−4.9375
5	4.9375	−2.46875	−.90625	−1.812	−4.281
6	−4.281	−2.1405	.3285	.657	−1.4835
7	−1.484	−.7420	1.3985	2.797	2.055
8	2.055	1.0275	1.7695	3.539	4.5665
9	4.566	2.2830	1.2555	2.511	4.794
10	4.795	2.3975	.1145	.229	2.6265
11	2.626	1.3130	−1.0845	−2.169	−.856
12	−.856	−.428	−1.741	−3.482	−3.910

[a] Equal to expenditure of preceding period, owing to time lag.
[b] $C = .5Y$.
[c] ΔC is the excess of this period consumption over that of the preceding period
[d] $I = 2 \cdot \Delta C$.
[e] The $C + I$ value of one period equals the Y value of the next. Rounding errors in the $C + I$ column are corrected by more exact computations, causing the entries in the Y column to differ from the corresponding $C + I$ values in one or two instances.
[f] Arbitrary, given values.

discussion. In short, the "explanation" of a phenomenon consists in the formulation of a model the laws of whose operation correspond to the author's observation of reality. Since additional insight is often desired by the student, the following pages will consist of an attempt to "explain" some of the properties of the model in the hope that the laws of its operation will be more understandable.

Let us recall two of the assumptions made with regard to the timing of events. First, this period's income is equal to last period's expenditure. Second, investment is proportional to the difference between this and last period's consumption. Here are two time lags which stretch out into a two-period lag when combined. Such time lags make the functional relations looser and permit more things to happen.

Let us consider this point briefly. Recall that through the *accelerator* this period's consumption helps determine next period's investment. In turn, next period's investment, operating through the dynamic *multiplier*, affects income in the terminal period, two periods removed from the start. In short, consumption acting through an accelerator–multiplier process affects income two periods away. But consumption working simply through the *multiplier*, affects income (and consumption) in the next period and (through consumption) income in the terminal period. In short, consumption levels produce an impact on income in two distinct ways; through an accelerator–multiplier process and through the multiplier alone. When these effects are combined, a rather large hump or depression of income may occur. In essence, it is this combination of effects which causes the fluctuations. To bring out some of the points discussed in this paragraph we will plot certain of the variables against time, and compare their variations.

INVESTMENT AND CONSUMPTION CHANGE

Corresponding to Table 45, we have drawn up Figure 58 showing income, consumption, and induced investment. In Figure 58 it is apparent that the highest level of induced investment is reached shortly after the steepest point of ascent on the consumption curve is attained, namely, at time T_2. In a similar way the sharpest drop in consumption and lowest induced investment are reached at time T_1. In Table 45 it is also shown that investment makes its greatest deviation from zero the period consumption makes its largest jump. At the same juncture that consumption makes its most rapid increase, income has the same experience, the two being functionally related without time lag.

Along segments of the consumption curve prior to the stretch giving the fastest increase, an increase at a slower rate is manifest, and the curve is convex ; at subsequent points the consumption curve is concave and the ascent slower. We find, then, a convex part of the income and consumption curve , followed by a part of steepest ascent, , followed by a concave part .
Investment is greatest at the time when consumption is increasing most rapidly. The reason is, we recall, that investment is equal to

twice the increase in consumption. New facilities are most needed when consumption is increasing most rapidly.

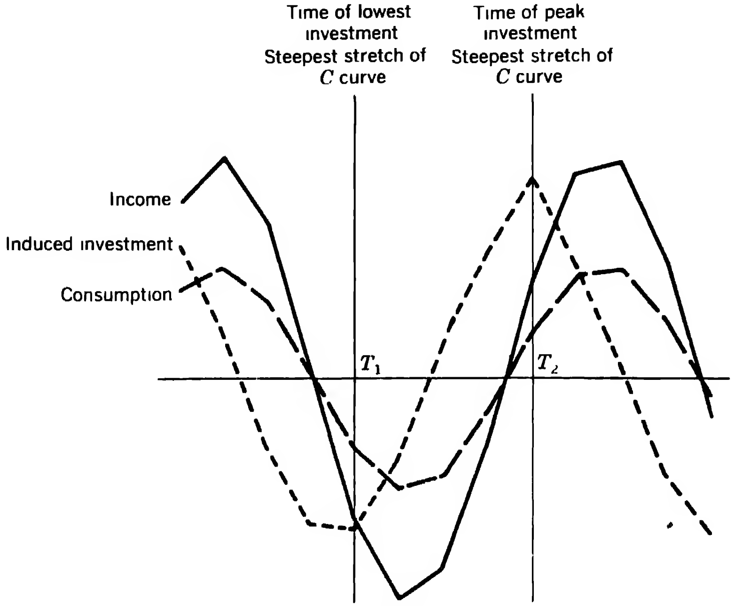


Figure 58. Interrelations of Consumption, Income, and Induced Investment.

UPTURN AND DOWNTURN

At what point does the upturn or downturn occur? In order to answer this question it is necessary to decide what the index of activity is to be. Common sense indicates that a measure of total production (income) or total expenditure, $C + I$, be used. Let us consider the downturn of each. When consumption is increasing most rapidly, investment is at its highest level. Obviously, consumption is *not* yet at its highest level, since it is increasing at its most rapid rate. At some juncture after this, when consumption is rising more slowly, but is greater in amount, and investment is less, expenditure will reach its highest point.

The peak in income is going to be reached one period later than that of expenditure. We recall that one period's expenditure

generates the income of the next. To summarize, peak expenditure will follow the point of most rapid *increase* of consumption (or income) and the highest level of investment. At some point of higher consumption, with a slower rate of increase and a lower level of investment, expenditure reaches a peak. One period later, income reaches a peak. These are elementary points, but constitute an endless source of confusion.

Dynamic equilibrium and the static figure

If there is anything the student of economics has intuitively in mind, it is that the economic system does not move smoothly to an equilibrium position when displaced from it. Typically, the student wants the system to overrun the equilibrium position and oscillate about it. By reason of time lags and the influence of increasing income on investment via consumption an overrun of this type may occur. In the case just discussed this is precisely what happens, and the sequence may be shown on a static (timeless) figure.

In Figure 59 we start from a position below the static equilibrium point, c. In this equilibrium, income equals consumption plus autonomous investment, I_a. Since autonomous investment is independent of income, the $C + I_a$ curve rises with income at the same rate as would the C curve, if present. Since this curve measures the *change* in consumption needed to compute induced investment, the consumption curve can be omitted.

The initial values given to income are shown at a and b; these incomes generate corresponding expenditures at A and B. The increase in income AZ produces an increase in consumption of ZB, I_a, the other component making up the curve $C + I_a$, being constant. Assuming an acceleration coefficient of 2, induced investment, labeled I_i, equals twice the increase in consumption, ZB. Using a compass, we mark off ZB twice in a vertical direction from B, ending at Y. Induced investment is thus BY by construction. When added to other spending at B, this yields total spending at Y in period one. Point B defines consumption plus autonomous investment, $C + I_a$; adding induced investment of BY defines total spending of $C + I_a + I_i$ at Y, as indicated in the figure.

Expenditure at Y in period one gives rise to income at c in period two; this represents an increase of BW in income which in turn generates an additional Wc of consumption. Laying off Wc twice above c we add induced investment to other spending to arrive at point X. In turn, following the arrows, we arrive at a new point,

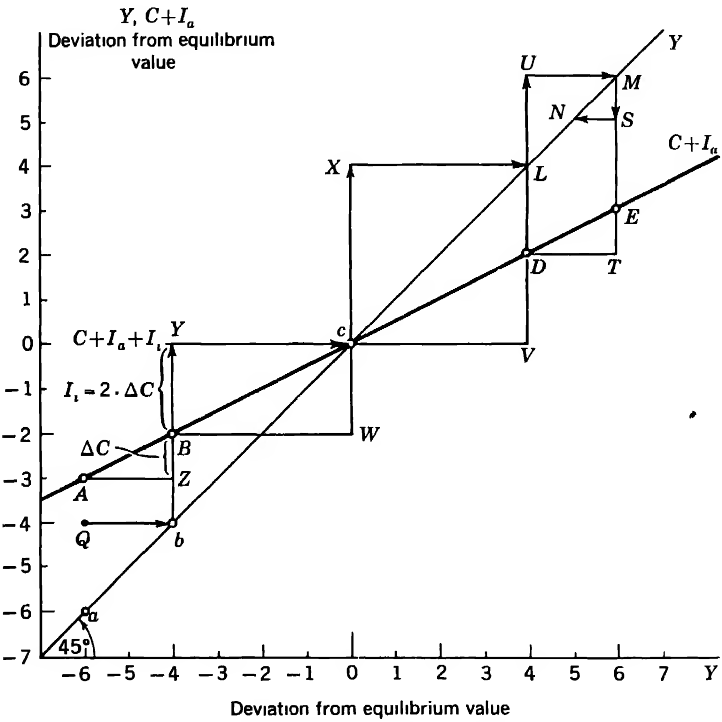


Figure 59. Acceleration Principle in the Equilibrium Diagram.

L, on the Y line. The successive jumps in consumption—ZB, Wc, VD, and TE—increase at first, become constant ($Wc = VD$), and then become smaller (TE is less than VD). As a result, induced investment increases from BY to cX, becomes constant as $cX = DU$, and declines from DU to ES, as the increase in consumption slackens.

When the increase in consumption drops from VD to TE, induced investment drops from DU to ES. Since the decline in

investment more than offsets the increase in consumption, expenditure falls to the point indicated at S. In turn, this generates income at N. Income having fallen by $SM = SN$, consumption will fall, and induced investment will become *negative*. We have not carried the lines in the diagram this far.

EQUILIBRIUM

Note that an "equilibrium" of income and expenditure, $C + I_a$, is reached at c, but that income does not stop its movement here. This indicates that expenditure does not depend on income alone. Rather, it is partly autonomous, I_a; it partly depends on income, C; and it partly depends on the *change* in consumption, I_i. For this reason an intersection of curves on a diagram will not suffice to determine the equilibrium. Rather there is a dance about the static equilibrium position. This dance starts with income at b, leads to expenditure at Y, income at c etc., following the path, $bYcXLUMSN$, indicated by the arrows. Only the beginning of the declining phase is shown. Table 46 records the results of the successive changes encountered.

TABLE 46. Changes in Income and Other Variables with an Acceleration Coefficient of 2

Period	Y	$C + I_a$[a]	ΔC	I_i[b]	E[c]
0	-6[d]	-3	—	—	-4[e]
1	-4[d]	-2	1	2	0
2	0	0	2	4	4
3	4	2	2	4	6
4	6	3	1	2	5
5	5	$2\frac{1}{2}$	$-\frac{1}{2}$	-1	$1\frac{1}{2}$

[a] $C + I_a = .5Y$ and MPC $= .5$.
[b] $I_i = 2 \times$ column four, and $g = 2$.
[c] $E =$ Column three plus column five.
[d] Values of income are given.
[e] Value may be omitted, but is consistent with rule $E_t = Y_{t+1}$, i.e., $E_0 = Y_1 = -4$.

INITIAL CONDITIONS, INCOME AND EXPENDITURE

One thing that is likely to be puzzling to the student is how the dance of income can begin with two arbitrary values of income. After all, income is supposed to be determined by the expenditure

of a preceding period. The point to keep firmly in mind is that these two values serve to determine an initial jump in consumption. Consequently, any arbitrary values of expenditure which will justify income of -6 in period zero and -4 in period one will be satisfactory. We write down -4 for E in period zero merely for the sake of consistency. Such a value satisfies the condition that the income of period one equal the expenditure of the preceding period. In practice, it is necessary to say how E came to have the value assigned in period zero. Let us consider this point briefly.

In period one income is shown at b in Figure 59. This implies that expenditure was at Q in period zero. Now the $C + I_a$ curve shows expenditure at A. Consequently, there is a deficiency of $QA(=1)$ in the expenditure. This could be brought about by: (1) a deficiency in normal autonomous investment of QA for this single period alone; (2) an autonomous reduction in consumer expenditure for this single period only. The cause of the deficiency in normal $C + I_a$ at that income level is unimportant. In some fashion the income -4 at b is determined. Note that it is possible to start the income expenditure sequence at Q. However, this is not an essential part of the logical development.

OSCILLATION OR EQUILIBRIUM?

As we have noted the level of income overshoots the static equilibrium value shown at c in Figure 59. Eventually, the level of income reaches a maximum and turns back towards lower values. Will it approach equilibrium without an overshoot now, or will it repeat its former course in the opposite direction? Once the decline in income sets in, consumption falls, the increment in consumption becomes negative, and investment collapses to a negative figure. As this happens, income falls greatly, producing a larger drop in consumption and an even larger negative value of investment. Eventually, income overshoots the level found at c and declines below the level indicated there.

Is equilibrium ever attained, or will income dance ceaselessly around the equilibrium point? In the particular case under discussion a ceaseless variation will occur. This is not to say that all such sequences behave in the particular manner shown in the example; this one has the property that MPC $= .5$ and $g = 2$, g

being the acceleration coefficient. The outcome depends on the relative sizes of these two numbers. In fact, the following rule defines the outcome of the situation.

THE STABILITY CONDITION

The condition that the sequence of incomes described above should converge to an equilibrium is that MPC $\cdot g < 1$. The condition for endless oscillation with no pronounced change in the fluctuations is MPC $\cdot g = 1$. This condition is satisfied in the case just discussed. Finally, the condition for divergence from equilibrium is MPC $\cdot g > 1$. We will try in the following paragraphs to make some sense of these conditions.

TABLE 47. Interaction of Accelerator and Multiplier: Explosive Cycles (Figures Given as Deviations from Original Equilibrium Values)

Period	Y^a	C^b	ΔC^c	I^d	$C + I^e$
0	4^f	2.4	—		
1	5^f	3.0	.6	1.20	4.20
2	4.2	2.52	−.48	−.96	1.56
3	1.56	.936	−1.584	−3.168	−2.232
4	−2.232	−1.3392	−2.2752	−4.5504	−5.8896
5	−5.8896	−3.5338	−2.1946	−4.3892	−7.9230
6	−7.9229	−4.7537	−1.2199	−2.4398	−7.1935
7	−7.1937	−4.3162	.4375	.8750	−3.4412
8	−3.4412	−2.0647	2.2515	4.5030	2.4383
9	2.4383	1.4630	3.5277	7.0554	8.5184
10	8.5184	5.1110	3.6480	7.2960	12.4070
11	12.4072	7.4443	2.3333	4.6667	12.1110
12	12.1109	7.2665	−.1778	−.3556	7.0887

[a] Equal to expenditure of preceding period due to time lag.
[b] $C = .6Y$.
[c] ΔC is the excess of this period consumption over that of the preceding period.
[d] $I = 2 \cdot \Delta C$.
[e] The $C + I$ value of one period equals the Y value of the next. Rounding errors in the $C + I$ column are corrected by more exact computations, causing the entries in the Y column to differ from the corresponding $C + I$ values in one or two instances.
[f] Arbitrary, given values.

Take the unstable case first with MPC $= .6$ and $g = 2$, so that MPC $\cdot g = 1.2 > 1$. Table 47, worked out by the same principles as the preceding one, illustrates this case. To illustrate the instability we may use the following line of argument. Suppose the

system is in equilibrium and an increment of autonomous investment equal to 1 occurs. The new investment generates income of 1 and induced consumption of .6 in the following period. Since investment is 2 times the increase of consumption, investment will

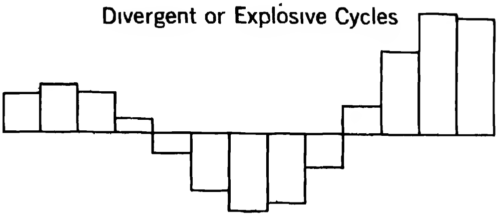


Figure 60. Explosive Cycle in a Multiplier-Accelerator Model.

rise by 1.2 in the period after that. This implies that an autonomous disturbance, such as a unit increase in investment, can magnify its initial force by increasing the value of investment itself from period to period. Under these circumstances the divergence from equilibrium, once set in motion, becomes larger and larger. To display this tendency visually, the income data of Table 47 are graphed in Figure 60.

NEUTRAL AND STABLE CASES

In the case where MPC $= 1/g$ the oscillation neither grows nor diminishes in size. Suppose MPC $= .5$ and $g = 2$, as in our first

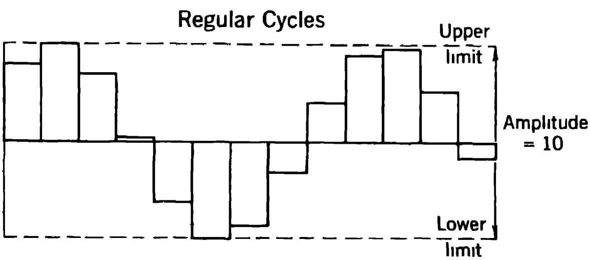


Figure 61. Regular Cycle in a Multiplier-Accelerator Model.

examples. An increase of 1 in autonomous investment will cause a rise of 1 in income, the following period. In turn, this causes a .5

rise in consumption which produces a rise of .5 $\times$ 2 = 1 in induced investment. This sustains, but does not increase, investment when the autonomous disturbance ceases. For this reason, a change in investment, autonomously produced, will neither exaggerate nor

Convergent or Damped Cycles

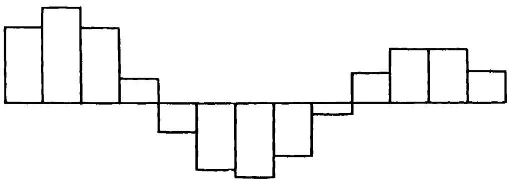


Figure 62. Damped Cycle in a Multiplier-Accelerator Model.

dampen the original increase. Figure 61 of the block variety shows this characteristic.

TABLE 48. Interaction of Accelerator and Multiplier: Convergent Cycles (Figures Given as Deviations from Original Equilibrium Values)

Period	Income[a]	C[b]	ΔC[c]	I[d]	$C + I$[e]
0	4[f]	2.40	—	—	—
1	5[f]	3.00	.60	.90	3.90
2	3.9	2.34	−.66	−.990	1.35
3	1.35	.810	−1.53	−2.295	−1.485
4	−1.485	−.891	−1.701	−2.5515	−3.4425
5	−3.4425	−2.0655	−1.1745	−1.7618	−3.8273
6	−3.8273	−2.2964	−.2309	−.3464	−2.6428
7	−2.6426	−1.5856	.7108	1.0662	−.5194
8	−.5194	−.3116	1.2740	1.9110	1.5994
9	1.5992	.9595	1.2711	1.9067	2.8662
10	2.8663	1.7198	.7603	1.1405	2.8603
11	2.8602	1.7161	−.0037	−.00555	1.7106
12	1.7106	1.0264	−.6897	−1.0346	−.0082

[a] Equal to expenditure of preceding period due to time lag.
[b] $C = .6Y$.
[c] ΔC is the excess of this period consumption over that of the preceding period.
[d] $I = 1.5\Delta C$.
[e] The $C + I$ value of one period equals the Y value of the next. Rounding errors in the $C + I$ column are corrected by more exact computations, causing the entries in the Y column to differ from the corresponding $C + I$ values in one or two instances.
[f] Arbitrary, given values.

In the case where MPC $< 1/g$ the oscillation diminishes in size. The autonomous investment here produces a diminished induced investment. In this case the fluctuation gets smaller and smaller, following the same argument. Ultimately, the equilibrium value is reached. This case is worked out in Table 48 and graphed in Figure 62. Both table and figure exhibit a gradual diminution of the fluctuation.

Cyclical and noncyclical behavior

Up to this point it has been tacitly assumed that the combination of an accelerator and a multiplier would produce oscillations. This is not necessarily the case. With the proper values of the accelerator and multiplier the level of income will (1) converge steadily to equilibrium, (2) diverge steadily from equilibrium. From the viewpoint of explaining cycles it is a matter of prime importance to know the conditions under which the system will oscillate. Without mathematics this condition appears to defy a simple explanation.

$$\text{Oscillation Condition: MPC} < \frac{4g}{(1+g)^2}.$$

When this condition holds, income fluctuates in a wavelike manner. If the inequality is reversed, the level of income either converges or diverges steadily. The question whether convergence or divergence obtains depends on whether g and MPC satisfy in addition the condition of stability or instability. For example, an MPC of .6 and an acceleration coefficient of $\frac{1}{5}$, satisfying the reversed inequality, will lead to steady convergence. On the other hand, an MPC of .9 and an acceleration coefficient of 2 produces steady divergence from equilibrium. These situations are, respectively, stable and unstable. The cases analyzed by table and figure all satisfy the oscillation condition.

THE REGULARITY OF CYCLES

From the example given, and particularly from the figures, it is evident that one cycle is not precisely like another. This is true even where the oscillation is of the very simple type given here. As one can well imagine, cycles will differ even more when random

economic disturbances enter the picture. In the model sequences shown there is regularity in one regard. This has to do with the period of the cycle or the length of time which elapses between one trough and the next. One other regularity is present. This consists in the maximum height or depth to which economic activity might go. This is entitled the "amplitude of the cycle." Consideration of these topics is beyond the scope of this book.

MULTIPLIER–ACCELERATOR SEQUENCES AND CYCLES

Do the sequences of income arising from the interaction of multiplier and accelerator account for cycles? It would be pleasant to be able to answer "yes." As the situation now stands, however, certain things remain to be explained. As we recall, the sequences exhibit cycles of three main types: explosive, steadily recurrent, and "damped" or convergent. Of the three types which one best explains the cycles in the real world? Theories of cycles have been built on each. Let us consider how the situation seems to stand with respect to each possibility.

STEADILY OSCILLATING SEQUENCES ON THE BORDERLINE BETWEEN STABILITY AND INSTABILITY

At first glance, the steadily recurrent sequence of cycles appears to be most like the situation in the real world. In such sequences the period or duration of the cycle is constant. Furthermore, the amplitude of the fluctuations falls within unchanging outer limits. Both of these characteristics accord well with observed properties of cycles. In fact, there is no observed tendency for cycles to change in amplitude or duration.

In other respects the case is not so good. In order that this case should occur it is required that the product of the multiplier and the acceleration coefficient be exactly equal to 1. This is not a very likely occurrence, because it requires certain specific values for these two coefficients. If the value of MPC is .8, the case requires that the acceleration coefficient be precisely 1.25—not 1.2 or 1.3 or some other value near by, but exactly 1.25. If this situation did not place such a sharp limitation on the values of either the multiplier or accelerator, it would be judged a more likely case. As it is, it seems unlikely that the acceleration coefficient would

consistently assume the precise value, say 1.25, required to produce a regular business cycle. By reason of the remarkable conjunction of values required, this case is not currently considered to constitute a valid explanation of cycles.

CYCLE CONVERGENT TO AN EQUILIBRIUM

In the case of convergent cycles the swings of the cycle steadily diminish in amplitude or severity. Immediately on laying down this condition we face the following problem. If cycles diminish in amplitude, and income converges on an equilibrium value, cycles will tend to disappear. Clearly, this pattern of events will not suffice to account for sustained cyclical behavior. To keep the swings of activity from dying away occasional random shocks must be introduced. This will set the system to oscillating again. In short, outside forces provide the energy to keep the system oscillating, while the inner structure converts these forces into cyclical movement.

If this pattern is to be accepted, more convincing evidence of the source and cause of the erratic shocks needs to be provided; at the same time the shocks must be incorporated in some formal way into the structure of the explanation. This can be done. Even if these conditions were to be satisfied, economists would not be likely to look with favor on such an explanation. What is desired is a fairly self-contained set of forces whose interaction produces fairly regular cycles. For the reason that outside shocks constitute a critical element in the explanation, this model will not be absolutely satisfying. Too much of the explanation depends on chance.

CYCLE INCREASING INDEFINITELY IN AMPLITUDE

At first glance the explosive theory of cycles seems the least promising of the three. From an empirical point of view the facts give no support to the contention that cycles are increasing in amplitude. Clearly, this explanation needs to be modified considerably if it is to hold water. Consider the modified account of the course of events given in the following paragraphs.

Suppose the system oscillates for a number of periods with increasing amplitude. Finally, the level of income reaches the full

employment level. Such a development interrupts the growth of (real) consumption. As a consequence, capital need not grow at the same rate. At this juncture investment falls. As a result, income falls, by the multiplier, and the expanding tendency is interrupted. In such a case, the swings might be so reduced in amplitude for a time as not to touch the full employment level.

On the other hand, if the swings grow in magnitude, income may fall to an extremely low level. At the same time investment falls to negative figures, since consumption declines (on the downward phase of the cycle). Again a lower limit to investment is indicated here. Negative net investment implies that capital is being reduced, as consumer demand shrinks. If consumer demand shrinks to the point where the existing capital equipment is not required to maintain an adequate output, the management does not take a pickaxe to this equipment. It neither adds new equipment nor replaces old. Under such a regime the capital stock declines annually by the amount of depreciation. Whatever the annual depreciation on the capital may be, this amount is the maximum disinvestment or negative net investment possible at a given time.[2] Nor is investment the only magnitude which has a lower limit. From our study of short-run consumption functions we know that consumption is somewhat rigid in the short run. Even if National Income were reduced to zero in the short run, consumption would not decline below some positive value. Evidently, there is a lower limit both to consumption and investment. From this it follows that there is a lower limit to the sum of consumption and investment. Since this sum equals total demand, and demand determines subsequent income, it is evident that a lower limit to income exists.

If, in the course of cycles of increasing amplitude, income strikes the lower boundary, its course is interrupted. Some time may elapse before the swings gather enough momentum to strike the upper or lower limit. As the cycle continues, the curve of income

[2] Suppose the acceleration coefficient equals 2, and consumption drops by 10. This suggests a decline of 20 in the capital stock or negative net investment of 20. If the capital stock is 80 with a life of 5 years, 16 units will be allowed to wear out and fail to be replaced. What about the other 4 units of capital, existing but not needed? The owner will find them superfluous for the moment, but will keep them, hoping demand will rise sufficiently to justify their use in the near future.

continues its irregular path, occasionally striking an upper or lower boundary in its course. When income reaches the full employment level, an increase in consumption produces no increase in investment. This implies that the acceleration coefficient drops to zero at the full employment level. As investment drops to its minimum figure, a reduction in consumption will cause no further change in investment. Again, the acceleration coefficient drops to zero at the lower limit of income.

SUMMARY OF THEORY BASED ON THE EXPLOSIVE SEQUENCE OF INCOMES

Evidently, the explosive multiplier–accelerator sequence combined with a full employment upper limit and a maximum disinvestment lower limit can be used to develop a theory of cycles. The most desirable feature is that the energy to maintain the fluctuation of income comes from within the system. This property is in distinct contrast to those theories which rely on the "damped" cycles which must be renewed by outside shocks.

This model has not been considered by formal geometrical or tabular methods. In fact, it does not lend itself readily to such treatment. For the most part, this arises out of the limits placed on income which in turn lead to varying values of the acceleration coefficient. Such variation in the basic structure of the system makes it difficult to attack the subject systematically. This may be regarded as a leading defect of the system.

A second defect of the account given lies in the fact that it is worked out wholly in real terms. The possibility of a continuation of the multiplier–accelerator interaction beyond full employment by inflation is not considered. Neither is the possibility of deflation at a lower level.

The Kaldor model—demand for capital[3]

Perhaps the leading difficulty in the preceding model lies in the variability of the acceleration coefficient around the limits of income. Something can be done to improve the method by relapsing

[3] N. Kaldor, "A Model of the Trade Cycle," *Economic Journal*, March 1940, pp. 78–92. See also R. M. Goodwin, "Econometrics in Business Cycle Analysis," in Alvin Hansen, *Business Cycles and National Income*, New York, Norton, 1951, pp. 434-460.

to a somewhat simpler form of analysis. Let the rate of investment depend on the difference between the quantity of capital desired and the existing quantity. While consumer demand may increase and cause desired capital to increase greatly, the translation of this additional need for capital into capital goods production takes time. When the gap between desired and actual capital widens, the quantity of capital produced will increase, to the end that the gap may be filled without the lapse of additional time. Let us note that the actual stock of capital at a *moment* of time is constant. In any *short period* of time it is *approximately* constant owing to the huge stock in existence.

The quantity of capital desired depends on the level of consumption and the rate of interest. In turn, the level of consumption depends on income and interest. Finally, then, desired capital depends on income and interest. By the same token the difference between desired and actual capital, the excess capital demanded, depends on income and interest. Investment, being proportional to excess capital demanded, depends on income and interest. In short, the production of capital or investment increases with income and decreases with the interest rate.

In the analysis that follows saving, investment, and income are deflated for price change. In ordinary terminology the discussion is cast in terms of real income, saving, and investment. Also the real rate of interest is used as a variable.

INVESTMENT AND INCOME IN THE KALDOR MODEL

In the model that follows the rate of interest is held constant. This is in line with the preceding multiplier–accelerator analysis. It implies a willingness on the part of the banking system to supply indefinite amounts of money, large or small, at a fixed rate of interest. This being the case, investment depends on income alone. Let us consider the precise way in which investment depends on income.

At low levels of income, investment will be negative and equal to the maximum rate of disinvestment. As income picks up, and consumption rises, a point is reached at which it will pay to replace some part of the capital equipment. (Investment will still be negative, but less negative than previously.) Eventually,

income increases to the point at which existing capital needs to be fully replaced as it wears out. At higher income levels investment is positive. As income rises, investment continues to be stimulated until full employment levels are reached. Beyond this point a rise in income leads merely to a competition between investment and consumption. In real terms the level of investment can rise no further. For this reason the level of investment does not rise with income beyond the full employment level. Considered together,

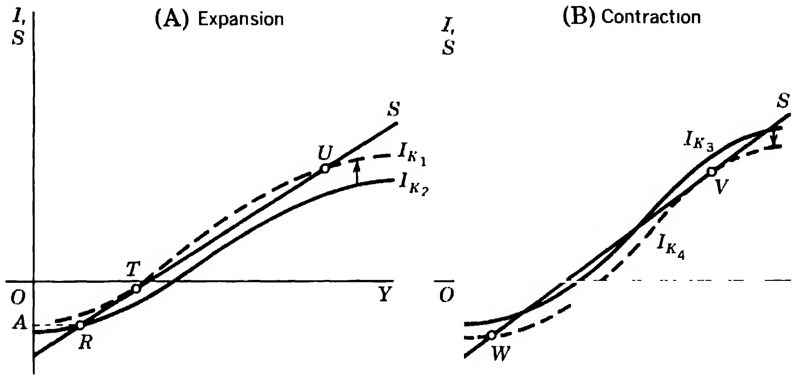


Figure 63. Kaldor Model of the Cycle.

the above considerations indicate that the investment curve has the drawn out S shape indicated by (say) curve I_{K_2} in Figure 63(A).

In accordance with our short-run approach we draw a short-run saving curve of the type shown in Figure 63.

One further assumption used here needs to be mentioned. The rate of investment is assumed to depend on the stock of capital in existence. At any given level of income an increase in the quantity of capital reduces the difference between desired capital and actual capital. In the first instance, the difference is reduced because actual capital has increased. In the second place, an increase in the capital stock reduces the productivity of further additions to that stock. The rate of interest being assumed fixed throughout, the marginal efficiency of capital (measuring this productivity) will be equal to the rate of interest at a lower level of investment. Consequently, the difference between actual and desired capital

is narrowed at both ends by an increase in the capital stock. The result is a decline in the rate of investment at the given level of income as capital stock increases. In Figure 63(A) the I_{K_2} lies below the I_{K_1} curve, capital stock K_2 being greater than K_1.

Let us start this account of the cycle from a period of deep depression. As shown in Figure 63(A) the level of income is determined originally at R by the intersection of the S and I_{K_2} curves. At this juncture, income is so low that OA disinvestment occurs. The capital stock is being reduced at the rate OA per year. As time passes and the quantity of capital is reduced, the I (investment) curve will rise. This means that, although the existing stock of capital exceeds the desired amount, the gap is being narrowed. As explained above, this is accomplished partly by reducing the amount of the stock by letting capital wear out and partly by the tendency of this action to increase desired capital. Since the excess of actual over desired capital at the given income level is being reduced, businessmen will want to let less of their capital wear out per year. In graphical terms this is registered by a movement upward of the investment curve, as shown in Figure 63(A). Although investment is still negative at low income levels, this negative amount is smaller in absolute value.

At point T in Figure 63(A) the investment curve just touches the saving curve. At such a position the economic system is in equilibrium with saving exactly equal to investment. However, if the level of income remains at T, further disinvestment will occur with the effect of shifting the I curve upward still further. This would cause investment to exceed saving at the given income level. The consequence of an excess of investment over saving is a rise in the level of income. At T, therefore, the level of income tends to increase until the point U is reached. Let us consider now in more detail the forces which occasion the move from T to U.

As the I curve rises above the S curve at T, income increases. In turn, the rise in income causes an increase in saving. However, the rise in income sparks an even greater increase in investment. Prior to the full employment level it is necessary to expand facilities in the lines where bottlenecks first develop. In a general way the increase in income raises the marginal efficiency of capital

relative to the rate of interest (which is fixed), and inspires businessmen with the desire to expand and invest.

As income increases from point T to point U, capital accumulates. This process narrows the gap between desired and actual capital, and tends to shift the investment curve downward. Ultimately, the downward shift brings the point of equality between I and S from U down to V in Figure 63(B). As capital accumulates further and the I_{K_4} curve falls below S, income declines to W. Again the system is plunged into depression. At this point, capital destruction begins anew and the cycle begins again.

EVALUATION OF THE KALDOR MODEL

An obvious weakness of this approach lies in its failure to define a specific time path for income. With the aid of simplifying assumptions, however, the path of expansion can be shown. For maximum convenience it is necessary to employ an income–expenditure diagram. Since $I - S = (C + I) - (C + S) = E - Y$, the vertical distance from the Y to the E curve is the same as from the S to the I curve. Scaling off this distance at every income level from the Y or 45° line and plotting the corresponding points will yield the E or $C + I$ curve. This is shown as the E curve in Figure 64(A). Assume a unit time lag with discontinuous change, as in the dynamic multiplier analysis.

In period zero, equilibrium is achieved at A when the E curve (not shown in the position being discussed) was tangent to the Y line at A. During period zero some capital equipment wears out, causing the E curve to shift upward to the position shown in period one. This upward shift in the E curve is translated into a rise in expenditure from A to B. In turn, the additional expenditure leads to greater income in period two. *We now neglect further shifts in the* E *curve.* Expansion takes place in the way indicated by the arrows. In turn, this generates the time series in the diagram on the right.

The time path generated is a nonlinear super-multiplier sequence. It is to be contrasted with the dynamic multiplier sequence studied earlier in that it has two bends in it rather than one. Since the effects of capital accumulation are omitted, the

time path is incorrect in a measure. For greater precision the *E* curve could be shifted downward as capital accumulates. If the ascent of income to the upper intersection takes place rather rapidly, the capital accumulation taking place during the ascent will be slight. If this be the case, the *E* curve will shift downward only slightly, and could be taken as fixed for practical purposes.

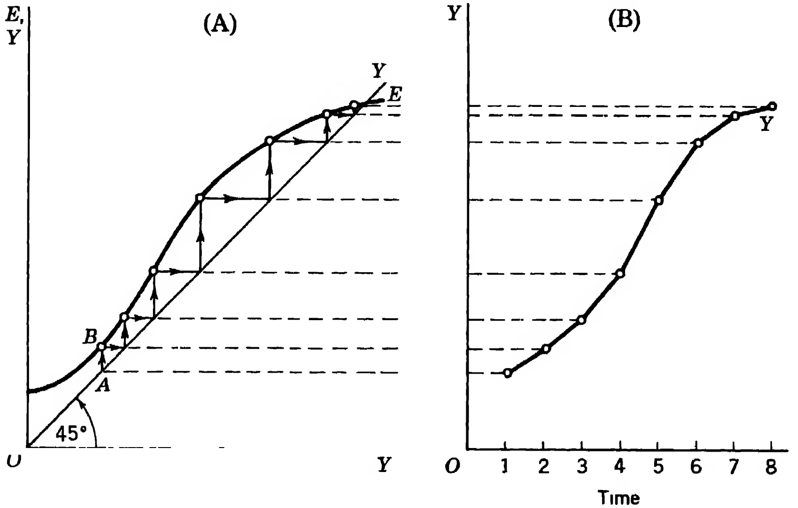


Figure 64. Income Path Generated by the Kaldor Model.

Let us indicate the modifications called for by the fact of capital accumulation. As income rises, investment becomes positive and the *E* curve moves downward. As the point of intersection of the *Y* and *E* curves is approached in the expansion process, the downward shift in the *E* curve brings the upper tangency point closer to the upper intersection point. Not long after the peak is reached, presumably, points *U* and *V* in Figure 63 will have been attained. At about the time the system moves to point *V* the downswing commences. The downward shift of the *E* curve will abridge the time the economy maintains a high income level. We may note that the downward time pattern of income will be roughly the reverse of the upward pattern. Clearly, an approximately wavelike pattern of income is generated.

Since the E curve is falling while income is rising, it might be thought that this complication would alter the character of the place at which the main contraction begins. Can we say definitely, in the light of this complication, that the critical point of the contraction occurs at a point like V in Figure 63(B)? At this point the I curve just touches the S curve from below. The answer to the question is "yes." If a sufficiently small time interval is considered, the quantity of capital accumulated will be negligible; as the time interval in use tends to zero, the quantity of capital tends to a fixed amount. Making use of this assumption, the I curve does not shift, and the development of events at any moment is governed by the relation between a fixed I and an S curve. Evidently, the critical point is reached when the tangency is attained between the fixed I and the S curve. As some finite interval of time elapses, the I curve shifts downward and the principal contraction commences.

A comparison of models

Of all the models dealt with prior to the last the most satisfactory perhaps arises from the explosive multiplier–accelerator sequence with upper and lower bounds which constrain the income changes, but provide sufficient internal energy for their continuance. Let us compare this with the Kaldor model.

In the first place, both these models are real; changes in price levels are not considered explicitly. In the second place, both models are built up in such a way that the turning points of the cycle arise from a relationship between consumption (or saving) and investment. In the first model the accelerator plays the vital role; in the second model a relation between the saving and investment curves performs this function. Third, both models produce an up-and-down motion of income which is internally generated.

One of the more important distinctions lies in the assumptions about investment, particularly in the constancy of the acceleration coefficient in the first model and the nonlinearity of the investment curve in the second (indicating variability in the response of investment to demand changes). Is it necessary to introduce a

nonlinear investment curve to secure a wavelike motion of income, or does the acceleration coefficient have to change over the cycle? We can secure wavelike movements of income with a constant acceleration coefficient. All the multiplier–accelerator sequences show this. However, our desire for an internally generated cycle led us successively to the explosive multiplier–accelerator sequence, to restraints on the fluctuations of income, and finally to an acceleration coefficient which drops off to zero around the ceiling and floor of the cycle. In effect, it is necessary to abandon a strict hypothesis of a constant acceleration coefficient, if the multiplier–accelerator sequences are to conform to reasonable ideas about the cycle. This line of thought is in precise accord with the second model, and we may conclude that the two agree in this essential respect.

In the Kaldor model, using simplifying assumptions, it can be seen that the principal expansion and contraction sequences have a wavelike shape. Is the nonlinear form of the investment curve essential to the attainment of this shape? We know that a super-multiplier sequence with linear I and S curves will produce an income buildup through time, but not one with the half-bell shape of the nonlinear variety. Such a shape is essential to moderately symmetrical cycles. Without the extended S shape of the investment curve, moreover, the contraction phase would not be symmetrical with the expansion phase. In consequence, nonlinearity is necessary to provide the required symmetry in the path of income through time in the Kaldor type model.

Bringing the preceding remarks together, we can make some final comments. The explosive multiplier–accelerator sequence is asymmetrical in that the cycles become successively larger. To avoid this type of asymmetry a variable acceleration coefficient is employed at the upper and lower limits of income. In the Kaldor model asymmetry in the super-multiplier sequence is minimized by the nonlinear shape of the investment curve. In the first case the symmetry required was in the amplitude of the cycle over successive cycles. In the second case the symmetry required lay within a single cycle. Evidently, the desire for over-all symmetry leads to the use of variable acceleration coefficients or nonlinear investment curves.

The monetary factor

Up to this point it has been assumed that the interest rate is constant throughout the cycle.[4] Furthermore, it has been assumed that the quantity of money expanded and contracted automatically to make this possible. Patently, the assumption is untrue, and it may be instructive to see what difference the monetary factor makes.

Let us continue the assumption that the investment curve has an extended *S* shape. To permit the monetary factor to enter let us return to the general equilibrium analysis of Chapter 5. This made use of a curve in the interest–income plane giving combined values of income and interest at which the money supply satisfies liquidity preference. Such a path is entitled the *L,M* curve. For the sake of convenience let us review its derivation.

The rate of interest is determined by the demand for money and the supply of money. In graphical terms the interest rate is found in Figure 65(A) at the point of intersection of the supply curve of

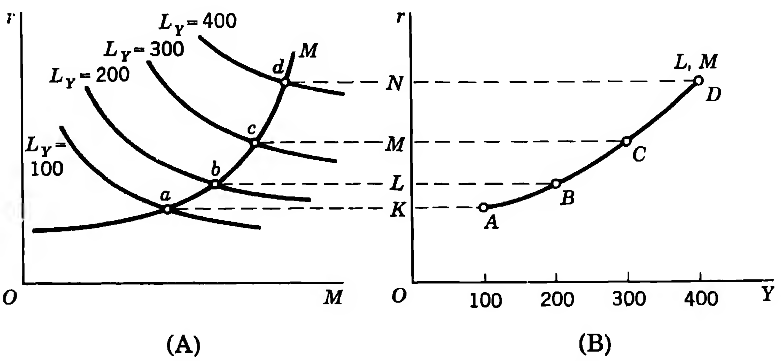


Figure 65. Derivation of *L, M* Curve Reviewed.

money with the liquidity preference (demand) curve for money. Since the demand for money depends partly on the level of income, it is necessary to draw in the *L* curve for representative

[4] Kaldor, *op. cit.* p. 70, does note a monetary factor, but he does not accord it formal analysis.

values of income; say 100, 200, 300, 400. In Figure 65(A) this procedure generates four L curves, appropriately labeled with subscripts. In Figure 65(B) the four given values of income—100, 200, 300, and 400—are laid out along the income or horizontal axis. In the left-hand figure the intersection of the L curve generated by an income of 100 with the M curve determines an equilibrium at a. The corresponding rate of interest is represented by a horizontal line extended over to the right from a. By extending this line over to the right-hand figure from a, and finding the intersection with the vertical line extended upward from the income value of 100 we determine the point A. The point A represents the interest rate, OK in the figure, which provides equilibrium in the money market when the level of income is 100. By the same technique we find points B, C, and D giving the interest rates corresponding to incomes of 200, 300, and 400, respectively. Connecting points A, B, C, and D with a smooth curve we find the L,M curve.

In order to complete the basic apparatus an I,S curve is also necessary. Such a curve marks out combinations of income and interest at which saving and investment are equal. By reason of changed assumptions about investment let us reconsider the L,S curve.

As in the original analysis of the I,S curve, assume the influence of interest on saving to be negligible. On the other hand, assume that a *decline* in the rate of interest *increases* investment by a significant amount. In the upper part of Figure 66 are represented the saving and investment curves, considered as functions of income. In this upper figure a reduction in the rate of interest raises the investment curve, considered as a function of income, while a rise in the rate lowers the curve. When the rate of interest is 6 percent, the solid I curve is generated by varying the level of income. Owing to the extended S shape of the investment curve and the linear form of the saving curve, no less than three points of equality between I and S are to be found—a, b, and c. Each such point defines a level of income at which I and S are equal. A vertical is dropped from each of these points to the figure below. At an interest rate of 6 percent a horizontal line meets the vertical lines in points A, B, and C. By varying the rate of interest the investment curve is shifted and new points of equilibrium generated in

the figure below, as shown. The resulting I,S curve resembles an S looking down at the Y axis. Note that points at which the I curve touches the S curve correspond to horizontal or flat points of the I,S curve. Thus point x in the upper part of the figure

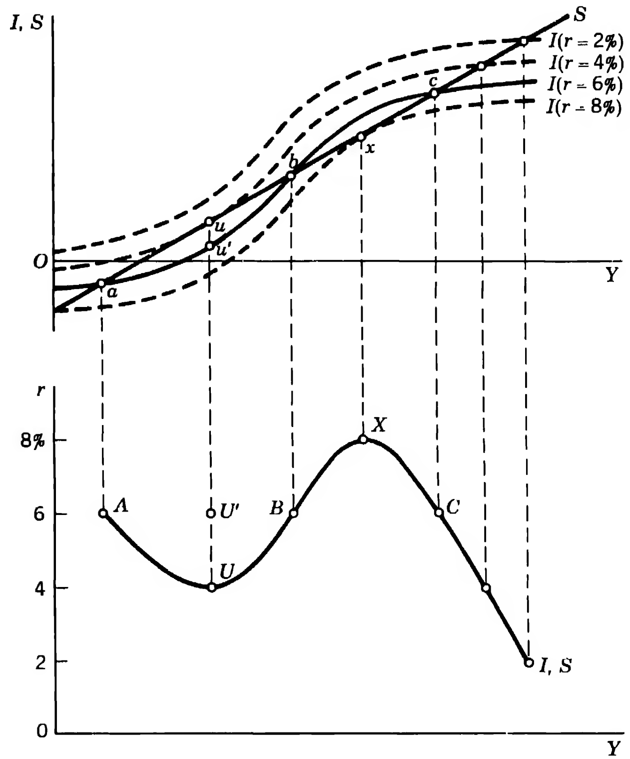


Figure 66. Derivation of I,S Curve for Modified Kaldor Model.

corresponds to X in the lower part. In the Kaldor theory such points of tangency between the I and S curves marked the principal turning points of the cycle.

The I,S curve in the income–interest plane defines that set of combinations of income and interest at which I and S are equal. Points which lie above the I,S curve are characterized by higher interest rates and an excess of saving over investment. Why?

Since a rise in the interest rate reduces investment without affecting saving, investment falls short of saving. Let us illustrate this relation graphically.

Consider point U in the lower diagram which represents a combination of income and interest at which I and S are equal. In the upper diagram an interest rate of 4 percent generates the investment curve $I(r = 4\%)$, which touches the saving curve S at u directly above U on the I,S curve. If the rate of interest is raised to 6 percent at the same level of income, the point U' above U is generated in the lower figure, income being the same but the interest rate 6 percent instead of 4 percent. With an interest rate of 6 percent the solid investment line $I(r = 6\%)$, is generated; this curve lies below the $I(r = 4\%)$ curve throughout. At the level of income represented by the vertical line $UU'u'u$ actual investment on the $I(r = 6\%)$ curve is found at u'. On the other hand saving is found at u on the S curve. Since the saving point, u, lies above the investment point, u', an excess of saving over investment equal to $u'u$ develops. Accordingly at point U' in the lower figure an excess of saving over investment exists.

By corresponding reasoning a reduction in the rate of interest below the one found on the I,S curve causes investment to exceed saving. To summarize, a point below the I,S curve is characterized by an excess of investment over saving. Let us now consider the application which can be made of this set of relations.

Consider a point U' lying above the I,S curve, and characterized by the same income and a higher rate of interest than point U on the I,S curve. Point U' corresponds to an excess of saving over investment equal to $u'u$ in the upper diagram. But an excess of saving over investment leads to a decline in the level of income. As we recall, saving can be regarded as an outflow from the "pool of income," while investment is a corresponding inflow. When the outflow represented by saving is exactly equal to the inflow represented by investment, income or the level of water in the pool remains constant. If more water should drain out of the pool through saving than flows back in through investment, the level of water in the pool or income will fall. Since point U' which lies above the I,S curve generates just such a situation, income will tend to fall.

To summarize, a point lying above the I,S curve is characterized by a contraction of income. By reversing the reasoning a point lying below the I,S curve is characterized by an expansion of income. Finally any point on the I,S curve is marked by the equality of saving and investment and a stable level of income.

In the following analysis it is assumed that the money market is always in equilibrium. On the other hand, saving is assumed to lag behind investment. For this reason the income–interest combination always lies on the L,M curve, but on the I,S curve only at points of equilibrium. Consequently, the points mentioned in the discussion which follows—T, U, etc—lie on the L, M curve, but not necessarily on the I,S curve. By combining these assumptions with the results of the preceding paragraph we can proceed to a discussion of the business cycle.

In a depression the economic system may find itself in equilibrium at the low level of income found at T in Figure 67(A). At this low income level capital is wearing out. As capital wears out the tendency to invest increases, since the remaining capital is scarcer and the rate of return increases. As a result investment rises relatively to saving at any given level of income. In order to equate saving and investment at a given level of income it is necessary to raise the rate of interest. Presumably, there is some definite advance in the rate of interest which will reduce the level of investment to equality with saving at the same level of income. As this advance in interest occurs, the I,S curve shifts upward in the direction of higher interest rates.

When the I,S curve shifts upward, the equilibrium point, marked by the intersection of the L,M and I,S curves, also shifts. Eventually, the I,S curve rises until it is just tangent to the L,M curve at point U in Figure 67(B). At this juncture the model is teetering on the brink of an expansion phase which would be brought into being by an arbitrary increase in the level of income. If this arbitrary increase does not occur, disinvestment will continue and the I,S curve experience a further upward shift. At this juncture the I,S curve lies above point U on the L,M curve. Since point U lies below the I,S curve, investment exceeds saving and income expands. In our model, then, the equilibrium at U

in Figure 67(B) depicts the situation which exists immediately prior to the expansion phase.

By our assumptions the path of expansion lies along the L,M curve, as depicted by the arrows in Figure 67(B). Since the I,S curve lies above the L,M curve at incomes immediately to the

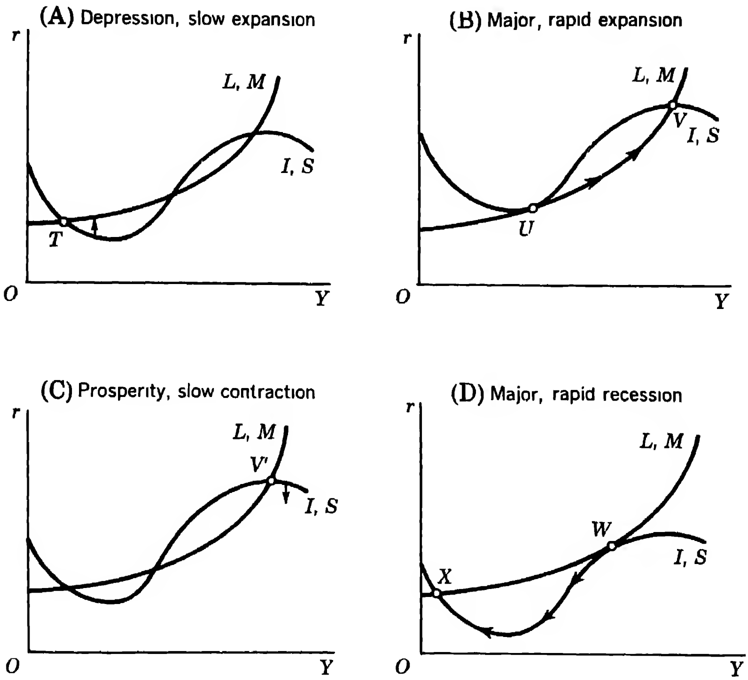


Figure 67. Kaldor Model with Monetary Factor.

right of U, income expansion continues until a new point of intersection is reached between the L,M and I,S curves. Such an equilibrium is represented by point V in Figure 67(B). During this phase expansion of income takes place in a relatively rapid manner to a higher income level.

At the new and higher income level capital accumulation occurs and the I,S curve begins to shift slowly downward. Such a condition is represented in Figure 67(C) with an equilibrium at V'. As capital continues to accumulate, and the I,S curve falls further,

the equilibrium point finally shifts to the critical point W shown in Figure 67(D). At this juncture the slightest decline in the level of income will cause the I,S curve to fall below the L,M curve with the result that income will continue to fall. If this accidental event fails to happen, a further increase in the capital stock will cause the I,S curve to fall, the I,S curve will lie below the L,M curve and contraction of income to X in Figure 67(D) will take place. We neglect changes in capital stock during the contraction, since the contraction time is presumed to be fairly short.

COMPARISON WITH KALDOR THEORY

It is clear that the Kaldor theory with monetary factor yields results essentially similar to the original theory. In one significant respect the theories differ, however. In the Kaldor theory the rate of interest is tacitly assumed to remain constant at a rate fixed by the monetary authority. In the diagram this is reflected in a horizontal L,M curve, the rate of interest being the same at any income level. The critical points of tangency between the I,S curves and the now horizontal L,M curve take place at R and T in Figure 68. With the assumption that the level of income affects the demand for money and thereby the interest rate the L,M curve is positively inclined, being shown as L,M_1 in the diagram. With the altered assumptions the tangencies occur at points such as R' and T'. Dropping perpendiculars to the income axis, we see the relation between the two cases. In the Kaldor case the range between critical points is AB, expressed in terms of income. In the revised theory with monetary factor this range is contracted to CD, lying within the old stretch.

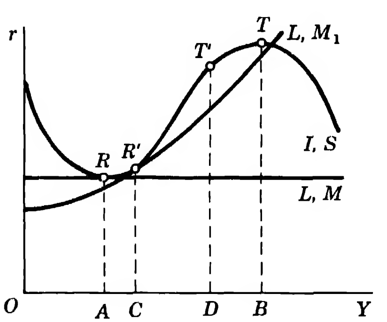


Figure 68. Effect of Monetary Factor on Kaldor Model.

Note that the range between critical points is not the range between the trough and the peak of a cycle. In fact, the trough comes at a lower level of income than the critical point in each

case, and the peak comes at a higher level. Consequently the range between critical points is less than the range between extreme points.

DEADLOCK

Referring back to Figure 67(A), we find the economic system in a depression equilibrium at T. Since the demand for goods is low, firms allow their capital equipment to wear out without replacement. As this happens the I,S curve shifts upward. Supposedly this upward shift continues until point U in Figure 67(B) is reached. If the circumstances are such that a point like T is reached which is "locally stable," and if demand is sufficient under the circumstances to support full replacement, the I,S curve may assume a fixed position of (locally) stable equilibrium. In this event the economic system may become stalled in a situation of underemployment equilibrium. Only some set of outside forces will serve to raise the I,S curve or lower the L,M curve to the critical point exemplified by U in Figure 67(B).

What is to bring the economy out of a deadlock of this kind? Perhaps the most promising possibilities lie in certain dynamic trends such as technological progress and population growth. These forces tend to raise the profitability of investment by lowering cost or raising demand. By the same token they will operate to raise the L,S curve. Unfortunately, the conditions of a depression militate against both these phenomena. Nowadays most of the inventions which are put to use in business are developed by corporations with the aid of corporate income. If a deep depression develops, high corporate officials may be more and more reluctant to provide research funds out of shrunken net income. At the time when the economy most needs this stimulus both the means to support it and the inclination will probably be weakest.

What about population growth? If population growth should take place at some steady rate, consumption out of a given total income would tend to rise, saving to fall, and the I,S curve to rise. (A higher rate of interest would be required to reduce investment to a level of equality with reduced saving at a given level of income.) Such a development would provide the desired movement of the economy toward the critical point. But not so fast! If the evidence of the 1930's means anything, population trends

may be related to the general framework of anticipations existing in society. When the economic picture is dark and uncertain, marriage tends to be postponed, education is extended, and married couples consider carefully the drawbacks of childbirth in a troubled world. In short, we must not take too much comfort from population trends, because these trends may be interrupted by a modification of basic economic conditions.

GROWTH

Is a "deadlock" possible at the upper end of the cycle? In short, it is possible for the economy to attain and remain at a high level of income for a long period of time? Consider several factors which may affect the situation. The short-run saving function tends to shift downward with time; this tendency corresponds to the upward shift characteristic of the short-run consumption function. In addition, technological innovation may cause investment to increase at any given rate of interest. As we know, it is the accumulation of capital which causes investment to decline after the prosperity phase has been attained. If investment is buoyed up by inventions or innovations and saving declines at a given level of income, the two may be equalized at an unchanged or higher level of income. In this event the I,S curve either remains stationary or shifts upward in the direction of a higher interest rate. This being the case, the equilibrium point will *not* move to a critical point followed by a major contraction.

A completely stationary condition would be symbolized by the equation, $\Delta i/\Delta t = 0$, where i is the "natural rate of interest," t is time, Δi is a change in the "natural rate," Δt is an interval of time, and $\Delta i/\Delta t$ is the rate at which the "natural rate" changes through time. (The natural rate of interest is the rate which equates saving and investment at a given level of income; it is the interest rate on the I,S curve at a given income level.) If the natural rate rises through time, and $\Delta i/\Delta t > 0$, an expansion of real income occurs. In the usual case analyzed, where innovation and the decrease in the propensity to save are less important than the effect of capital accumulation, $\Delta i/\Delta t < 0$. In this event the I,S curve shifts downward to the critical point, leading to a major contraction.

RESTATEMENT OF THE ARGUMENT

The argument which we have just reviewed can be recast in possibly more convenient terminology. Let us refer to the interest rate which equates saving and investment as the "natural rate of interest." On the other hand, let us entitle the interest rate which equates the supply of money with liquidity preference "the money rate." At a given level of income the natural rate is found on the I,S curve at a point immediately above that income, and the money rate is found on the L,M curve.

When the natural rate exceeds the money rate, it pays to borrow in the money market in order to invest in the capital market. Such investment increases income through the multiplier and leads to expansion. At the same time such a discrepancy involves the creation of additional amounts of money. Conversely, when the money rate exceeds the natural rate, funds which would normally flow into the capital market are used to retire bank debts, investment is constrained for lack of funds, and income falls through the operation of the multiplier. At the same time the repayment of bank debts leads to an extinction of a part of the money supply.

In the expansion phase of the cycle the natural rate exceeds the money rate. In turn, this leads businessmen to borrow when funds are cheapest, to increase investment, and to promote a general expansion of income. Since an expansion of income extends the opportunities for investment, the natural rate rises further, still exceeding the money rate. Eventually, the money rate begins to rise rather sharply, owing to the extension of credit to that point at which excess reserves of banks become exhausted. By this time, too, capacity production has been reached and the pressure on the capital goods industry has caused a rise in the price of capital goods. Such a development tends to lower the return available from investment, and the natural rate tends to fall. Eventually, the expansion phase ends when the natural rate falls or rises more slowly than the rising money rate.

In the declining phase of the cycle the natural rate falls below the money rate. In turn this leads individuals to direct their savings to paying off bank debts in preference to new investment. This development both reduces investment and, ultimately, income

and leads to a contraction in the quantity of money. As income declines, the consequent reduction in consumer demand reduces the opportunities for investment, and thus depresses the natural rate still further. At the same time the reduction in the level of income reduces the transactions and finance demands for money, leading to a decline in the money rate. As the natural and money rates decline with income, the money rate exceeding the natural rate, a stage is eventually reached where the natural rate levels out or rises. Certain long-term capital projects yield returns which are insensitive to current income, depending instead on conditions in the near and distant future. As income declines, such investment tends to be maintained, while saving drops off unabated. Ultimately, the decline in the natural rate with income must be checked or replaced with a rise in order to reduce investment to equality with saving. As the natural rate levels off or rises, it eventually becomes equal once again to the money rate, restoring equilibrium.

While we could restate the argument about the critical points in this terminology, it does not seem to be strictly necessary. The reader may, if he wishes, take this as an exercise in the use of these terms.

Expansion phase in time—Kaldor model with monetary analysis[5]

In the revised analysis the possibility of deriving a time path of income seems to have receded farther into the background. Using our assumption of continuous equilibrium in the money market, we can partially remedy this defect. Consider the saving–investment figure. The investment curve can be modified to take account of changes in the interest rate. Such changes occur when income (say) rises, increasing the demand for money and causing the interest rate in the money market to rise. Saving is not affected greatly by changes in the interest rate so that we can ignore changes from this direction.

In Figure 69 the curves labeled I represent investment as functions of income. Taking one of the curves, I_4, we may note that

[5] See the note in Kaldor, *loc. cit.*

it is drawn up on the assumption that the rate of interest is 4 percent. A similar statement holds good for each of the other I curves. Suppose we assume that the current level of income is OB, as indicated in the figure, while the rate of interest is 4 percent. In order to find the level of investment we proceed along the

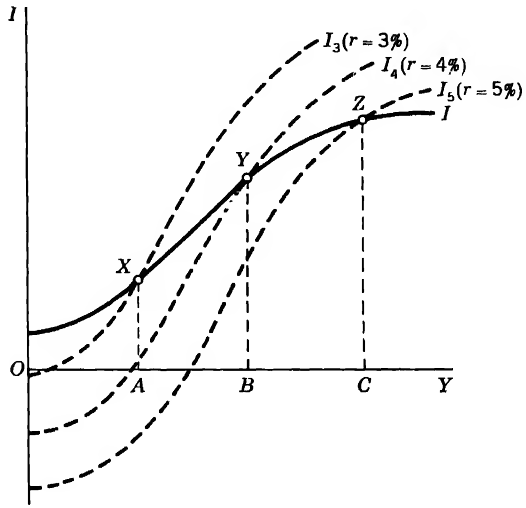


Figure 69. Investment Curve Generalized by Monetary Factor.

income axis to B and up to the investment curve drawn up for a 4 percent interest rate at point Y. To sum up the situation, a level of income OB and an interest rate of 4 percent lead to an investment rate of BY: all these conditions are realized at point Y on the I_4 curve.

In the analysis to follow the interest rate is regarded as a function of the level of income which is taken to be the independent variable. Let income increase by an arbitrary quantity BC from OB to OC. As a result of the increased income the transactions and finance demands for money increase. With a stronger demand for money and an unchanged supply the rate of interest is forced to a higher level, say 5 percent. In consequence, it is necessary to refer to curve I_5 to determine investment at the higher income level OC. In order to find the level of investment

we proceed along the income axis to C and up to the I_5 curve at Z. Point Z defines investment at an income level OC and an interest rate of 5 percent.

In exactly the same way a reduced income of OA leads to a reduced interest rate of 3 percent, a higher investment curve I_3, and investment AX. Also point X summarizes the relevant conditions. Passing a curved line through points X, Y, and Z, we secure a generalized investment curve. Such a curve takes account of both the direct effect of changes in income on investment and the indirect effect operating through induced changes in the rate of interest.

Let us assume that we are given the consumption function, stating consumption as a function of income. (We ignore the presumably weak effect of changes in the interest rate on consumption.) With the information presently in hand, we can superimpose the investment generated at each level of income on the consumption generated at this level to give total spending, $C + I$, at each corresponding level of income. Such a $C + I$ curve is generalized by including the consequences of repercussions produced by changes in income on the rate of interest.

In the discussion on evaluation of the Kaldor model we showed how a nonlinear $C + I$ curve, taken in connection with a Y line and a unit lag of $C + I$ behind Y, generated a half-bell-like time series of income rising to a peak. What we have now done is to provide a generalized $C + I$ line as an underpinning of the resulting time path of income. By comparing the generalized I curve in Figure 69 with the I curves drawn up at specific interest rates we see that the generalized I curve is flatter. By the same token the generalized $C + I$ line is flatter. In consequence, the generalized $C + I$ line will produce a somewhat flatter time path of expansion for income. This calls for a brief word of explanation.

In the first version of the Kaldor model the assumption is made that the rate of interest is constant. In effect, this means that the monetary authorities will add to or subtract from the money supply that exact amount necessary to keep the rate of interest constant. When expansion of income occurs in the Kaldor model, the monetary authorities are assumed to come forward with enough cash to satisfy additional demands for money induced by

the transactions and finance motives. Accordingly, these monetary needs offer no check to the expansion of income. In the later, generalized model the increase in the level of income is assumed to take place under conditions in which the monetary authority neither creates nor destroys cash except on a purely independent and autonomous basis. Consequently, as income increases, and the transactions and finance demands expand, the rate of interest rises. In turn the rise in the rate of interest checks the increase in the level of investment. In turn, this limits the rise in total spending and, thereby, the rise in the level of income.

In the generalized model, the monetary factor restrains the fall of income. A decline in the level of income reduces investment and total spending. However, the decline in income also reduces the transactions and finance demands for cash. Taken in conjunction with a fixed supply curve for cash this leads to a decline in the rate of interest. Since a decline in the interest rate tends to check somewhat the initial decline in the level of investment and total spending, the resulting decline in the level of income is moderated. If the monetary authority had insisted on a constant interest rate, subtracting cash as fast as the demand for cash fell, the indirect buoying up of investment through the interest rate would not have occurred.

The inclusion of the monetary factor in the analysis of the generalized Kaldor model produces two main effects. First, the magnitude of the income change from trough to peak of the cycle is reduced by the monetary factor. Second, the time path of income from trough to peak exhibits a more gentle increase as a consequence.

Summary of chapters 9 and 10

Much more could be said on the subject of fluctuations. Such materials as are included merely illustrate kinds of analysis commonly encountered in the field. Certain conclusions may now be stated.

1. When $C + I$ considered as a function of income is representable by a straight line, a uniform upward shift caused by a succession of equal doses of investment causes a rise in income through

time to a new, steady level. Income ascends quickly at first, then more gradually as the limit is approached.

2. The acceleration principle states that net investment depends on the change in consumption. When consumption begins to increase at a decreasing rate, investment decreases. On the other hand, investment increases, when consumption decreases at a decreasing rate (when its descent slows down).
3. Cycles of income result from a model which includes a multiplier and accelerator principle, the multiplier being one of the dynamic type with a unit time lag. These cycles are marked by symmetrical wavelike motions. According to the values of the marginal propensity to consume and the acceleration coefficient such models are marked by:
   a. regular oscillation,
   b. explosive oscillation,
   c. damped oscillation,
   d. steady divergence from or convergence to equilibrium.
4. The introduction of bounds to income, such as full employment and maximum disinvestment, are necessary to restrain income in the explosive oscillation case. Since the economic system generates the energy to produce cycles internally under this case, the case of explosive oscillation with constraints represents a promising attempt to explain cycles. Damped oscillations require outside erratic shocks to maintain fluctuations. The case of regular oscillations is considered unlikely, because MPC and the acceleration coefficient must have a product precisely equal to 1; this coincidence is not considered to be plausible.[6]
5. The Kaldor model makes use of the bounds mentioned above to set limits to the fluctuation of investment. With a nonlinear investment curve of the type used the acceleration coefficient becomes variable. With some simplifications made necessary by the nonlinear behavior of investment the Kaldor model provides an explanation of regularly recurring cycles.

[6] See J. R. Hicks, *A Contribution to the Theory of the Trade Cycle*, Oxford, the Clarendon Press, 1950. Here Hicks develops the notion in question and extends the generality of the multiplier–accelerator model in a number of ways.

6. The monetary factor may be adjoined to the Kaldor model, and the resulting model leads to the same general conclusions. The cycle appears to be constrained within narrower limits.

APPENDIX: The multiplier and economic cycles

In the chapter proper we have included only those theories of economic change which seem to flow directly out of the materials at hand. At least two other theories are not unrelated to the approach taken in this text. These are, respectively, the *reinvestment cycle*, and the *innovation theory*. These bear on the tools developed previously in that they can be used in conjunction with the multiplier to explain fluctuations in the level of national income. For the benefit of those who are interested in seeing what can be done with the multiplier in conjunction with some other assumptions these brief notes are included. By reason of brevity they do no more than indicate the outlines of the subject matter.

REINVESTMENT CYCLES

REINVESTMENT CYCLES AND THE MULTIPLIER

Perhaps the simplest theory of economic cycles is one based on replacement of capital. Suppose that an innovation occurs in a certain period; the innovation makes possible the expansion of some segment of industry on a really large scale. For some time thereafter businessmen invest in the capital equipment necessary to carry on production in this field. After a limited time the investment opportunities in this area are exhausted and investment ceases. If the capital equipment has a definite length of life, say ten years, replacement of the initial investment becomes necessary after the lapse of this period. Furthermore, the pattern of reinvestment will tend to follow the initial pattern of investment.

INVESTMENT AND INCOME PATTERNS IN TIME

Superficially, it would seem that the time pattern of income would strictly parallel that of investment. In fact, the curve of income in time should be a mere amplification of the investment curve—the amplifying factor being the multiplier. However, the time lag of income behind expenditure invalidates such an expectation. If the time curves of income and investment have the same form, a wavelike form for the time investment curve is necessary to explain the cycles of income actually

observed. As a result of the time lag, however, a regular stream of investment sets in motion a wavelike increase in income to a peak. If the investment ceases, a corresponding decline occurs. More precisely, if investment occurs regularly for a number of periods, ceases for a like number, resumes for the same number, and so forth indefinitely, a pattern of income change emerges. First, income builds up according to the multiplier pattern, declines by the same pattern, picks up by the same pattern, and so forth. The result is an alternation of high and low incomes having characteristics distinct from the alternations of investment.

A REINVESTMENT MODEL

Assume that an innovation is introduced and that in period zero entrepreneurs begin to demand capital on the basis of the innovation. In period one production or income first begins to rise. During periods zero through five investment demand is sustained at a steady level, at the end of which time investment opportunities are exhausted. Let us assume that the life of the capital equipment is twelve years. During periods six through eleven no investment demand occurs, and the old machinery wears out. In period twelve it is anticipated that the first equipment installed will soon be worn out. Consequently, replacement demand commences in period twelve and continues through seventeen. A lagged response to this demand in the form of increased income begins in period thirteen and builds to a peak in eighteen. This alternation in the investment demand pattern continues indefinitely and changelessly.

INCOME CHANGE

To show the pattern of income change we have designed a multiplier table and block diagram of the type previously used. Investment commences at a rate of 1 in period zero, and income begins to rise in period one. From periods one through six income builds up in the usual fashion. In period six investment drops to 0, which eliminates the corresponding income block of 1 in period seven. For simplicity, in Figure 70 all induced income blocks beyond the sixth in size and age are lumped together in a single residual element, labeled *R*. Thus the sixth block in order of size and "age" is $(.6)^5 = .07\overline{8}$, the top block in period six. "Older" and smaller blocks than this are generated in period seven and in subsequent periods. All induced blocks older and smaller than this are lumped together; this includes blocks which range from the seventh block onward in any given series. These elements form a triangular section marked out in Table 49.

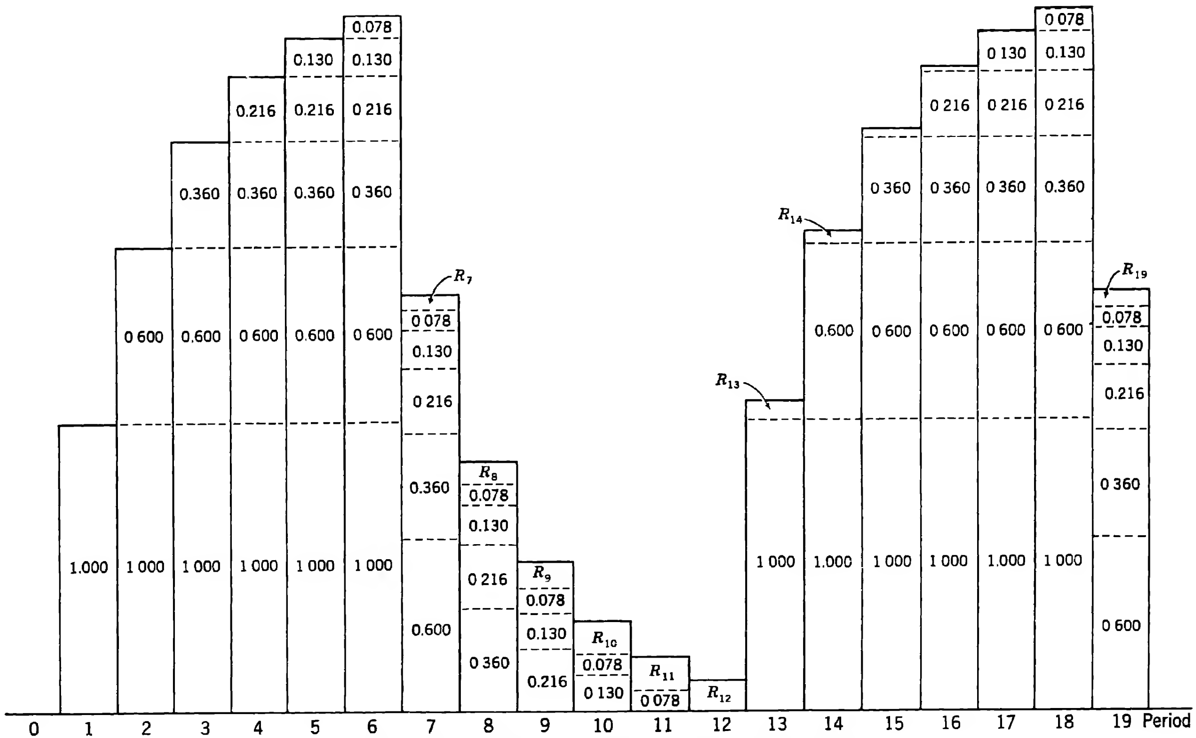


Figure 70. A Reinvestment Model of the Cycle.

TABLE 49. Multiplier Table with Alternations of Investment—Reinvestment Cycles

														.001
													.002	.002
												.004	.004	.004
											.006	.006	.006	.006
							Residuals							
										.010	.010	.010	.010	.010
									.017	.017	.017	.017	.017	.017
								.028	.028	.028	.028	.028	.028	
							.047	.047	.047	.047	.047	.047		
							R_7	R_8	R_9	R_{10}	R_{11}	R_{12}		
						.078	.078	.078	.078	.078	.078			
					.130	.130	.130	.130	.130	.130				
				.216	.216	.216	.216	.216	.216					
			.360	.360	.360	.360	.360	.360						
		.6	.6	.600	.600	.600	.600							.600
	1	1	1	1	1	1							1	1
Y	1	1.6	1.960	2.176	2.306	2.384	1.431	.859	.516	.310	.186	.112	1.067	1.640
Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14

Consider the sequence of events from period seven onward, autonomous investment having ceased in period six. In period seven the income block of 1 is removed, the investment which might have generated it having vanished in the preceding period. The only offset to this loss is the addition of the small residual block R_7. In consequence, income declines in period seven by an amount only slightly less than 1. In period eight the block of .6 is removed, the income block of 1 which would have induced it having vanished in period seven. Again income declines by nearly .6, the residual element R_8 being only slightly larger than R_7. During periods seven through twelve the blocks built up in periods one through six are successively removed in descending order of size—first 1, then .6 and so forth. Finally, only the residual element R_{12} is left in period twelve.

In period twelve reinvestment begins, leading to an increase of income in the succeeding period. From period thirteen onward the pattern begun in period one repeats itself. The only distinction lies in the presence of a small residual element of dwindling size, "placed on the shoulders" of the repeated income structure. The new buildup of income occurs like the old, with the new structure carrying a residual element which reaches negligible proportions by period eighteen. In practical effect period eighteen duplicates period six. Consequently the sequence of events from eighteen onward duplicates the pattern that began in period seven.[7]

ECONOMIC CYCLES

From the foregoing discussion it is evident that alternations of investment give rise to cycles of income. Although the pattern lacks perfect symmetry on the upswing and downswing, it is evident that a roughly wavelike motion is involved. It was stated earlier that a more even motion of investment would lead to increased symmetry in the time curve of income. Consider this point briefly.

If investment increases by one in each successive period up to a maximum and then decreases, a more rounded time curve of income will emerge. Without going into the details of verbal description we

[7] It can be proved that the residual element cannot exceed

$$(.6)(2.384)\,2.5 = (\mathrm{MPC})^6 K_6 K,$$

where 6 is the number of periods elapsing between the cessation of investment and its resumption, K_6 is the truncated multiplier or the sum of the first six blocks generated in the series, and K is the multiplier. This sum amounts to .27, but is considerably higher than the actual upper limit, and is attained only after an indefinite number of periods.

give the numerical information in Table 50. To permit a comparison of the time investment curve and the corresponding income curve we have plotted the two sequences. It is quite evident from a glance at Figure 71 that the peaked investment curve produces a more regular income curve than does the more jerky pattern also shown. In particular, troughs and peaks are smoothed out. However, all that is necessary for a regular set of peaks and valleys in the income curve is consistent alternations of investment.

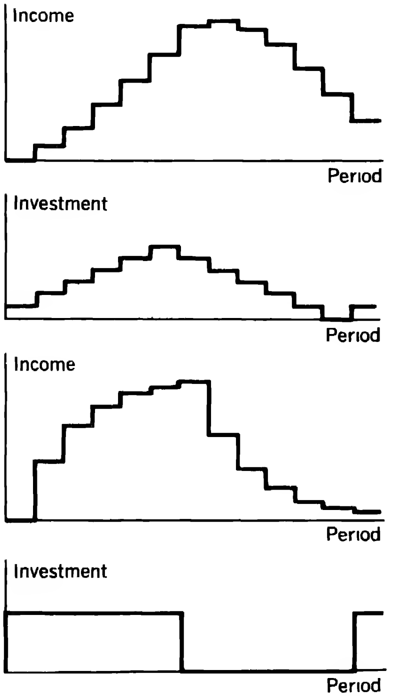


Figure 71. Investment Patterns and Corresponding Income Patterns.

MULTIPLIER AND INCOME FLUCTUATIONS

The case of reinvestment cycles is not outstandingly important in and of itself. However, it is worthy of study for two reasons. First, it affords a model with regularly recurring cycles not caused by intermittent external forces.[8] More precisely, the external forces impinge on the internal situation during just one period of time. Second, it provides an opportunity to show how time curves of investment, working through a lagged multiplier, generate cycles of income. Generally, any time shape of investment generates a corresponding income pattern through the dynamic multiplier. We have considered only two of the simplest investment patterns; the study is obviously capable of extension.

[8] In practice, the regularity tends to vanish because the machinery is not all replaced at the same time or following the same interval. The successive "echoes" of the original investment hump tend to exhibit more and more dispersion until the effects presumably are evened out completely over time. Also, owing to technological change the original machinery is replaced by other equipment having a different life span.

TABLE 50. Multiplier Table with Peaked Investment Pattern

											.01	.02	.03
										.02	.03	.05	.07
									.03	.05	.08	.11	.14
								.05	.09	.14	.19	.23	.28
							.08	.16	.23	.31	.39	.47	.39
						.13	.26	.39	.52	.65	.78	.65	.52
					.22	.43	.65	.86	1.08	1.30	1.08	.86	.65
				.36	.72	1.08	1.44	1.80	2.16	1.80	1.44	1.08	.72
			.6	1.20	1.80	2.40	3	3.60	3	2.40	1.80	1.20	.6
		1	2	3	4	5	6	5	4	3	2	1	0
I	1	2	3	4	5	6	5	4	3	2	1	0	—
Y		1	2.6	4.56	6.74	9.04	11.43	11.86	11.11	9.67	7.80	5.68	3.40
Period	0	1	2	3	4	5	6	7	8	9	10	11	12

INNOVATION THEORY

INNOVATION AND CYCLES

Closely related to the preceding analysis is a theoretical system known as the "Innovation Theory," which is due to Joseph Schumpeter. According to this theory there is a tendency for investment to occur in roughly wavelike patterns. In turn, each wave of investment is occasioned by an "innovation." An innovation is any change initiated by producers which leads to the large-scale development of an industry. On the one hand, the innovation may consist in an invention or other change in the technique of production which reduces cost and requires extensive new investment. On the other hand, it may consist in bringing out a new product or marketing an old product in an improved manner. In any event, the profitability of producing the product increases, new firms are attracted to the industry, new investment becomes necessary, and expansion occurs.

According to the innovation theory the introduction of the innovation follows a wavelike pattern. At first, only a limited number of daring, perceptive, and resourceful entrepreneurs dare to take the plunge. As they enter the field and reap large profits, others follow suit. Success rapidly breeds imitators and a wave of new firms go into the expanding field, working out the profit possibilities. Eventually, the relationship between cost and price is restored and expansion subsides. This argument defines and justifies a wavelike pattern of investment.[9]

As we have seen in the discussion of reinvestment cycles, a wavelike pattern of investment will cause an amplified and smoothed wavelike pattern of income. Evidently, the crucial question is whether the regularly recurring innovations required by the theory will in fact occur. Failing the occurrence of regular alternations of innovation, income will not follow the wavelike pattern thought to characterize the actual economic world. In the following paragraph certain objections to the analysis are listed.

DRAWBACKS TO INNOVATION THEORY

In the first place, the innovation theory depends upon a pattern of external forces. Unless these forces, innovations, behave in the required fashion the regular alternations of income will not occur. Ordinarily, economists prefer to deal with a self-contained set of variables whose interaction defines the event under consideration. In Schumpeter's

[9] See Joseph Schumpeter, *Business Cycles*, Vol. I, New York; McGraw-Hill, 1939, and *Theory of Economic Development*, Cambridge, Harvard University Press, 1934.

theory the analysis recedes ultimately to the wavelike motion of investment and its origin. This presents a new problem of equal difficulty. Therefore we have by no means come to the end of the analysis— a fact which renders the analysis somewhat unsatisfying. In all justice, however, it must be said that this account of business activity bears a distinct resemblance to behavior actually to be observed. By reason of its strong empirical content the theory has a great deal of importance.

PROBLEMS

1. Repeat the computations carried out in Table 45, with an MPC of .5 and a g of 2, so that $I = 2 \cdot \Delta C$, except that the initial conditions are: $Y = 6$ in period 0 and $Y = 2$ in period 1.
2. Repeat Table 47 with the changed initial incomes of problem 1.
3. Repeat Table 48 with the changed initial incomes of problem 1.
4. Draw block diagrams of these three cases.
5. Work out the sequence of incomes with initial incomes of 4 and 10 in periods 0 and 1, MPC (a) of $\frac{1}{3}$, and an acceleration coefficient (g) equal to 3. Why is this a case of regular oscillation?
6. Carry out the same operation with the same initial incomes as above, an MPC (a) of .6, and an acceleration coefficient (g) equal to 3. Why is this a case of explosive oscillation?
7. Carry out the same operation with the same initial incomes as above, an MPC (a) of .6 and an acceleration coefficient (g) of 1.5. Why is this a case of damped oscillation?
8. With 6 and 2 for initial incomes in periods 0 and 1, respectively, MPC (a) of .6 and an acceleration coefficient (g) of .2, calculate the sequence of incomes. Why does income decline? Check this case with the oscillation condition.
9. With initial incomes of 2 and 6 in periods 0 and 1, respectively, an MPC (a) of .8, an acceleration coefficient (g) of 3, calculate the sequence of incomes. Why does income expand? Check this case with the oscillation condition.
10. Assume that the shapes of the L,M and I,S curves are those shown in Figure 31. Assume that interest is set on day zero in the money market, while income remains constant at the level set on the preceding day. On day one, assuming interest to remain fixed, income is set in the goods (saving–investment) market and so on.
    a. Start with a given level of income on day zero. Show graphically the sequence of interest rates and incomes which follow. (Start

at a given income, go up to the L,M, line, over to the I,S line, up to the L,M line, etc.)

b. In the graph you drew does the pattern of income and interest rates converge on the intersection of the L,M and I,S curves? Is this always true?

c. Assume that the L,M and I,S curves are both straight lines. By experimentation find what slopes of these lines lead to "convergence" of the pattern onto the intersection, and what slopes to "divergence."

CHAPTER 11

Inflationary processes

When the economy reaches full employment, various forces come into operation which may lead to inflation. Most obvious of these is the existence of excess demand for goods at full employment. Let us define full employment as that level of employment at which the demand for labor is equal to the supply. Both the quantity of labor demanded and the quantity supplied are regarded as functions of the real wage = money wage/price of output. Only a change in the real wage is thought to change the quantity of labor demanded or supplied; in consequence, a proportional change in the wage and the price will affect neither the quantity demanded nor the quantity supplied. By somewhat similar reasoning the physical quantity of goods demanded for consumption depends on real income. If the level of money income and the price of consumer goods change in the same proportion, individuals will wish to purchase the same physical quantity of goods for consumption.

In the chapter on the theory of employment it was pointed out that in periods of prosperity an excess demand for goods might develop at full employment. At a static level of analysis it seems appropriate to assume that conditions in the labor market determine an equilibrium there, even at the expense of excess demand in the goods market. Owing to the existence of a sellers' market both employers and laborers are in a relatively strong bargaining position, compared to the buyer of goods. By making various kinds of assumptions of a dynamic character, however, we may achieve other results. Insofar as these assumptions weaken the supposedly superior position of labor, they involve a lower real wage for labor and a higher level of employment. Insofar as the assumed conditions strengthen labor's position, they involve a higher real wage and reduced employment. Consider a case which slightly strengthens the assumed position of the buyer of goods, and slightly weakens that of the laborer.

Wage–price spiral: A. The labor market has the last say

Assume that buyers and sellers of goods meet on day "zero" of the first week. In their competition the various parties take the money wage to be a constant, but the price and output of goods and employment are all subject to the bidding process. As a result of the bidding process contracts are made and remade until a situation results in which the aggregate demand for goods is equal to the aggregate supply. As the outcome of this situation we find the price of output, the corresponding output (real income), and level of employment. Of these three only the money price of output is subject to an unbreakable contract. Both output and employment will be subject to recontract on day "one."

On day "one" of the first week buyers and sellers of labor meet to discuss terms. In the ensuing trading process the money wage is determined. Since the money price was fixed on day zero, this leads to the determination of the real wage. During this competitive process employment and output are subject to recontract. In fact, the labor market has the last say on the real wage, employment, and output. But we have allowed the goods market a final say on the money price of goods in the first "week"; this week consists of days zero and one.

Assume that we start with a given money wage, money price, and real wage such that equilibrium prevails in the labor market, and excess demand in the goods market. In Figure 72 equilibrium prevails at E in the lower figure with real wage, $ON = W_0/P_0$. In the upper figure excess demand of RS prevails. In order to bring about equilibrium in the goods market, an increased price is necessary. Such a higher price, relative to the fixed wage, will make it worth while to the employers to expand employment and output. As a result of the competitive process, employment increases from OA to OB, the new equilibrium is reached at V, and the real wage gravitates to $OM = W_0/P_1$. The real wage is found by dropping a vertical dashed line from V to X on the demand curve for labor and horizontally over to the real wage axis at M. On day zero laborers enter into the competition only as

consumers. They take the fixed money wage and the employment figures determined as the basis for the provisional demand for goods.

On day one as employers and employees assemble in the market for labor, an excess demand for labor equal to YX exists. Such an excess demand generates a rise in the real wage. Since the money price has been fixed on day zero, this must be effected by a rise in the money wage. By the time the money wage has risen sufficiently, and the output and employment contracts remade in a final form for the week, the real wage has risen to $ON = W_1/P_1$ once more, and employment has returned to OA. The real wage, employment, and output determined in this model correspond precisely to the static results, given the strong bargaining power attributed to labor. However, some changes do occur from week to week in the money wage and the price.

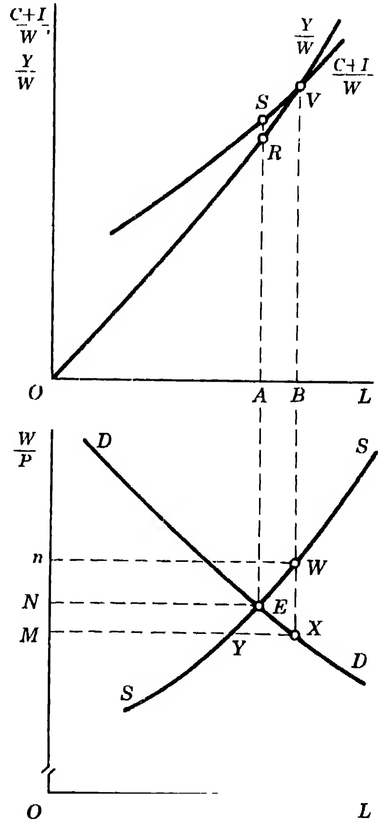


Figure 72. Basic Model for Wage-Price Spirals.

First consider the change in the money price. The real wage at the outset was equal to $ON = W_0/P_0$. In the first week on day zero the money wage remained the same, but the money price changed, to afford employers the real wage desired at the intersection of the aggregate supply and demand curves for goods. This real wage is $OM = W_0/P_1$. Then in the first week on day one the money wage is set to afford the full employment wage, $ON = W_1/P_1$, which leads to an equilibrium in the labor market.

In Table 51 this setup is illustrated. In column (1) weeks and days are listed. In column (2) the full-employment real wage ON is listed with the numerical value 10 (units of output). In column (3) the real wage OM corresponding to the intersection of the aggregate supply and demand curves for goods is listed as having

TABLE 51. Wage Price Spiral: Labor Market Has the Last Say

(1) Period		(2)	(3)	(4)	(5)	(6)	(7)
Week	Day	Full Empl. Real Wage $\frac{W}{p} = ON$	Supply Goods = Demand Real Wage $\frac{W}{p} = OM$	Money Wage	Money Price	Lagged Real Wage W_{t-1}/P_t	Real Wage W_t/P_t
				\$8	\$—		
0	Zero	10 ↓	8 ↓	↓ ( )	\$1 ↓	8	—
	One			10	( )	—	10
1	Zero			↓ ( )	1.25 ↓	8	
	One			12.50 ↓	( )	—	10
2	Zero			( )	1.5625 ↓	8	
	One			15.625	( )	—	10

the value 8 (units of output). In column (4) the money wage prevailing in any week is shown. The arrow brings the last week's wage up to the present week, so as to occupy the position shown by brackets. On the basis of last week's money wage the money price in the current week is set on day zero to make the lagged real wage equal to the value 8, which is desired by employers.

On day zero of week 0 the money wage of \$8 determined in the previous week is brought into the second row of column (4). The money price is then determined at that value which will make the

real wage equal to 8. Of course, the required money price is \$1, since \$8/\$1 = 8 (units of output).

On day one of week 0 the money price of \$1 determined on day zero is brought into the third row of column (5); the position is indicated by parentheses. On the basis of this price laborers ask a wage in money which will make the real wage equal to 10. The required money wage is \$10, since \$10/\$1 = 10 (units of output).

On day zero the employers did their best to attain the desired real wage, but gave way before the wishes of labor for a real wage of 10. In so doing they offer employment and set output in a way that is harmonious with this real wage. Their willingness to pay for labor, as expressed in the demand schedule for labor is fully taken into account by the real wage of 10. In fact, it is the buyer of goods who is not satisfied, for excess demand (equal to RS) exists in the goods market at the end of the bargaining process on day one.

In week 1 on day zero the money wage of \$10 is brought up from week 0 into row four of column (4). The money price is now bid up to \$1.25 to the end that the real wage be 8 = \$10/\$1.25. On day one of week 1 this price is brought up to row five of column (5). On the basis of this higher price a wage is set at \$12.50 to ensure a real wage of 10 = \$12.50/\$1.25. This process continues indefinitely under the assumed conditions.

Note that the wage rises from \$8 to \$10 to \$12.50; these successive values imply wage increases of 25 percent per period. At the same time price rises from \$1 to \$1.25 to \$1.5625—an increase of 25 percent per period also. Clearly, an inflationary process characterized by a 25 percent weekly increase in wage and price levels is under way.

To some extent this set-up can be illustrated with a diagram. In Figure 73 the money wage is represented along the vertical axis, the money price along the horizontal. Let the initial money wage, W_0, be scaled off on the vertical axis, having the value OA. Starting from point A let the initial price be scaled off horizontally from A, having the value AB. The ratio, $OA/AB = ON$ in the preceding diagram, and represents the value of the real wage at the intersection of the supply of and the demand for labor. Point B represents the combination of money wage and money price from which the discussion begins.

Next we construct a line representing all combinations of money wage and money price which give the real wage OA/AB corresponding to the intersection of the supply of and demand for labor. Such a line is found simply by drawing a straight line from the origin passing through the point B. Thus the real wage at D and

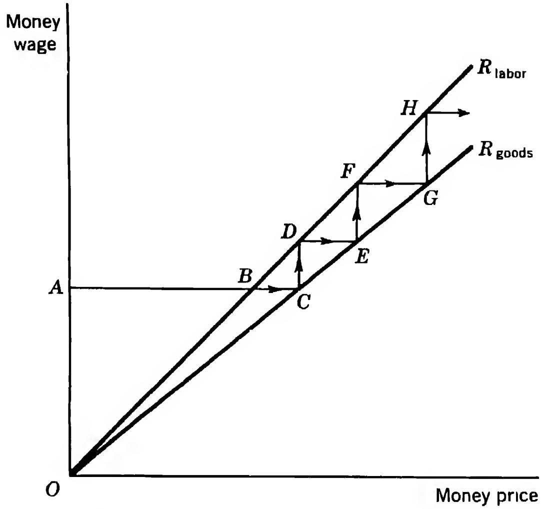


Figure 73. An Upward Wage-Price Sequence.

F is the same as it is at B. This can be proved by similar triangles. This line is labeled R_{labor} to indicate the *real* wage which affords equilibrium in the *labor* market.

Now we find the combination of money wage and price which gives rise to equilibrium in the goods market. We start again with the initial money wage, OA. From point A we scale off the price AC which affords equilibrium in the goods market. The ratio $OA/AC = OM$ in the preceding diagram. Point C represents the real wage at the intersection of the aggregate supply of and demand for goods. Drawing a straight line from the origin through C, we find the combinations of all money wages and money prices which yield this equilibrium real wage. This is labeled R_{goods}.

Starting from B on day zero in the first week, the money price increases as a result of the interplay of forces in the goods market.

This change is expressed by the movement from B to C. On day one of week 0 the money wage found at C is lower than that which affords equilibrium in the labor market. This implies the existence of excess demand in the labor market, a condition which causes a rise in the money wage represented by the movement from C to D. This movement is assumed to continue indefinitely, following a path $BCDEF$ . . . , as indicated by the arrows.

Without attempting to exhaust the implications even of this simple case, let us note two basic points. First, with strong demand for goods the labor market has the last say. For this reason the combinations of money wage and price which determine employment and output are points like B, D, and F, lying on the equilibrium real wage curve for the labor market. Second the entire area may be divided into three zones: (1) the area lying above the R_{labor} line, in which there is an excess supply of labor, and an excess demand for goods; (2) the area lying between R_{labor} and R_{goods} lines, in which there is an excess demand for both labor and

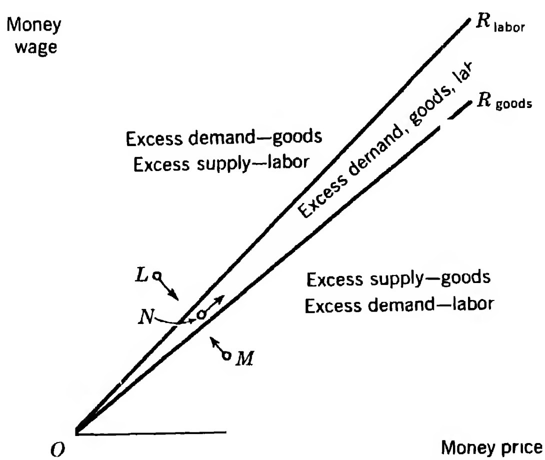


Figure 74. Background for Wage-Price Spirals.

goods; (3) the area lying below the R_{goods} line, in which there is an excess supply of goods and an excess demand for labor.

In Figure 74 these areas have been delineated. Recall that an excess demand for something causes a rise in its price, whereas an

excess supply causes a fall. Using this relationship it is not hard to see that the wage–price combinations found at L and M lead to movements, such as those illustrated, rather directly aimed at the middle zone between the curves. Generally, any point such as N *within* the curves generates excess demands in both markets, and a tendency for both price and the wage to rise. Note the following statement carefully. The tendency depicted in Figure 74 is quite independent of the particular pattern by which the rise occurs. It does not rely on the reaction pattern shown in Figure 73 and the particular assumptions on which that analysis was built.

When the demand for goods is low, and the equilibrium employment in the goods market is less than full employment, the preceding analysis may be reversed. The suggested analysis is more or less summarized in Figure 75. Note that the middle zone now

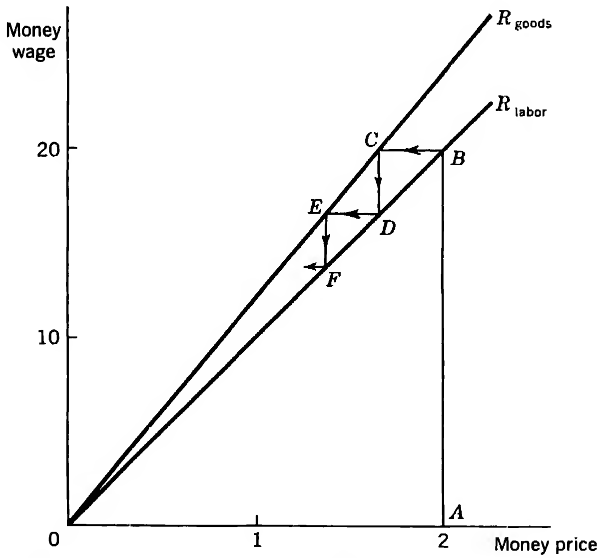


Figure 75. A Downward Wage-Price Sequence.

corresponds to excess supplies of both goods and labor. In consequence a declining pattern now ensues. Owing to wage and price rigidities this case is unlikely to find much application.

In Figure 73 it is possible to show by similar triangles that the wage changes *CD*, *EF*, etc. are proportionately the same. The same remark holds true for the successive price changes *BC*, *DE*, etc. In the ascending pattern the absolute changes grow in size. In the descending pattern, as the price and wage shrink, the absolute changes in each shrink. Furthermore, neither price nor wage reaches zero when following the descending pattern. Instead, both become very small and approach zero which is a different sort of thing.

SUMMARY OF CONDITION A. LABOR HAS THE LAST SAY

In the preceding analysis the laborer is assumed to exert an influence in wage determination at least equal to that of the employer, and superior to any influence emanating from the goods market. In each week some temporary influence of excess demand for goods on the price of goods is allowed, a procedure giving rise to an inflation of prices. In the end the real wage of the laborer is always the full employment wage and full employment is precisely maintained.

Under inflationary conditions signs of extreme economic activity are often to be noted in practice. Such conditions suggest that in some manner employment exceeds the full employment level. Unless either the employers or the laborers give way on their wage bids and askings the situation cannot arise. Under one set of conditions this is possible, namely, "the money illusion." In this situation either the employers or the laborers are slow in registering the wage and price changes which occur. The employer is expected to be sensitive to changes in the price of the product he sells, but may lag somewhat in his recognition of wage changes. The laborer is expected to be sensitive to changes in money wages, but may lag somewhat in his recognition of price changes. In either case we assume that, as buyers of goods, all parties in the economy formulate purchases on the basis of real income. This implies that everyone is sensitive to price changes as buyers of goods.

We may entitle this situation the "Somebody Gets Fooled Hypothesis." It may be applied to either employer or laborer in a formal manner. Let us start by "fastening the donkey's ears"

on the employer. We do this because the case in question bears a close formal analogy to the one just covered.

B. The "somebody gets fooled hypothesis"—employers' money illusion

Very briefly, the hypothesis is that laborers respond immediately to changes in money wages and prices, whereas employers fail to react immediately to changes in money wages. The main factor which influences their decision to produce goods and hire labor is assumed to be the price of goods. After a one-period time lag employers are assumed to recognize changes in the money wage and adjust their prices accordingly.

On day zero of successive weeks buyers and sellers of goods meet and arrive at a final settlement of the price of goods. Provisional settlements for employment and output are also made with sellers (employers) who assume that last week's money wage will prevail in the current week. Operating under this assumption, employers offer that level of employment found at the intersection of the aggregate demand for goods with the aggregate supply. This generates an offer of OB employment, as indicated in Figure 72. Dropping a vertical line from this employment level to the demand curve for labor at X and over to the real wage axis at M, we find the real wage expected by employers to be OM.

Since employers regard the money wage as constant at the level of the last period, and since the money price is determined on day zero, they regard the real wage as constant on day one. For these reasons they ignore the changes in money wages which take place during day one of each week. On day one a final settlement for the money wage, real wage, employment, and output is determined. By virtue of the employers' attitude toward money and, hence, real wages, their demand curve for labor is equivalent to the vertical line dropped downward from the intersection of the aggregate demand curve and the aggregate supply curve for goods. This line passes through B and through X on the employers' (long-run) demand curve for labor.

Since laborers are alert to changes in money wages and prices, their supply curve, consisting of the curve labeled SS in Figure 72,

is not altered. To compensate themselves for the large quantity of labor desired by the employers laborers insist on a real wage found at the intersection of the employers' demand line BX with their supply curve at W. In turn, this determines a real wage On.

TABLE 52. Wage-Price Spiral: Employers' Money Illusion

(1) Period Week	Day	(2) Desired Real Wage of Laborers, W/P	(3) Desired Real Wage of Employers, W/P	(4) Money Wage	(5) Money Price	(6) Lagged Real Wage, W_{t-1}/P_t	(7) Real Wage, W_t/P_t
				$8			
				↓			
0	Zero	12[a]	8	( )	$1	8	—
		↓	↓		↓		
	One			12	( )	—	12
				↓			
1	Zero			( )	1.50	8	—
					↓		
	One			18	( )	—	12

[a] The full employment wage is 10. Compare column (2), Table 51.

Let us now trace out the first few steps of the wage–price spiral, using specific numbers. Table 52 summarizes the steps to be discussed. Note first that the desired real wage of the laborer is higher than the full employment real wage. Since employment exceeds the full employment level, laborers ask a real wage of 12 which exceeds the full employment wage of 10. The desired real wage of employers has the value assigned in the last problem, namely 8. Compare columns (2) and (3) of Table 51.

As in the preceding case, assume that the money wage in the week preceding 0 was $8. On day zero this value is brought up into the second row of column (4). Operating under the illusion that this money wage is to be the current wage, employers determine a price on day zero such that the real wage is 8. This determines the money price of $1 and the lagged real wage, $8 = \$8/\$1$.

On day one laborers bargain for a real wage of 12 (units of output) which entails a money wage of \$12, since \$12/\$1 = 12. Employers overlook the rise in money wages, operating under the assumption that the real wage is still 8 = \$8/\$1. In short, they are satisfied to carry through employment, output, price, and money wage on this basis, overlooking the rise in the real wage.

In week 1, however, they wake up to the fact that the wage has risen to \$12 and act accordingly. To bring about the desired real wage of 8, they force the price of goods up to \$1.50. Since the ratio of the money wage of week 0, namely \$12, to the price of this week, \$1.50, is 8 = \$12/\$1.50, the solution desired by employers is apparently achieved. On day one of week 1 laborers bargain for and achieve a wage of \$18 which affords a real wage of \$18/1.50 = 12 (units of output). Again, both parties are satisfied—laborers getting what they want and employers being under the illusion that they have gotten what they want. This process may continue indefinitely.

Although the formal analysis in this case is almost the same as in the last, three differences may be noted. First, the level of employment has been pushed to a point which exceeds full employment. Second, the real wage actually paid is higher than the full employment wage. Third, the magnitude of the wage–price change is greater. The first of the results is revealed by an inspection of Figure 72. The second two points may be brought out by inspection of the preceding table and figure and the construction of another diagram in the wage–price plane.

By examining Figure 72 we see that the real wage found on the supply schedule for labor at *W*, *On*, is higher than the real wage *ON*, which corresponds to the intersection of the demand schedule for labor with the supply schedule. Pursuant to this conclusion we took the desired real wage of laborers to be 12 instead of 10 as in the preceding table.

Note that in Table 52 the money price rises from \$1 to \$1.50 between weeks 0 and 1, a rise of 50 percent. Likewise, the wage increases from \$8 to \$12 which also represents a rise of 50 percent. This increase exceeds the rise of 25 percent per period in the last case. In general the wage–price spiral is more rapid in this case. By drawing a figure in the wage–price plane we may illustrate this point.

In Figure 76 the line labeled R_{goods} represents the real wage which corresponds to the intersection of the aggregate demand curve for goods with the aggregate supply which in turn defines wage OM in Figure 72. The dashed line labeled R_{labor} represents the real wage ON found at the intersection of the demand curve for labor with the supply. The R'_{labor} line represents the real wage found on labor's supply curve at the level of employment corresponding to the intersection of the aggregate demand curve for goods with the aggregate supply. The real wage in question is On in Figure 72. Since the actual level of employment exceeds the full employment level, the real wage on the supply curve of labor is higher than the full employment wage. For this reason the R'_{labor} line lies above and to the left of the R_{labor} line in the direction of higher money wage and lower money price.

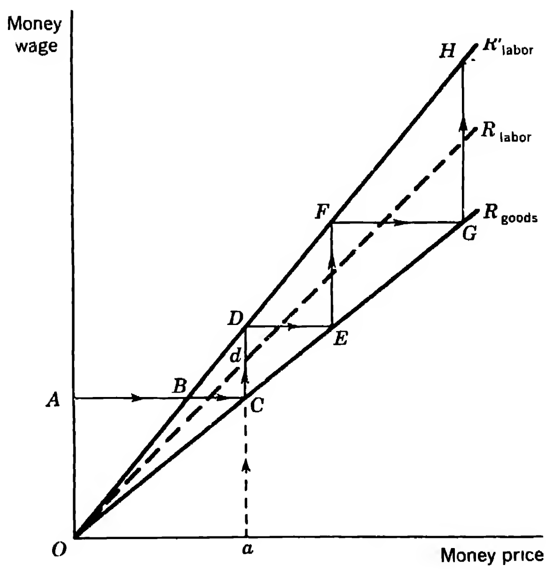


Figure 76. Wage-Price Sequence with Money Illusion.

In Figure 76 the R_{goods} and the R'_{labor} lines represent the outer boundaries for the settlements. As compared with the situation in which "labor has the last say," the R'_{labor} line is displaced upward

and to the left. The R_{goods} line is exactly the same, and the dashed R_{labor} line is the same as the R_{labor} line in the earlier case. It is inserted for the sake of comparison, but is not a factor in wage–price determination. Finally in the case previously analyzed it was the R_{goods} and the R_{labor} lines which defined the resulting wage–price spiral. Compare Figure 73.

At the outset the money wage taken from the preceding period is laid off along the vertical or wage axis, having the value OA. From A the price in the preceding period is laid off in a horizontal direction, ending at point B on the R'_{labor} line. Operating under the illusion that the money wage is to remain constant throughout week 0, employers tend to push the wage over to point C on the R_{goods} line on day zero. On day one laborers push the money wage up to secure the real wage acceptable to them at D on the R'_{labor} line. Employers offer no resistance, being under the impression that the money wage has the value OA taken from the preceding period.

In week 1, day zero employers wake up to the fact that the money wage has risen by CD and push the money price over to E on the R_{goods} line. This move temporarily brings them the desired real wage. Laborers then push the money wage up again, employers again clinging to their illusion that wages are fixed at the level of the preceding week. The result of this process is a divergent seesaw pattern $ABCDEF$ . . . .

Let us note the properties of the solution. In the first place the wage–price spiral is characterized by the same percentage wage and price changes in each successive week. By similar triangles, for example, we can prove that price changes CD and EF are proportionately the same. In the second place, all final settlements for the week are reached on day one, and are found at points like B, D, and F on the R'_{labor} line.

Finally, let us note that the rate of wage–price inflation is greater when employment is pushed past the full employment level. In this case the proportional wage change in week 0 is the wage change of CD divided by the money wage of the preceding week, OA. In the case where full employment is maintained the wage increase is only Cd, being limited by the full employment real wage line, R_{labor}.* Since CD/OA exceeds Cd/OA, the proportional

rate of wage and price inflation is greater in the present case, being 50 percent instead of 25 percent. Consequently, it makes a good deal of difference what sort of wage–price spiral is being sustained. One which is maintained to afford a spread between the supply and demand curves for labor will require a greater rate of inflation.

C. The "somebody gets fooled hypothesis"—Laborers' money illusion

Here employers are responsive to changes both in money wages and prices, whereas laborers are immediately responsive only to changes in money wages. When making their wage bargains, laborers are assumed to be slow in taking account of changes in money prices. Both laborers and employers are assumed to be attentive to changes in money prices when engaging in the purchase of goods. After a one-period time lag laborers are assumed to take cognizance of changing prices in arriving at wage bargains.

To present this case it is necessary to change the order of activities occurring on the two days of the week. On day zero of each week employers and laborers meet and arrive at a final settlement of the money wage. Provisional settlements for employment and output are also made, with laborers assuming that last week's price for goods will prevail in the current week. Operating under this assumption, laborers offer any quantity of employment which affords the desired "lagged" real wage. Employers want to produce a quantity of goods and offer a level of employment such that the aggregate demand for goods is equal to the aggregate supply. In doing so employers will wish to pay a real wage found at the corresponding employment on the demand schedule for labor.

In Figure 72 the level of employment *OB* corresponds to the intersection of the aggregate demand schedule for goods with the aggregate supply. Dropping a vertical to the supply curve of labor at *W* and over to the real wage axis at *n*, we find the lagged real wage to be *On*. Since this real wage is based on the money price of the last period, the value *On* determines the money wage necessary to elicit a supply of labor equal to *OB*.

If the money price were to remain unchanged, this real wage would exceed the level acceptable to employers at this level of employment. However, on day one the price of output is determined, and contracts for employment and output are definitely set for the week. If all goes well, the values of output and employment determined tentatively on day zero will be confirmed. Employers wish to offer employment OB in order to produce the desired output. At the same time they would like to obtain labor on more favorable terms. Dropping a vertical line downward to the demand curve at X and over to the real wage axis at M, we find the desired real wage of employers to be OM.

The money wage has been set on day zero, so that the difference between the real wage On sought by laborers (on day zero) and the real wage OM sought by employers does *not* consist in a money wage difference. Evidently, the realization of the lower wage OM desired by employers is contingent on a higher price of output than laborers expected. In short, the price must rise on day one as compared with the price prevailing in the preceding week, if the real wage is to fall to the level OM. When employers make their offer of output on day one, they seek to attain a price high enough to afford the desired real wage OM. Since laborers ignore the change in the money price when deciding on their offer of employment, the lower real wage which develops on day one produces no withdrawal of labor. In brief, the offer of labor is conditioned wholly by the money wage.

In week 1 laborers take note of the fact that the price of goods has risen. Belatedly, laborers recognize that a rise in the money wage is necessary. At the same time they continue to suffer a lag in their response, because they ignore changes in the money price which occur in week 1. Thus the process described for week 0 is repeated, but the operation requires a rise both in the money wage and price.

Assume that the money price in the week preceding week 0 is \$1. In order to induce labor to offer the quantity of labor OB in Figure 72 employers must offer a money wage of \$12, as shown in the second row of column (4). In turn, this generates the desired (lagged) real wage of 12 and satisfies the desires of the laborers. On day one, employers bargain for a price of \$1.50 in order to

secure a desired real wage of 8 = \$12/\$1.50. Since laborers are under the spell of the money illusion, they ignore the rise in price and resulting decrease in the real wage. During week 0 both parties are satisfied, employers actually getting what they want and laborers being under the illusion that they are doing so.

TABLE 53. Wage–Price Spiral: Laborers' Money Illusion

(1) Period Week	Day	(2) Desired Real Wage of Laborers, W/P	(3) Desired Real Wage of Employers, W/P	(4) Money Wage	(5) Money Price	(6) Lagged Real Wage W_t/P_{t-1}	(7) Real Wage W_t/P_t
					\$1		
0	Zero	12[a] ↓	8 ↓	\$12 ↓	↓ ( )	12	—
	One			( )	1.50	—	8
1	Zero			18 ↓	↓ ( )	12	—
	One			( )	2.25	—	8

[a] The full employment wage is 10. Compare Tables 51 and 52.
The basic supply and demand situation is assumed to be the same as in the case in which employers experienced the money illusion. Thus, the desired real wage of employers is 8, the full employment wage is 10, and the desired real wage of laborers is 12. Compare Table 52.

On day zero of week 1 laborers belatedly comprehend that price has risen from \$1 to \$1.50. Consequently, they bargain for and secure a rise in the money wage from \$12 to \$18. This causes the lagged real wage to rise to 12 = \$18/\$1.50. On day one of week 1 employers secure a price of \$2.25 which provides them a real wage of 8 = \$18/\$2.25. As usual, this change passes unnoticed by laborers when making their offer of labor. The process can continue indefinitely with no cessation under the conditions assumed. The wage and price rise 50 percent in each successive period, but the real wage actually paid is low, in this case, 8.

Figure 76 may again be used, with modifications which may be indicated verbally. The R_{goods} and R'_{labor} lines represent, respectively,

the desired real wage of employer and laborer, and the R_{labor} line represents the real wage at full employment. On day zero we start with the knowledge that the money price in the week preceding O was equal to Oa. Starting from point a the laborer seeks a money wage which will cause his real wage to lie on the R'_{labor} line. Consequently, on day zero the wage tends to shift to D on the R'_{labor} line. On day one of week 0 employers push the money price higher so that the settlement moves over to E.

The sequence follows the pattern $DEFGH$. . . . This is essentially the same sequence as we found in the case of "employers' money illusion." In this case the settlements all lie on the R_{goods} line, since points E, G etc. represent the final situation for each week as worked out on day one of successive weeks. The actual real wage, represented in this case by the R_{goods} line, is lower than the full employment wage found on the R_{labor} line. The lagged real wage, represented by the R'_{labor} line, is higher, but represents merely the illusion of a high real wage. The low actual real wage and the illusion of the high real wage persuade both employers and laborers to join in offering a quantity of labor in excess of the full employment value.

Algebraic note

Although the substance of this note is simple and intuitively obvious, it may not be entirely superfluous.

A. Labor Has the Last Say. On day zero of (say) week 1 a price P_1 is set, entailing a lagged real wage, $W_0/P_1 = OM$, in Figure 72. W_0 stands for the money wage set in week 0 and accepted by employers as the basis for the determination of price. Also, $W_0/P_1 = OM$ is the real wage acceptable to employers at an employment level corresponding to the intersection of the aggregate demand schedule with the aggregate supply schedule. On day one a wage W_1 is set, determining a real wage, $W_1/P_1 = ON$ in the same figure. This real wage corresponds to the intersection of the demand curve for labor with the supply. Since the settlement on day one of week 0 also produced the same real wage, we have: $W_0/P_0 = ON$.

For ease of notation we define two quantities: (1) $k = P_1/P_0$, the price index in week 1, week 0 being the base week; (2) $r = (P_1 - P_0)/P_0$, the proportional increase in price in week 1 over week 0. Let us note the relation between these quantities derived by the following obvious sequence of manipulations:

$$\begin{aligned} k = P_1/P_0 = (P_0 + P_1 - P_0)/P_0 &= P_0/P_0 + (P_1 - P_0)/P_0 \\ = 1 + (P_1 - P_0)/P_0 &= 1 + r \end{aligned} \tag{11–1}$$

Verbally, relation (11–1) states that the price index k equals 1 plus the proportional price increase.

Since $ON = W_0/P_0$ and $OM = W_0/P_1$, we find that:

$$ON/OM = (W_0/P_0)/(W_0/P_1) = P_1/P_0 = k. \tag{11–2}$$

Since ON also equals W_1/P_1, we also find that:

$$ON/OM = (W_1/P_1)/(W_0/P_1) = W_1/W_0. \tag{11–3}$$

Since $P_1/P_0 = k = ON/OM = W_1/W_0$, and since the letter W may be substituted for P in (11–1), it is clear that k and r are identical for P and W. In what follows we work with P only.

From the figure we note that:

$$k = ON/OM = (OM + MN)/OM = 1 + MN/OM.$$

Comparing this result with (11–1), we find that:

$$r = MN/OM \tag{11–4}$$

$$k = 1 + MN/OM \tag{11–4a}$$

We are now in a position to derive the algebraic relations between the prices in successive periods. From (11–1), (11–2), (11–4) and (11–4a), it follows by simple algebra that:

$$\begin{aligned} P_1 &= P_0(P_1/P_0) = P_0k = P_0(1 + r) = P_0(1 + MN/OM), \\ P_2 &= P_1k = P_0kk = P_0k^2 = P_0(1 + r)^2 = P_0(1 + MN/OM)^2, \qquad (11\text{–}5) \\ & \cdots\cdots \\ P_n &= P_{n-1}k = P_0k^{n-1}k = P_0k^n = P_0(1 + r)^n = (1 + MN/OM)^n \end{aligned}$$

This set of equations indicates that P grows at compound interest, that is, at a constant percentage increase per period. This percentage rate of increase is equal to r. In the numerical example

price rose at the rate of 25 percent per week. The same conclusions hold for wage changes so that the letter W may be substituted for P in equations (11–5).

B. Employers' Money Illusion. The algebra is unchanged throughout, but there is some change in what the symbols represent. In this case employers actually think that wage W_0 prevails during week one, owing to the money illusion. Also On stands for the real wage on the supply curve of labor at greater-than-full employment. Since OM is unchanged, the spread Mn is bigger, Mn/OM is bigger, $(1 + r)$ is bigger, and the rate of price increase is larger than in the first case.

C. Laborers' Money Illusion. Essentially, the algebra is the same as in the last case and with the same meanings attached to the terms as in the last case. Now on day zero laborers seek a wage On, assuming that last week's price remains unchanged. This leads them to identify the real wage for week 1 with the lagged real wage, W_1/P_0. This value is found on their supply curve at the value of employment defined by the intersection of the aggregate demand curve for goods with the aggregate supply. In short, in Figure 72 $On = W_1/P_0$.

On day one employers seek a real wage OM found on their demand curve at the level of employment defined above. Since the wage has been determined as W_1, the real wage sought is $OM = W_1/P_1$. Since this was the case during week 0, it is also true that $OM = W_0/P_0$.

We now find that the analogues of equations (11–2) and (11–3) run as follows:

$$On/OM = (W_1/P_0)/(W_1/P_1) = P_1/P_0, \tag{11–2a}$$

$$On/OM = (W_1/P_0)/(W_0/P_0) = W_1/W_0. \tag{11–3a}$$

As we can see, this is essentially the same result as before, though the values of On/OM are different from those of ON/OM.

Conclusion

It would be possible to develop this subject at considerably greater length. For example, the passage to equilibrium is described as taking place in each week. When an excess demand for goods

develops at full employment, however, it may take time for the economy to reach an equilibrium. We might describe the last two cases by the same stairstep approach used in describing the simple dynamics of National Income. Each period's expenditure would generate the income (and employment) of the next. Such an approach would generate a convergent stairstep pattern between the aggregate supply and demand curves. This approach results in a pattern of price increase which approaches as a limit the value found in the last two cases analyzed. In this case the algebraic solution breaks down, except for the limiting value, and geometry does all of the work.

We might attempt to set up assumptions leading to a wage–price spiral with rising prices at less than full employment. This would involve making assumptions about price anticipations in relation to current and past prices.

Tempting as these topics may be we must eschew them. This chapter is intended merely as an introduction to the subject. The subject matter suggests that a variety of conditions exist under which a wage–price spiral may occur. Some benefit one party, some another. The outcome in the first case is satisfactory to laborer and employer but leaves demand for goods partly unsatisfied, the standard case. The outcome in the second satisfies the demand for goods, gives labor their desired real wage at the expense of the employer with his money illusion. The outcome in the third case again satisfies the demand for goods, while giving employers the desired low real wage at the expense of the laborer with his money illusion. The several cases serve to illustrate the variety of results obtainable with different assumptions.

PROBLEMS

1. Work out a wage–price spiral table similar to the one in Table 51. Assume the full employment *real* wage is 9 and the *real* wage at which the supply of goods equals the demand is 6. Assume the money wage brought up from the preceding period is \$9. What is the percentage wage increase per period? What is the percentage price increase?
2. Draw a figure like Figure 73 illustrating problem 1.

3. Discuss the basis for assuming the particular lag used when "the labor market has the last say." Can you illustrate this situation from our economy?
4. Referring to Figure 73, prove by similar triangles that the wage increases, *CD*, *EF*, and *GH*, represent the same percentage change. Prove the same for the price increases, *BC*, *DE*, and *FG*. Finally, show that price change *BC* is proportionately the same as wage change *CD*.
5. Draw a freehand figure like Figure 72 which generates Figure 75.
   a. Show the real wage at which the supply of goods is equal to the demand.
   b. Show the full employment real wage.
   c. Considering the nature of the economic situation shown in the figure who "has the last say" in determining the real wage? Explain. In the light of this reasoning where and on whose terms are the first or tentative settlements made (on day zero) and where and on whose terms are the final settlements made (on day one) for the week?
6. Repeat for Figure 75 the zoning into excess supply and demand etc. of goods, labor, etc. carried out in Figure 74 for another situation.
7. Construct a table similar to Table 51 except that it represents a downward spiral as in problem 5. Assume that the real wage which corresponds to the equality of the supply of and demand for goods is 12, while the full employment real wage is 10. Let the initial money price existing prior to week zero be $2. On this basis work out the pattern of money wage which ensues.
   a. Will the money wage reach zero? Explain. The money price?
   b. If the money wage and the money price become small, will this impair the functioning of the economy? Explain.
8. Discuss the basis for considering that the employer lags behind the laborer in his recognition of price and wage changes. Compare problem 3.
   a. Does the employer recognize what has happened to the price and wage on day one after the final settlement has taken place? (Compare the preceding case.)
   b. Compare the real wage and employment with the preceding case (and with the full employment values).
   c. Compare the rate of price and wage inflation in this and the preceding case. Use Figure 76 to prove your statement.
9. Discuss the basis for considering that the laborer lags behind the employer in his recognition of price and wage changes. Compare problems 3 and 8.

a. Does the laborer recognize what has happened to the price and wage on day one after the final settlement has taken place? (Compare problem 8a).
b. Compare the real wage and employment with those found in problem 8b and with full employment values.
c. Compare the rate of price and wage inflation with the case in problem 8c and the case in the text. Use Figure 76 to prove your statement.

CHAPTER 12

Notes on the value of capital and the marginal efficiency of capital

In this text we do not attempt to deal systematically with problems of capital. To do so would require a broadening of the entire subject matter, with consequent lengthening of the book. The focus of attention in the main is the short run under the assumption that the quantity of capital is a constant. Such an assumption sharply restricts the topics in capital theory which require treatment.

The treatment which has been given of investment determination is fairly complete. For some students, however, a sketch of a few elementary problems in accumulating and discounting values in time may prove useful. To this end the following notes are included.

Interest and capital growth

By its very nature investment in fixed capital involves the element of time. When money is invested in capital goods, an interval will elapse before the funds invested can be recovered. If the money is used to acquire an inventory of goods to be sold, only a few months will elapse before the goods pass into the hands of customers and yield their returns. Such stocks of goods are entitled *circulating capital*. In the case of money fixed in buildings and machinery a much longer period is involved. For example, a building designed to house a factory, the life of which is expected to be twenty years, may be erected. Only after this period has elapsed will the investment yield its full return. Such investments are entitled *fixed capital*.

By the very fact that capital yields returns which are deferred in time, its value depends on these deferred returns. Furthermore,

the question arises whether more distant returns have the same value as closer ones. In our society the distant returns are less valuable than today's returns. To find the value of future returns it is necessary to *discount* these future returns to the present. In order to discount future returns it is necessary to have in hand the relationship between present and future income. Clearly, this involves the question of how money accumulates at interest through time. Conversely, it involves how future sums can be discounted to the present by a relationship involving interest.

HOW SUMS ACCUMULATE AT INTEREST

Let us consider how a sum accumulates at interest. Suppose we picture a man who invests his money for a one-year period and draws interest for the period. At the end of the year he will have recovered his original capital plus interest on his investment. Symbolically, if a is the sum invested and i is the rate of interest, the individual will begin the year with a sum a and will end the year with a sum $a + ai$. For example, if a is \$100 and i is 4 percent or .04, then $a + ai$ is \$100 + \$100(.04) = \$104. Let us note that in the sum, $a + ai$, a can be factored out and the sum written $a(1 + i)$ = \$100(1 + .04) = \$100(1.04). The factor $(1 + i) = 1.04$ is the accumulation factor or the proportion in which the capital sum is magnified by reinvested interest in the course of a year.

At the end of the first year the individual reinvests the entire sum, principal and interest, for another year. Writing $a(1 + i)$ as a_1, we see that by the end of the second year the sum a_1 will have grown to $a_1 + a_1 i = a_1(1 + i)$. Since $a_1 = a(1 + i)$, $a_1(1 + i) = a(1 + i)(1 + i) = a(1 + i)^2$, it is clear that a has grown to $a(1 + i)^2$ at the end of the second year. Evidently, the capital sum attained at the end of the third year is derived by multiplying $a(1 + i)^2$ by the accumulation factor $(1 + i)$, giving the amount $a(1 + i)^3$. Let us arrange the result in tabular form.

TABLE 54. Capital Sum Attained in Successive Years

Year	0	1	2	3	—	n
Capital Sum	a	$a(1 + i)$	$a(1 + i)^2$	$a(1 + i)^3$	—	$a(1 + i)^n$

Summary of Capital Accumulation

This discussion can be summarized rather briefly. A capital sum, a, which accumulates for any number of periods, n, at a rate of interest, i, attains a value, $V = a(1 + i)^n$ at the end of the nth period. Intuitively, it seems obvious that this process can be reversed. In so doing we come to consider the problem of discounting future values back to the present.

DISCOUNTING FUTURE VALUES BACK TO THE PRESENT

Suppose a sum b is to accrue to the individual n years from today. What is the value of the sum b, due in n years, *today*? At first blush we might consider its value as b, but reflection reveals that this is not the case. Some amount P, smaller than b can be invested today and grow at interest through time, attaining the value b in n years. Exactly what sum would have to be invested to obtain b in n years? To answer this is tantamount to answering the following precise question. What sum of money P, invested today at a rate of interest i, will attain the value b in n years?

The above question can be thrown into the form of an equation. We know that any sum P, invested at a rate i, will achieve the value $P(1 + i)^n$ after the lapse of n years. Set this value equal to the known amount b, giving:

$$P(1 + i)^n = b.$$

In this equation $(1 + i)^n$ and b are known, because i, n, and b are given, and the values in question are given or can be calculated. Evidently, the unknown quantity is P. Solving for the unknown in terms of the known elements we get:

$$P = b \frac{1}{(1 + i)^n} = b(1 + i)^{-n}.$$

The second expression on the right is merely an alternative way of stating the first such expression. The equation states that the present value, P, of a sum, b, due in n years, is b times the discount factor, $\frac{1}{(1 + i)^n}$.

In Figure 77(A) we see how a sum $1 accumulates at an 8 percent rate of interest for 20 years until it reaches $4.66.[1] In Figure 77(B) we see that the future sum $1, accruing fifteen years from now takes on a present value which depends on the rate of

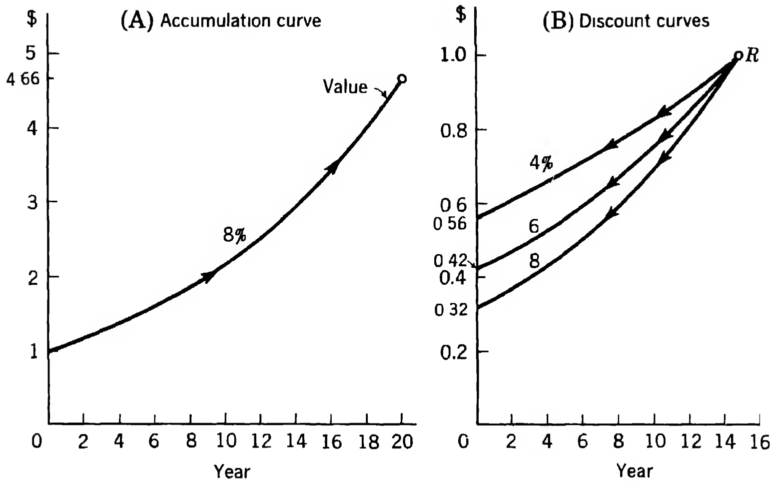


Figure 77. Accumulation and Discount Curves.

interest. If the rate is 8 percent, the present value of $1 is $.32; if the rate is 6 percent, the value is $.42; if the rate is 4 percent, the value is $.56. This data may be arranged in tabular form, as in Table 55.

TABLE 55. Present Values and Interest Rates

Present Value of $1 Deferred Fifteen Years at the Corresponding Rate of Discount $= 1/(1+i)^{15}$	Rate of Discount $= i$
$.32	8%
.42	6
.56	4

SOURCE *Mathematical Tables*, pp. 207–208.

[1] *Mathematical Tables from Handbook of Chemistry and Physics*, 3d ed. Cleveland, Chemical Rubber Publishing Co., 1933, p. 200.

FORM OF THE ACCUMULATION CURVE

Perhaps the student may wonder why the accumulation curve is not a straight line. A straight line curve is characterized by the property that as the independent variable is changed by successive equal increments, the dependent variable also changes by equal amounts. If the capital sum stayed the same throughout the fifteen periods, or if the return were computed as a percentage of the initial investment, this would be the case. Say the interest rate is 4 percent and the capital sum is \$1. In each year 4 percent of \$1 would yield \$.04, the amount by which capital would increase. In fact, however, after the first year interest is earned on interest. By the end of the first year the capital sum has grown from \$1 to \$1.04. Since the \$.04 is reinvested at interest, it earns \$.04(4%) = \$.04(.04) = \$.0016 or 16/100 of a cent during the second year. Thus interest is compounded, and capital accumulates at a faster and faster absolute rate, as measured in dollars. However, the rate of percentage growth is always the same, 4 percent.

PRESENT VALUE OF AN ANNUITY

If capital assets yielded their returns in one year alone, the treatment of the subject which we have given would be rather complete. In this case the value of a capital asset would be the return, b, discounted back to the present. For example, suppose a man stores some grape juice in a keg and allows it to mature for twenty years until it becomes a good grade of wine worth (say) \$100. If the rate of interest is 5 percent, the present value of the wine equals $\$100(1 + .05)^{-20} = \$100(.37) = \$37$. We can also infer that the purchase and storage of the grape juice is worth while, if the initial cost of the asset (grape juice and keg) is less than \$37.

Unfortunately for such simplicity, it is common for assets to yield returns in each of a number of successive years—b_1 in one year, b_2 in two years, and so forth. In such a case the present value of the capital is the sum of the several returns—each discounted back to the present. Thus the present value, P, of this set of returns can be written:

$$P \doteq \frac{b_1}{(1 + i)} + \frac{b_2}{(1 + i)^2} + \cdots + \frac{b_n}{(1 + i)^n}. \qquad (12\text{–}1)$$

If all the b's have the same value, call it b, the term b can be factored out, giving to the expression the following simpler form:

$$P = b\left[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \cdots \frac{1}{(1+i)^n}\right]. \qquad (12\text{–}2)$$

The above formula values the returns from an investment (say a machine) on the assumption that the returns cease when the capital good acquired wears out, no deduction being made for depreciation. When depreciation enters the calculations explicitly, the machine is treated as though it were endowed with perpetual life. To put the matter another way, when the returns are reduced by depreciation allowances, the sums thus accumulated may be reinvested so as to extend indefinitely the period over which these smaller returns accrue.

It is possible to compute the value of the sum in (12–2) by a more compact formula. To do so, however, requires the formula for a geometric progression with n terms.

Abridged Formulas for Annuities

The formula for a geometric progression with n terms is:

$$S_n = 1 + r + r^2 + \cdots + r^{n-1} = \frac{1 - r^n}{1 - r}, \qquad (12\text{–}3)$$

where r is the common ratio of each term to the term preceding it. If the geometric progression continues without limit, so that it is an infinite series, its value is:

$$S = 1 + r + r^2 + \cdots \text{indefinitely} = \frac{1}{1 - r}. \qquad (12\text{–}4)$$

In a later paragraph (p. 344) a proof of these formulas is given.

Before applying the formulas above it is necessary to put equation (12–2) in the proper form. By factoring out $1/(1 + i)$ the equation may be written in the form:

$$P_n = \frac{b}{1+i}\left[1 + \frac{1}{(1+i)} + \cdots + \frac{1}{(1+i)^{n-1}}\right].$$

The terms within brackets form a geometric progression of n terms

with common ratio $r = 1/(1 + i)$. Applying formula (12–3) to this expression, we secure:

$$P_n = b\frac{1}{(1+i)}\left[\frac{1 - \dfrac{1}{(1+i)^n}}{1 - \dfrac{1}{(1+i)}}\right] = b\frac{1}{(1+i)}\left[\frac{1-(1+i)^{-n}}{\dfrac{i}{(1+i)}}\right]$$

$$= b\,\frac{1-(1+i)^{-n}}{i} = b\left[\frac{1 - \dfrac{1}{(1+i)^n}}{i}\right]. \tag{12–5}$$

Equation (12–5) is the formula for an annuity of amount b paid in equal installments for n years. This formula is important in economics, insurance, and in business generally.

A special case of formula (12–2) is important. If the annual return b is paid in perpetuity, the asset resembles land. Securities entitled "consols" and having this property have been issued by such governments as the British. In this case n, representing the number of years for which the annuity is paid, becomes infinite. Accordingly, the series (12–2) is infinite and can be written:

$$P = b\left[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \cdots \text{indefinitely}\right].$$

$$= b\cdot\frac{1}{(1+i)}\left[1 + \frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \cdots \text{indefinitely}\right]. \tag{12–6}$$

The expression in brackets has the form of (12–4), with $r = 1/(1 + i)$ being the common ratio. Applying formula (12–4) to the expression above, we secure:

$$P = b\cdot\frac{1}{(1+i)}\left[\frac{1}{1 - \dfrac{1}{(1+i)}}\right] = b\cdot\frac{1}{(1+i)}\left[\frac{1}{\dfrac{i}{(1+i)}}\right]$$

$$= \frac{b}{i} \tag{12–7}$$

This is the well-known formula for capitalizing an annual return.

The marginal efficiency of capital

The marginal efficiency of capital may be defined as that rate of discount which will make a given set of returns have a discounted value equal to the cost of a capital asset acquired today. To find the marginal efficiency the following data must be given: (1) the cost of the capital, (2) the net return expected at the end of each year. On any given piece of capital equipment the marginal efficiency of capital is not necessarily equal to the rate of interest.

Pursuant to the definition given above, we may write the equation expressing the marginal efficiency as follows:

$$C = \frac{b_1}{1 + m} + \frac{b_2}{(1 + m)^2} + \cdots + \frac{b_n}{(1 + m)^n} . \qquad (12\text{-}8)$$

In this equation C, the cost of the machine, and the bs, the net returns in the several years, are known. Note that the bs accrue at the end of each year. To find the marginal efficiency of capital, m, it is necessary to solve this equation for m. In general, this is difficult to do. Let us note just two cases in which this operation is rather easy to perform.

If the machine lasts just one year, the equation takes the form:

$$C = \frac{b_1}{(1 + m)} .$$

Solving this equation for m, we get:

$$m = \frac{b_1 - C}{C} .$$

Since C and b_1 are known, we can compute the value of m. For example, suppose the cost of the capital C is \$100, and the return b_1, coming in at the end of the year, is \$108. In this event, the value of m is (\$108–\$100)/\$100 = .08 = 8 percent. If there is a succession of machines to be purchased whose cost and returns are known, a schedule of the marginal efficiency of capital can be computed by making successive calculations of this sort.

A second case rather easy to work with is a capital improvement which is expected to be virtually permanent. Appropriate

examples are canals or drainage projects. Assume that the returns accruing at the ends of the several years have a common value b and that the sequence of returns continues indefinitely. In this situation formula (12–7) applies. Rewriting this formula in our present notation, we get:

$$C = \frac{b}{m}.$$

Obviously, the expression can be written

$$m = \frac{b}{C},$$

which states simply that the marginal efficiency of capital equals the annual net return divided by the original cost. If the annual return, b, is \$6 and the machine costs \$150, m is \$6/\$150 = 4%.

To find the value of m in more general cases is not easy from a practical viewpoint. Recourse must be had to compound interest tables which tabulate the expression given in brackets in (12–5). Because we are seeking i or m, and not the expression in brackets, it is necessary to work backwards. This is not easy, particularly when accuracy is desired. From a theoretical viewpoint, however, the calculation can always be made, and it does not seem worth while to work out further examples.

GRAPHICAL EXPLANATION OF THE MARGINAL EFFICIENCY OF CAPITAL

Something can be done with graphs to show the meaning of the marginal efficiency of capital. In Figure 77(B) on p. 337 the investor expects a return of \$1 in fifteen years. Suppose that his initial investment was \$.42. To find the marginal efficiency of capital it is necessary to find that discount curve which, starting at \$1, passes through the value \$.42 at time zero. Each discount curve is associated with a particular discount rate—4 percent, 2 percent, and so forth. In this case 4 percent is associated with a discount rate which ends up with \$.56 at time zero. One's eye proceeds backward down the curve from R. On the other side, an 8 percent discount curve ends up at \$.32, which is too low. However, a discount curve associated with 6 percent passes backward through \$.42. This means that \$.42 invested at compound

interest for fifteen years will accumulate to \$1. Hence the marginal efficiency of capital is said to be 6 percent. The rate of interest prevailing on the market is not considered in this discussion. It may be 4 percent, 6 percent, or some other value.

SHORT- AND LONG-TERM INTEREST OR GROWTH RATES

It is possible that the rate of interest will vary from year to year, each rate prevailing for just one year. In the first year the rate is i_1, in the second, $i_2, \cdots$, and in the nth year, i_n. An individual investing a sum a in the first year would receive $a + ai_1 = a(1 + i_1) = a_1$ at the end of the first year. Reinvesting this sum at the end of the first year at i_2, he would accumulate $a_1 + a_1 i_2 = a_1(1 + i_2) = a(1 + i_1)(1 + i_2)$. Carrying this process forward, we arrive at the result that the value of the capital at the end of n periods is:

$$V = a(1 + i_1)(1 + i_2) \cdots (1 + i_n). \tag{12–9}$$

If the individual invests a sum of money in the short-term loan market at i_1, reinvests in the same market at i_2 in the second year, and so on, he will accumulate the sum indicated by V. However, an alternative is open to the investor. He may choose to invest in the long-term market at a fixed rate i for the whole n years. In this case he will emerge with the sum $V = a(1 + i)^n$. Assume that the individual has no liquidity preference, and that the speculative motive does not operate.

If the investor felt that he could accumulate a larger sum by investing in long-term securities than in short-term loans, or that $a(1 + i)^n$ exceeded $a(1 + i_1) \cdots (1 + i_n)$, he would do so. A general tendency on the part of investors to purchase long-term assets would raise their price and lower the annual return. Therefore, the two expressions should eventually come to be equal, or:

$$(1 + i)^n = (1 + i_1)(1 + i_2) \cdots (1 + i_n), \tag{12–10}$$

implying that,

$$(1 + i) = \sqrt[n]{(1 + i_1)(1 + i_2) \cdots (1 + i_n)}. \tag{12–11}$$

The accumulation factor, $(1 + i)$, is the nth root of the product of the short-run accumulation factors. By definition the nth root

of the product of n terms is known as the "geometric mean" of those terms. Therefore, the long-term accumulation factor is said to be the geometric mean of the short-run accumulation factors.

As an example, suppose the short-term interest rate for years one and two were $4\frac{1}{6}$ percent and 50 percent, respectively. The accumulation factor would be

$$(1 + i) = \sqrt[2]{1.04\tfrac{1}{6} \times 1.50} = \sqrt[2]{1.5625} = 1.25 = 1 + .25.$$

Clearly, the two-year interest rate is 25 percent. It is not the arithmetic mean of the rates $4\frac{1}{6}$ and 50 which is $27\frac{1}{12}$.

In general, the rate expressed by i can be regarded as a long-term growth rate, when the growth rates are different in every year. So the i_1 and i_2 might represent the percentage growth in autos in two successive years, while i would represent the average growth rate. It is proper in certain economic problems to work with an average growth rate of this type which defines a uniform percentage rate of growth for the entire period.

In the case of interest rates there is a tendency for long-term or average rates to move when short-term rates move. However, we are neglecting a preference for shorter-term securities, which arises from their lower possible fluctuation in value. By reason of the greater stability in short-term loans one can expect these assets to bear a lower interest rate than those of longer maturity. By a short-term security or asset is meant one which yields its full return plus any return of principal in a limited period of time, such as a year.

FORMULAS FOR GEOMETRIC PROGRESSIONS

The material presented here has been covered in substance in Chapter 9. By definition a geometric series is a set of terms each of which bears the ratio r to the preceding term. The sum of a geometric series of n terms may be written:

$$S_n = 1 + r + r^2 + \cdots + r^{n-1}.$$

If we multiply S_n by r and subtract from S_n in the way shown below, we get a simplified result.

$$\begin{aligned} S_n &= 1 + r + r^2 + \cdots r^{n-2} + r^{n-1} \\ {}_rS_n &= r + r^2 + \cdots r^{n-2} + r^{n-1} + r^n \\ \hline S_n - {}_rS_n &= 1 - r^n \end{aligned}$$

Factoring out S_n on the left-hand side, and then dividing out both sides by $(1 - r)$, we get successively:

$$S_n(1 - r) = 1 - r^n$$

$$S_n = \frac{1 - r^n}{1 - r} . \tag{12-12}$$

This is formula (12–3) on page 339.

If the number of terms, n, in S_n is increased indefinitely, S_n approaches a value which may be entitled S. S is the sum of a geometric series with an infinite number of terms:

$$S = 1 + r + r^2 + \cdots \text{ indefinitely.}$$

However, S is simply the value of S_n when the value of n is allowed to increase indefinitely. Considering the formula for S_n above, we find that if r is less than 1, r^n diminishes as n increases. If $r = .6$, $r^2 = .36$, $r^3 = .22$, $r^4 = .13$, and so forth. As n becomes very large, r^n approaches zero. Consequently, this term may be neglected in (12–12) above, and we can write:

$$S_n \text{ approaches } S = \frac{1}{1 - r} \text{ as } n \text{ becomes large.} \tag{12-13}$$

PROBLEMS

1. What is the capitalized value of a canal whose life is indefinitely long, costing \$1,000,000, yielding a return of \$60,000 annually, when the long-term rate of interest is 5 percent? Was it worth while to build the canal?
2. What is the present value of a keg of wine which can be sold for \$105 one year from now? The initial cost of putting the grape juice up into the keg totaled \$100, and the rate of interest is 6 percent.
   a. Is it worth while to make this investment?
   b. Suppose the rate of interest drops to 4 percent. Is the investment worth while? Explain.
3. Using the formula $P = A/(1 + r)^n$ and straight arithmetic find the present value of \$100 due in two years when the rate of interest is 6 percent. Check the answer from a compound interest table.

4. Using formula (12–5) as a guide, look up in a compound interest table the value of an annuity of \$1 for thirty years with an interest rate of 6 percent. Suppose the annuity is \$2 in amount. What is the answer?
5. Using a compound interest table, find the value of \$100 due in twenty years, when the rate of interest is 6 percent. Suppose the sum due becomes \$200. What is the answer?
6. Look up in a set of compound interest tables the value of an annuity of \$5 a year for twenty years, when the interest rate is 4 percent. Compare the correspondng values at thirty-five years and fifty years. Discuss the growth in value with the number of years.
7. Is there much difference in the outright sale price of a piece of property and its value under a ninety-nine year lease? Explain.
8. What is the MEC in problem 1? Which is greater, the MEC or the rate of interest? How does the answer to this question relate to the last sentence in problem 1?
9. Compare the MEC which you find in problem 2 to the interest rate. How does this bear on the answer to question 2a? How does it relate to question 2b?
10. Suppose we are given two assets, one having a one-year life with a present value given by the formula, $P = a/(1 + i)$, and the other having perpetual life with a value given by the formula, $P = a/i$. Let $a =$ \$100 and let i change from 4 percent to 6 percent.
    a. Compare the change in present value in the two cases.
    b. Which asset exhibits the greater percentage variation and why?
11. Ignoring risk considerations, what is the long-term (two-year) rate when the short term rates are 2 percent and 40 percent? (Does not come out even.)

CHAPTER 13

Investment growth and its effect on income

Since World War II the remarkable development of the Western world has overshadowed all other economic phenomena. Among the striking features of the situation is the steadiness with which growth has occurred. In large part steady growth of a system arises from a set of outside influences which impinge on the determinants of an economic system. Suppose that external forces such as population growth and innovation operate directly on the level of investment in such a way as to cause that level to rise in a definite manner. The resulting pattern of investment will work out its repercussions through the multiplier. Here we entirely ignore the acceleration principle, assuming that all investment is determined by outside forces. By considering various patterns of investment growth we will discover the corresponding patterns of income growth.

Equilibrium concepts

In dealing with the problem of economic growth it is convenient to have in hand certain concepts of equilibrium. Equilibrium means simply a balance of forces. For example, two persons exactly balanced on a seesaw are in equilibrium. In economics the most commonly used example is the equilibrium of supply and demand. Given the supply and demand curves, an equilibrium of supply and demand is found at that price and quantity where the curves cross. At this point exact equality between quantity supplied and demanded is found.

STATIC AND MOVING EQUILIBRIUM

In essence, the balance of supply and demand, as pictured above, is a static idea. That is, if all relevant economic forces are held constant, an equilibrium is reached. Furthermore, the

element of time does not enter the picture explicitly. Let us define a *static equilibrium* as that outcome which will emerge if all forces of change are impounded, and the results are allowed to work themselves out for an indefinite period of time.

An obvious generalization of static equilibrium arises when something is allowed to change at a constant rate. Suppose a wind blows at a constant rate against a propellor. The propellor will begin to revolve. All other things remaining the same, the propellor will eventually tend to revolve at a constant rate. Such a rate would constitute a moving equilibrium between the wind and the propellor. Let us define a *moving equilibrium* as that outcome which will emerge if one force (or set of forces) changes at a constant rate indefinitely. Clearly, the same proviso is made about things remaining the same.

ACTUAL DEVELOPMENT AND EQUILIBRIUM

In the real world the changes which occur in underlying variables are jerky rather than constant. It is necessary to keep in mind that actual economic variables are usually in a state of transition, rather than in equilibrium—static or dynamic. In some sense or other the actual values assumed by these variables can be said to be in equilibrium. For example, we assume in the following multiplier analysis that current production is always adjusted to last period's expenditure. This situation may be referred to as temporary equilibrium, but it defines a purely transitional state of affairs.

THE MULTIPLIER AND INCOME GROWTH

As an introduction to the theory of growth it is convenient to work with the multiplier. From this point onward let us assume that investment is purely autonomous, but grows according to some pattern. It may or may not be possible to justify the assumption of such a growth pattern for investment. Our purpose here is not to discuss this, but to consider the consequences of such growth patterns. As in the dynamic multiplier analysis we assume that there is a lag of one period of income behind expenditure. If investment increases in one period, income will increase in the next.

Growth of investment by a constant amount per period

Let us consider first the simplest case. Here investment increases by the same amount in every time period, beginning with period zero. For simplicity, assume this amount to be 1. If it is 2, the numbers are multiplied by 2 throughout.

PATTERN OF ACTUAL INCOME IN TIME

In Table 56 the growth of investment in arithmetic progression 1, 2, 3, 4, · · · at times 0, 1, 2, 3, · · · is shown. Owing to the lag of income behind expenditure income does not pick up until period 1. Each dose of investment produces an effect which can be followed along a diagonal of the table. As always, this effect diminishes with time. Each element along the diagonal is .6 of the preceding element, since MPC is .6. At any time the level of income attained is the sum of the blocks in a column, and is listed in a row entitled Y^a.

The course of income through time, under the given assumptions, turns out to have the form shown by the curve labeled Y^a (for actual income) in Figure 78. It is convex to the time axis, starting slowly, increasing more and more rapidly with time until it begins to increase at an approximately steady rate. As we shall see later, income is very nearly in moving equilibrium at the later times shown in the diagram.[1]

In the case given investment is increasing at the rate 1 per period, a process which began in period zero. If the level of investment became steady at some fixed figure, say the figure 6 attained in period five, income would tend to the level found where expenditure, $C + I$, equals income. This is the static equilibrium level of income, labeled Y^s (for static equilibrium income) in Figure 78. This static equilibrium level at Y^s is considerably greater than the actual income level at Y^a. In Figure 78 the last step shown is taken when investment rises to 6 and the $C + I$ curve rises to the position shown by $C + 6$. In the right-hand diagram, actual income, expenditure, and static equilibrium income are

[1] Compare Hicks, *A Contribution to the Theory of the Trade Cycle*, Oxford, The Clarendon Press, p. 25.

Table 56. Multiplier Table, Showing Income Growth with a Constant Investment Growth Rate

											.01
										.02	.03
									.03	.05	.08
								.05	.09	.14	.19
							.08	.16	.23	.31	.39
						.13	.26	.39	.52	.65	.78
Incomes Generated					.22	.43	.65	.86	1.08	1.30	1.51
				.36	.72	1.08	1.44	1.80	2.16	2.52	2.88
			.6	1.20	1.80	2.40	3.	3.60	4.20	4.80	5.40
		1	2	3	4	5	6	7	8	9	10
I	1	2	3	4	5	6	7	8	9	10	11
t	0	1	2	3	4	5	6	7	8	9	10
Y^a	0	1	2.6	4.56	6.74	9.04	11.43	13.86	16.31	18.79	21.27 $\rightarrow n(2.5)-3.75$
Y^s	2.5	5	7.5	10	12.50	15.00	17.50	20.00	22.50	25.00	27.50 $= (n+1)(2.5)$
Y^m	—	−1.25	1.25	3.75	6.25	8.75	11.25	13.75	16.25	18.75	21.25 $= n(2.5)-3.75$

shown at Y^a, E, and Y^s. If the growth process continues, the elements will grow in the way indicated by the directions of the curves.

From our study of the static and dynamic multiplier we know that income *ultimately* tends to increase by the increase in investment times the multiplier. For this reason, when investment rises from 0 to 1, income rises from 0 to 2.5, 2.5 equalling the multiplier. As investment rises to 2, 3 and so forth, income rises to 5, 7.5 and so forth. In short, as investment rises by 1, static equilibrium income rises by 2.5. This requires no demonstration beyond what we have already carried out in previous work on the multiplier. The values of investment and the corresponding static equilibrium values of income are listed in Table 56. By reason of the rigid character of the relation the static equilibrium income curve plotted against time has the form of a straight line.

THE DYNAMIC MULTIPLIER PRINCIPLE

In the case under discussion investment increases at the rate of 1 per period. When investment increases at a constant rate through time, income tends ultimately to increase at a rate several times as fast. In fact, the rate at which income ultimately tends to increase is the multiplier times the increase in investment per period. In this case, income ultimately should tend to increase by 2.5 times 1 or by 2.5 per period. Immediate support for this conclusion is given by the diagram. Y^a, which is supposed to obey this rule, follows a path rising only gradually at first, but ultimately increasing at about the same rate as Y^s. Since Y^s increases by 2.5 without time lag, and Y^a comes to parallel Y^s, it is pretty clear that our rule is correct.

A little consideration reveals clearly what is going on here. As investment increases, income rises only by the amount of investment at first. As the increase continues, the derived increments of income begin to pile up. Thus an increase in investment of 1 produces income of 1 in the following period. In the next period a derived increase of .6 occurs, in addition to an amount 1 from the continued increase of investment. In the next period this .6 generates an increase of .36, the 1 of this period generates .6 and the continued investment increase generates 1 of income. Ultimately, the increase of 1 in investment for each of a large number

of preceding periods gives rise to a sequence of increases, $1 + .6 + (.6)^2 + \cdots = 2.5$. In short, income ultimately rises at a rate of 2.5 per period, when investment rises by 1 per period.

Static and actual equilibrium income

When investment increases at a steady rate through time, the time lag of income (production) behind demand causes actual income, Y^a, to rise by less than the static equilibrium income, Y^s. In consequence, actual income is less than static equilibrium income. This raises some questions. Will actual income ever catch up with the static equilibrium value of income? Under our assumptions, the answer to this question is "no," a conclusion revealed clearly in Figure 78. Why does actual income fail to keep

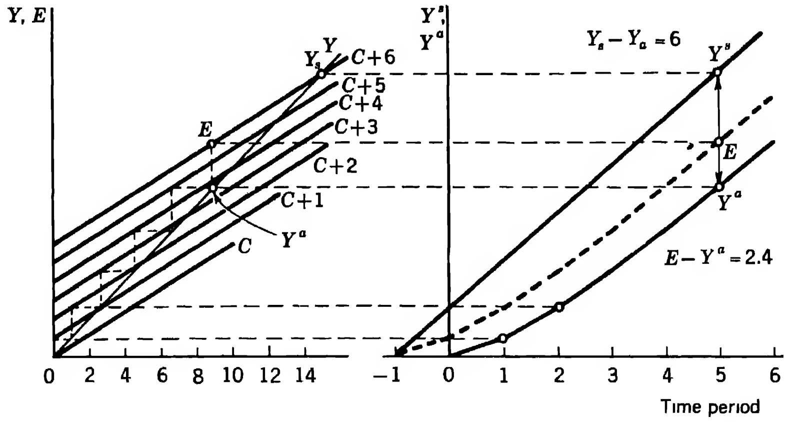


Figure 78. Income Paths When Investment Increases Steadily.

pace? It is not easy to answer this question. Income ultimately rises as fast as it can be expected to rise. Allow enough time and income will rise as fast as if no time lag existed. In the meantime, however, the lag in transforming demand into production (income) causes actual income to fall below the static equilibrium level.

Since the rate at which actual income increases never equals the static rate, the difference can never be made up. However, the

difference approaches an upper limit. In this particular case the greatest difference or the one toward which the two curves, Y^a and Y^s, tend is 6.25. The reason the difference increases but approaches this limit is that actual income eventually tends to increase as fast as static equilibrium income. Consequently, the deficiency can ultimately not be further increased. Neither can it be narrowed, as explained above.

If it is impossible for actual income ever to equal the static equilibrium level in a condition of moving equilibrium, could this rate be attained under any other conditions? Yes, if the progression of investment is broken off and a constant level maintained indefinitely, the static equilibrium level is attained. This follows from the definition of the static level as that quantity achieved when the process of change is suspended.

Accordingly, if we break off the advance of investment at some point and allow it to remain at a constant level, income will tend toward a fixed static equilibrium level. As the advance in invest-

TABLE 57. Income Pattern when Investment is Stabilized Following Steady Advance

Blocks built up from						.02	
steady progression					.03	.05	
of investment				.05	.09	.14	
			.08	.16	.23	.31	
		.13	.26	.39	.52	.52	
	.22	.43	.65	.86	.86	.86	Blocks built up
	.72	1.08	1.44	1.44	1.44	1.44	from steady invest-
	1.80	2.40	2.40	2.40	2.40	2.40	ment rate of 4
	4.00	4.00	4.00	4.00	4.00	4.00	
Inv.	4	4	4	4	4	4	
Time	4	5	6	7	8	9	
Y^a	6.74	8.04	8.83	9.30	9.57	9.74	
Y^s [a]	10	10	10	10	10	10	

[a] Y^s = Investment × 2.5 = 4 × 2.5 = 10

ment breaks off, all blocks of primary income, depending directly on investment, as well as the derived sequence of incomes, tend to build up to a sequence in which all income elements depend on the fixed investment level, say 4 as in Table 57. Thus the primary

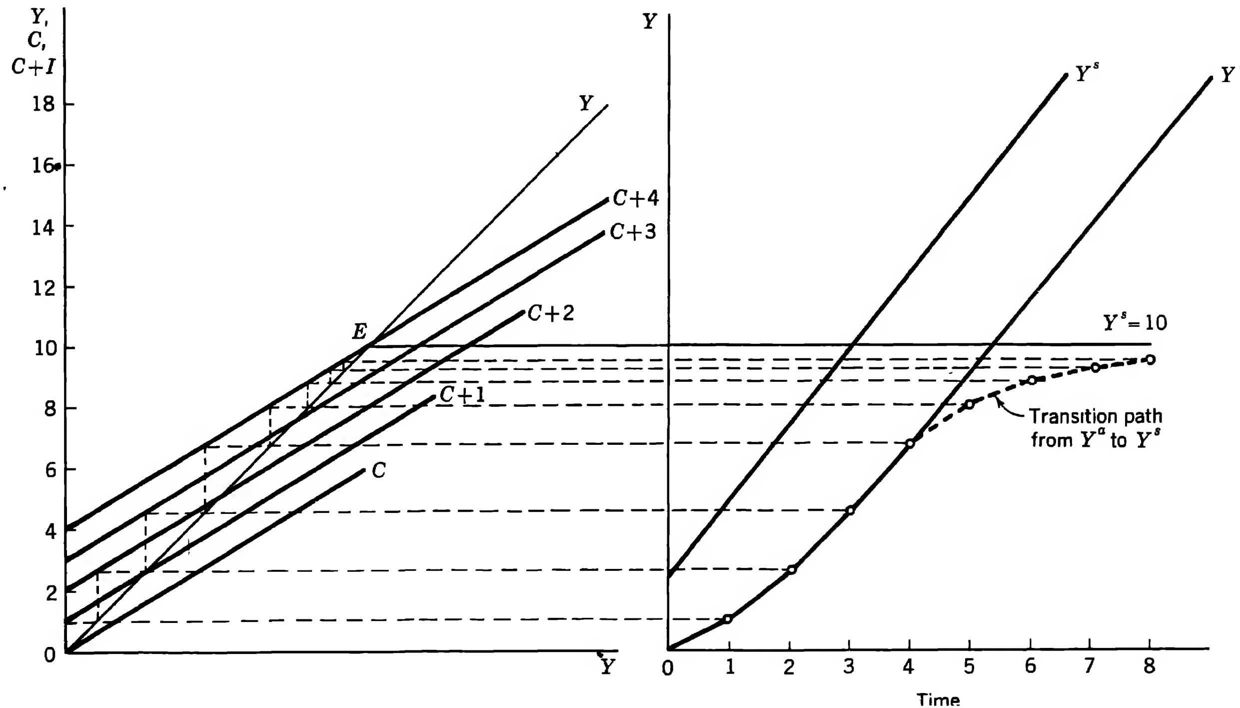


Figure 79. Path Relating Rising Income to Constant Income.

income would be 4, from investment of 4 in the preceding period, while derived incomes would be $(.6) \cdot 4 + (.6)^2 \cdot 4 + \cdots$, resulting from expenditure on consumption for successive periods in the past.

To illustrate this notion we have worked out Table 57, the results of which are plotted in Figure 79. The income data are developed by breaking off investment growth in period three, giving investment a value 4 which it maintains thereafter. Prior to period four (not shown) Table 57 is identical with Table 56; it is also the same in period 4, when the alteration begins, except for investment. In period four, indeed, investment fails to advance to 5. Owing to the time lag this fact occasions no income change. As time passes, more and more of the blocks are built up from the steady level of investment, equal in amount to 4. In consequence, the value of income gets closer and closer to the static level. In Figure 79 the transition path from growth to a static level is shown by a dashed line.[2]

SUMMARY OF THE RELATION BETWEEN ACTUAL AND STATIC INCOME

To summarize, the static equilibrium line (labeled Y^s) represents that value toward which actual income (Y^a) would tend *if* investment were to be held constant. This level is attained only when investment remains constant for a long enough period of time for income and expenditure to come into equilibrium. Theoretically, this would require an infinite time. Practically, a very close equalization of income and expenditure will occur in ten periods provided the marginal propensity to consume is not very high. It cannot be too strongly emphasized that actual income (Y^a) does *not* tend toward the static level (Y^s) under conditions of steady change. It does tend toward a moving equilibrium. Such an equilibrium path may readily be defined.

[2] This path is described by an equation of the form:

$$Y_t^a = c^{(t-t_0)} Y_0 + K_{t-t_0} I_0,$$

where Y_t^a is income at period t, c is the marginal propensity to consume, K_{t-t_0} is the truncated multiplier with $t - t_0$ blocks, Y_0 and I_0 are income and investment, respectively, in period t_0.

Moving equilibrium income

Extending our earlier definition of moving equilibrium, let us define *moving equilibrium income* as that path which would be followed by income if an indefinite time were allowed to elapse during which a steady rate of increase of investment took place. From the preceding discussion it emerged that actual income could never attain the static level under progressive conditions. The time lag gives a start to the static equilibrium income which cannot be overcome. However, despite the time lag actual income soon begins to increase at approximately the same rate as the static equilibrium income. As we concluded earlier, this rate of increase eventually tends to bear a ratio to the rate of increase of investment equal to the multiplier. By the time this is true the actual income path is: (1) virtually parallel to the static income path; (2) practically linear in shape, since it has become virtually parallel to the straight-line static income path.

It seems evident that the actual income path eventually tends toward a line which expresses the working out of the given rate of increase of investment. Such a line must exhibit precisely the same rate of increase as the static equilibrium path, but lie below it. The immediate reason it must lie below it is that moving equilibrium income (call it Y^m) can never catch up with static equilibrium income. The ultimate reason lies in the time lag, permitting static equilibrium income to gain a head start.

Moving Equilibrium Path and Actual Income Path

The moving equilibrium path is never attained by actual income. If the given rate of increase of investment is maintained for a significant number of periods, however, the difference $(Y^a - Y^m)$ between the actual and moving equilibrium incomes approaches nothing. The reason for the persistence of the small disparity is that actual income merely *tends* to increase at a given rate equal to the multiplier. As the number of periods increases, this rate of increase gets closer and closer to the equilibrium rate. After a fairly large number of periods the difference between the rate attained and the limit is negligible. By this time the actual income

line verges on the moving equilibrium line. In due course the latter line will be illustrated.

Quantitative Aspects of the Different Paths

It is clear enough that static equilibrium income is equal to investment times the multiplier. Under the assumptions used investment is 1 in period 0, 2 in period 1, and so forth. Evidently, investment is numerically equal to 1 more than time. Therefore,

$$I_t = t + 1, \tag{13–1}$$

where I_t is investment at time t, and t is the time period expressed as an integer. Since the static equilibrium income is simply the multiplier times investment, we find that:

$$Y_t^s = KI_t = K(t + 1), \tag{13–2}$$

where Y_t^s stands for the static equilibrium income.
In period zero, when $t = 0$, $Y_0^s = K \cdot 1 = K$ or 2.5 in our example.

Let us now consider the difference between static and moving equilibrium incomes. Considering Table 56, we see that investment of 1 in period 0 produces no actual income, while static equilibrium income goes up by the full amount of the multiplier. In Table 58, Table 56 has been restated for the first five periods. Incomes generated are broken down into elements which result

TABLE 58. Income under Conditions of Steadily Rising Investment

											$(.6)^3$
							$(.6)^2$			$(.6)^2$	$(.6)^2$
				.6		.6	.6		.6	.6	.6
		1	1	1	1	1	1	1	1	1	1
Inv.	1	2	3		4					5	
Time	0	1	2		3					4	

SOURCE: Table 56 with the blocks broken down or decomposed into elements of 1 or less.

from each separate investment of 1 in a given period. In period 1 the investment of 1 in the previous period has generated a single block of 1. In period 2 the extra block of investment of period 1 results in a corresponding income block of 1. In addition the income block of 1 in period 1 generates a block of .6. Altogether this constitutes a little triangular block of incomes. As time passes

the income of any one period forms a triangular income block like an entire multiplier sequence. Our problem is to find the sum of this combined sequence.

The sum of these terms can be calculated directly, but an indirect computation is simpler. Consider first the static equilibrium income in period four. In this period investment is 5, exceeding the time period by 1, as explained earlier. We can arrange all the blocks of income which the investment of 5 will *ever* generate, the static equilibrium income, in the pattern shown in Table 59.

TABLE 59. Static Equilibrium Income Blocks and Actual Income Blocks, Period Four

	-	-	-	-	-	
Difference	$(.6)^3$	$(.6)^3$	$(.6)^3$	$(.6)^3$	$(.6)^3$	
between →						
static and	$(.6)^2$	$(.6)^2$	$(.6)^2$	$(.6)^2$	$(.6)^2$	
actual						←Actual income in fourth
income	.6	.6	.6	.6	.6	period
	1	1	1	1	1	←Blocks of investment

Static income—period four

The sum of all the blocks indicated within the rectangle, including the ones indicated by dashes, is easy to find. The sum of all the blocks in one column $(1 + .6 + (.6)^2 + (.6)^3 + \cdots)$ is equal to the value of the multiplier. In general the sum can be designated as K; in this case $K = 1/(1 - .6) = 2.5$. Since there are five columns, one for each block of investment, the sum of all the blocks is $5 \cdot (2.5) = 12.5 = (t + 1) \cdot K$. This is just the static equilibrium level of income at time t, and has already been written down in (13-2).

The sum of the income blocks in the lower right-hand triangular area is actual income in the fourth period. It is this which we want to find. The rounded triangular area in the upper left hand corner

represents the disparity between static equilibrium income and actual income. Consider the elements in the *first* column in this sector of the table. Plainly, the sum is: $(1 + .6 + (.6)^2 + \cdots) = K = 2.5$. In the second column we find: $(.6 + (.6)^2 + \cdots) = .6(1 + .6 + (.6)^2 + \cdots) = .6K = .6(2.5)$. In the third column the sum is: $(.6)^2 + (.6)^3 + \cdots) = (.6)^2(1 + .6 + \cdots) = (.6)^2K = (.6)^2(2.5)$. Plainly, the sum of all five columns is: $2.5 + 2.5(.6) + 2.5(.6)^2 + 2.5(.6)^3 + 2.5(.6)^4 = 2.5\{1 + .6 + (.6)^2 + (.6)^3 + (.6^4)\} = K(1 + c + c^2 + c^1 + c^4)$, where c stands for the marginal propensity to consume, .6. By formula (9–3) of Chapter 9 the expression in parentheses can be written as follows:

$$K_5 = 1 + c + c^2 + c^3 + c^4 = \frac{1 - c^5}{1 - c},$$

where $c = \Delta C/\Delta Y$. Evidently, the difference between actual and static income in period four is KK_5, where K is the full static multiplier and K_5 is the "truncated multiplier," cut off at five terms. In general the difference between static equilibrium and actual income is:

$$Y_t^s - Y_t^a = KK_{t+1}. \tag{13–3}$$

Taking the static equilibrium income and subtracting its excess over actual income, $Y_t^s - (Y_t^s - Y_t^a) = Y_t^a$, we find actual income to be:

$$Y_t^a = K(t + 1) - KK_{t+1} = K \cdot t - K \cdot K_t \cdot c.^3 \tag{13–4}$$

This is the equation for actual income. If the period of time is extended sufficiently, K_{t+1} approaches K, actual income approaches moving equilibrium income, and the formula becomes:

$$Y_t^a \to Y_t^m = K(t + 1) - K^2 = K \cdot t - K^2 \cdot c, \tag{13–5}$$

where Y_t^m denotes moving equilibrium income. See formula (13–4).

INITIAL INVESTMENT AND THE INCOME PATH

Up to this point we have assumed a perfectly even progression of investment. We may have, however, an initial investment

[3] $K(t + 1) - KK_{t+1} = K \cdot t + K - K \cdot K_{t+1} = K \cdot t + K(1 - K_{t+1})$
$= K \cdot t + K[1 - (1 - c^{t+1})/(1 - c)] = K \cdot t + K[(1 - c - 1 + c^{t+1})/(1 - c)]$
$= K \cdot t - K \cdot c[(1 - c^t)/(1 - c)] = K \cdot t - K \cdot K_t \cdot c.$

TABLE 60. Multiplier Table, Concave Line Case

														.6872
													.8590	.9448
												1.0738	1.1810	1.2885
											1.3422	1.4763	1.6106	1.7448
										1.6777	1.8454	2.0133	2.1810	2.3488
									2.0971	2.3068	2.5166	2.7262	2.9360	3.1458
								2.6214	2.8836	3.1458	3.4078	3.67	3.9322	4.1943
							3.2768	3.6045	3.9322	4.2598	4.5875	4.9152	5.2429	5.5706
						4.096	4.5056	4.9152	5.3248	5.7344	6.144	6.5536	6.9632	7.3728
					5.12	5.632	6.144	6.656	7.168	7.68	8.192	8.704	9.216	9.728
				6.4	7.04	7.68	8.32	8.96	9.6	10.24	10.88	11.52	12.16	12.8
			8	8.8	9.6	10.4	11.2	12	12.8	13.6	14.4	15.2	16	16.8
		10	11	12	13	14	15	16	17	18	19	20	21	22
I	10	11	12	13	14	15	16	17	18	19	20	21	22	23
t	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Y^a	0	10	19	27.2	34.76	41.808	48.4464	54.7571	60.8057	66.6445	72.3155	77.8524	83.2819	88.6256
Y^s	50	55	60	65	70	75	80	85	90	95	100	105	110	115
Y^m	25	30	35	40	45	50	55	60	65	70	75	80	85	90

which is not equal to the uniform increase which follows it in each subsequent period. Let us refer to the distinct autonomous quantity invested in period zero as A. Thus, if 10 is invested in period zero and the amount invested increases to 11, 12, and so forth in successive periods, the quantity is taken to be 9. Evidently, $9 + 1 = 10$, the quantity 9 being the distinct initial amount, and the 1 corresponding to the increase which occurs in each successive period.

This element adds something new to the problem.[4] Clearly, the burst of investment in period zero will tend to produce a rapid

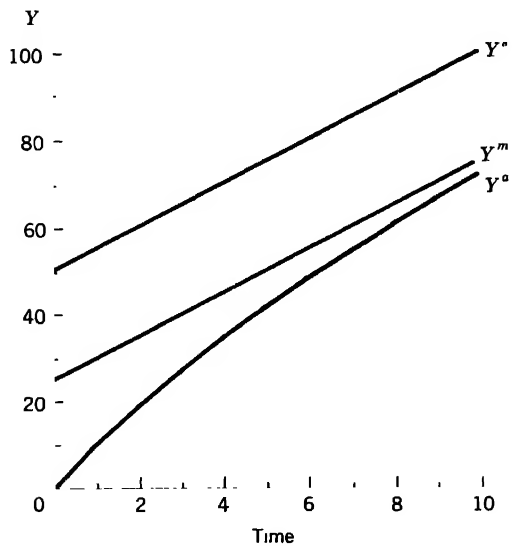


Figure 80. Actual and Equilibrium Income Paths.

[4] An additional modification gives the increase in investment as *a* per period instead of 1. Taking account of these modifications, we find our revised formulae to be the following:

$$I_t = A + a(t + 1) \tag{13–1a}$$

$$Y_t^s = A \cdot K + a \cdot K(t + 1) = K(A + a(t + 1)) \tag{13–2a}$$

$$Y_t^s - Y_t^a = A(K - K_t) + a \cdot KK_{t+1} \tag{13–3a}$$

$$\begin{aligned} Y_t^a &= A \cdot K_t + a \cdot K(t + 1) - a \cdot KK_{t+1} = A \cdot K_t + a \cdot K \cdot t - a \cdot K \cdot K_t \cdot c \\ &= A \cdot K_t + a(K \cdot t - K \cdot K_t \cdot c) = K_t(A - a \cdot K \cdot c) + K \cdot a \cdot t \end{aligned} \tag{13–4a}$$

increase in income in the first few periods. As this effect plays out, the rate of increase in income will become more gradual. In Figure 80 the actual income curve (Y^a) is shown to be concave to the time axis.[5] The data for this curve are shown in Table 60. In this table MPC is .8 for the sake of variety. In Figure 80 the moving equilibrium line is inserted for the first time. Clearly, the actual income line tends to approach the moving equilibrium income line.

Concave and Convex Actual Income Paths

Evidently, there is a difference in the actual income line according to the presence or absence of the initial, autonomous element of investment, A. In the absence of it the actual income curve is convex to the time axis. In Table 56 and Figure 78

$$\begin{aligned} Y_t^m &= A \cdot K - a \cdot K^2 + a \cdot K(t+1) = A \cdot K + a \cdot K \cdot t - a \cdot K^2 \cdot c \qquad (13\text{–}5a) \\ &= A \cdot K + a(K \cdot t - K^2 \cdot c) = (A \cdot K - a \cdot K^2 \cdot c) + a \cdot K \cdot t. \end{aligned}$$

Let us note also:

(a) $Y_t^m - Y_t^a = (A - a \cdot K \cdot c)(K - K_t)$,

(b) $Y_t^s - Y_t^m = a \cdot K^2$.

Since $(K - K_t)$ is always positive, we see from (13–3a) that $Y_t^s - Y_t^a > 0$ at all times. By the same token from (a) we find that $Y_t^m - Y_t^a \gtrless 0$, according as $(A - a \cdot K \cdot c) \gtrless 0$. Finally, by (b), we find that $Y_t^s - Y_t^m > 0$.

[5] To find out whether the actual income curve is concave to the time axis or convex it is first necessary to find the change in actual income. Thus,

$$\begin{aligned} \Delta Y_t^a = Y_{t+1}^a - Y_t^a &= K_{t+1}(A - a \cdot K \cdot c) + K \cdot a \cdot t + K \cdot a - K_t(A - a \cdot K \cdot c) \\ &\qquad - K \cdot a \cdot t \\ &= K \cdot a + (A - a \cdot K \cdot c)(K_{t+1} - K_t) = K \cdot a + (A - a \cdot K \cdot c) \\ &\qquad \times \left(\frac{1 - c^{t+1}}{1 - c} - \frac{1 - c^t}{1 - c}\right) \\ &= K \cdot a + (A - a \cdot K \cdot c)\left(\frac{c^t - c^{t+1}}{1 - c}\right) = K \cdot a + (A - a \cdot K \cdot c)c^t. \end{aligned}$$

If the rate of increase of income, ΔY_t^a, increases with time, the curve is convex, ascending more and more steeply. Since c^t diminishes with t (i.e., .6,(.6)^2 etc.), income can increase at an increasing rate if and only if $A - a \cdot K \cdot c < 0$. If $A = a \cdot K \cdot c$, $\Delta Y_t^a = K \cdot a$, and Y_t^a is therefore a straight line. Comparing (13–4a), we can see that Y_t^a is a straight line through the origin in this case. If $A - a \cdot K \cdot c > 0$, the increase will decline with time, and the curve will be concave. In any event c^t tends to become small, so that ΔY_t^a tends toward the value $K \cdot a$.

investment followed the pattern: 1, 2, 3, $\cdots$, A was zero and the actual income curve was convex. In Table 60 and Figure 80 investment followed the pattern: 10, 11, 12, $\cdots$, and the actual income curve was concave. In this case an element A equal to 9 was present. Somewhere in between these cases in which the values of A were 0 and 9, respectively, there should be a value of A which would lead to a straight line actual income curve. If it does exist, it will cause income to increase from the beginning at a rate equal to the multiplier.

We now desire to find that value of initial investment, A, separate from the increase of 1, which will cause the path of actual income to be one of steady increase. Graphically, this would indicate a straight-line path through the origin of income and time. What caused the actual income path to be convex when A was 0 was the fact that income increased by less than its later rate, which equalled the multiplier. On the moving equilibrium curve, the level of income increases by the multiplier (2.5) in every period except the first. By examining Table 56 we see that in period 1 only the first block of the multiplier is present. We find 1, but none of the succeeding blocks, namely, .6, $(.6)^2$ and so forth. Suppose the increase in investment for each period *after* zero is to be 1 and the rise in income is to be 2.5 *from the beginning*. What initial investment would be necessary to compensate for the lack of the derived blocks, .6, $(.6)^2 \cdots$, which are needed to fill out the increase of income from its figure of 1 to the full multiplier?

Evidently, the missing blocks are .6, $(.6)^2 \cdots$, which add up to $.6(1 + .6 + \cdots) = .6(2.5) = 1.5$. This implies that 1.5 is lacking to flesh out the increase of 1 to the full 2.5 necessary. If the sum of 2.5 were invested in period zero, a sum $1 + 1.5$ would be present in period one, equal to the whole desired set of blocks. In period two investment would be 3.5, because our problem has it that investment is to increase by 1 after period zero. The rise of 1 in investment compensates for the drop in derived as compared with primary income to 1.5 from 2.5, as the 2.5 is translated into derived income by consuming $.6(2.5) = 1.5$. This new derived income of 1.5 plus new investment of 1 produces the income rise of 2.5. Of course, added to the "old" 2.5 of investment, the increase gives a total income of 5. In each successive period this

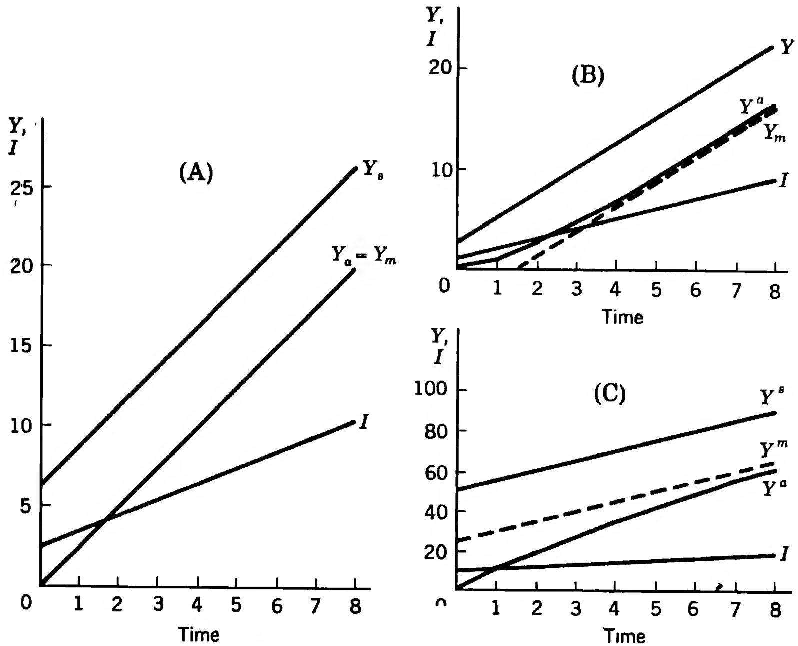


Figure 81. Concave, Convex, and Straight Line Income Paths.

result is repeated, and income rises steadily by 2.5 per period. This construction is detailed in Table 61.

TABLE 61. Income Pattern when the Initial Investment Is Such as to Equate Moving and Actual Equilibrium Income

					.54
				.9	1.26
			1.5	2.1	2.70
		2.5	3.5	4.5	5.50
$Y_t^a = Y_t^m$	0	2.5	5.0	7.5	10.00
Y_t^s	6.25	8.75	11.25	13.75	16.25
I_t	2.5	3.5	4.5	5.5	6.5
t	0	1	2	3	4

Our conclusion is this: an initial autonomous amount, A, of $K - 1$, in addition to 1 in period zero, making a total of K invested in period zero, with an extra 1 in each succeeding period, leads to an actual income curve which is a straight line through the origin. This coincides with the moving equilibrium line precisely. In Figure 81 the three types of income curves are shown, together with the corresponding moving equilibrium lines.

MOVING EQUILIBRIUM REINTERPRETED

As we have explained moving equilibrium so far, it is merely a sort of trend to which actual income ultimately conforms. In a certain sense, however, it may be said to represent a mutual adjustment of economic forces. The argument shows that income ultimately begins to increase in successive periods by constant amounts. In short, income finally tends to increase at a steady rate. (See footnote 5.)

In explanation, we recall that one of our basic assumptions states that income or production lags behind demand by one period. If expenditure exceeds income (production) in one period, a corresponding increase in income occurs in the following period. If the system is in moving equilibrium, the increase in income from one period to the next is constant. By the same token the cause of this change, the excess of expenditure over income, remains constant through time.[6] This gap between demand and production does not change through time in moving equilibrium. Although exact equality of supply and demand, income and expenditure, may be absent, each of the opposing elements changes at the same rate through time. There is a presumption that under dynamic conditions the adaptation process is more important than an exact balance of amounts. On this ground we justify calling attainment of a moving adjustment of the sort described a moving equilibrium.

[6] $\Delta Y_t{}^a = Y_{t+1} - Y_t = E_t - Y_t$. By the preceding footnote, as t becomes large,

$$\Delta Y_t{}^a = E_t - Y_t \rightarrow K \cdot a$$

We could also define moving equilibrium as a condition marked by a constant rate of increase of income.

MONETARY REQUIREMENTS FOR GROWTH

During a growth process increasing demands are made on the money supply. Consider the components of the money supply. First, there is money held for precautionary purposes which we have considered to be autonomous. Since this varies neither with income nor interest, it does not vary as a result of the process which is occurring. Second, the money held for speculative purposes depends on the rate of interest. In the present argument the rate of interest is regarded as constant, and as a result the speculative demand for money can also be regarded as constant. Third, the demand for finance was assumed to be actuated by changes in the rate of interest operating on investment. This factor does not act here, since investment is treated as autonomous. In actuality, the demand for finance may be incorporated under the transactions demand for cash, since it operates in the same way under the assumed conditions. Fourth, the transactions demand for money, now including the need to finance autonomous investment, depends on the level of income. Since income varies under the assumed conditions, the transactions demand for money (now including investment) also varies. To summarize, additional money is needed during the growth process to finance transactions, including investment.

Now we need to consider how much additional money is needed to finance the additional income transactions. In the argument which precedes this section we assumed that the expenditure of one period generates the income of the following period. This assumption implies that money makes a complete circuit from the buyer of output to producer and back to buyer as factor income in one time period. This condition implies that the time interval is so chosen as to permit this circuit to be made exactly. This time interval may be entitled the period of *unit circuit velocity*. The length of this period is assumed to remain constant.

In the period of unit circuit velocity a dollar of transactions money travels the income circuit once, generating a dollar of income. If the period in question is one quarter, and quarterly income is \$100, exactly \$100 of transactions money will be required to effect the necessary transactions. In short, the required

volume of transactions money is equal to income flow during the period of unit circuit velocity of money.

If income generated in a quarter is equal to \$100, the annual income rate is \$400. The \$100 stock of transactions money which can manage one circuit in a quarter can effect four circuits in a year, and generate \$400 of income. The number of circuits made by transactions money in generating income during a year may be called the *circuit velocity of transactions money*. In the example under consideration this velocity is 4. Clearly, income at an annual rate of \$400 equals transactions money of \$100 times the circuit velocity of transactions money which is 4.

We may now formulate certain conclusions, making use of algebraic notation. Let Y_t^a represent income calculated for the period of unit circuit velocity, $Y_t'^a$ represents income on an annual basis, M_t^T the quantity of transactions money, and V the circuit velocity of transactions money. In our model the level of income started from 0, so that Y_t^a represents the total increase in the level of income. Since each dollar of Y_t^a must be matched by a dollar of M_t^T, we may conclude from (13–4) that:

$$Y_t^a = M_t^T = K(t+1) - KK_{t+1} = K \cdot t - K \cdot K_t \cdot c. \quad (13\text{–}6)$$

From the last statement of the paragraph which precedes the one above, we may conclude that:

$$Y_t'^a = V \cdot M_t^T = V[K(t+1) - KK_{t+1}] = V[K \cdot t - K \cdot K_t \cdot c]. \quad (13\text{–}6a)$$

The advantage of the last expression is that it puts income on an annual basis. Note that we may generalize this slightly by the use of equation (13–4a) in an earlier footnote.

Constant Percentage Increase In Investment

ARITHMETIC PROGRESSION

In the case dealt with so far investment increases after the first period by constant amounts equal to 1. This is an arithmetic rate of increase, going, say, 6, 7, 8, 9, $\cdots$. The case in which the increase goes 6, 8, 10, 12, $\cdots$ is of essentially the same character. Here the increase is 2, and a very minor adjustment in the argument will solve the problem.

GEOMETRIC PROGRESSION

A case of quite a different nature occurs when investment increases by a constant percentage each succeeding year. Thus, a sequence of investment 1, 2, 4, 8, 16, $\cdots$, representing a doubling of investment every year, requires separate analysis. In this case a 100 percent increase in investment occurs every year. Clearly, the increase may be more moderate, as 3 percent. In this event the sequence runs 1, 1.03, $(1.03)^2$, $\cdots$. In this form it is seen to be identical with the compound interest problem. Generally, then, the sequence of investments can be written in the alternative forms:

$$1, M, M^2, \cdots, M^t$$
$$1, (1+r), (1+r)^2, \cdots, (1+r)^t,$$

where t stands for the time period and $M = 1 + r$, while r is the yearly proportional increase in investment.[7]

Before plunging into further abstractions let us consider a multiplier table of the usual type. In Table 62 investment is assumed to increase at the rate of 10 percent a year. Thus $r = 10$ percent and $M = (1 + .10) = 1.1$. It is not hard to see that, starting with investment of 1 in period zero, investment will become $1 \cdot M = 1(1 + r) = 1(1.1) = 1(1 + .10) = 1.1$ in period one. In the next period investment will become $1.1(1.1) = (1.1)^2 = 1.21$, and so on.

The successive blocks of investment generate a sequence of derived income blocks. Each of the derived blocks is found, as usual, by following a diagonal. And these blocks build up in the usual triangular pattern. Static equilibrium income is found, as before, by multiplying investment by the multiplier, $K = 2.5$. In this case MPC is assumed to be .6. New investment is 1 in period 0, 1.1 in period 1, $(1.1)^2$ in period 2, $\cdots$, $(1.1)^t$ in period t. More generally,

$$I_t = M^t = (1 + r)^t. \tag{13–7}$$

Clearly, static equilibrium income simply equals I_t times K, so that

$$Y_t^s = K \cdot I_t = K \cdot M^t = K(1 + r)^t. \tag{13–8}$$

Since investment increases at compound interest, as it were,

[7] From (13–7) we find that $I_t = M^t$. We then find that $\Delta I_t/I_t = (I_{t+1} - I_t)/I_t = (M^{t+1} - M^t)/M^t = M^t(M - 1)/M^t = (M - 1) = 1 + r - 1 = r$.

so does static equilibrium income. Consequently the static equilibrium income curve shown in Figure 82 is simply 2.5 times as high

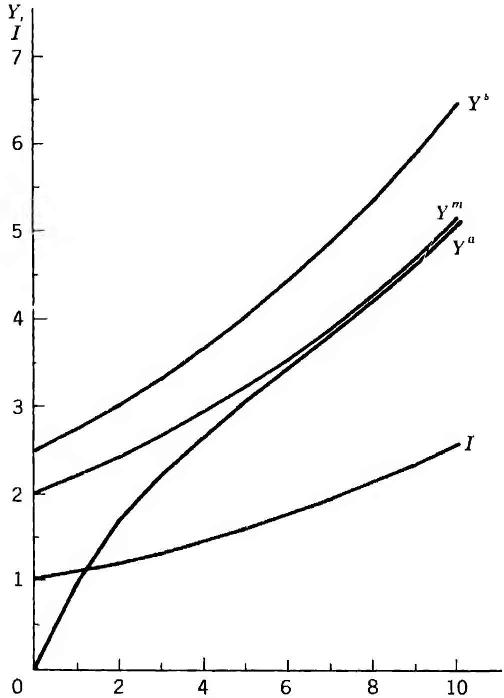


Figure 82. Income Paths with Constant Percentage Increases in Investment.

as the investment curve. Both are accumulation curves, representing the increase of the original amounts of income and investment, K and 1, respectively, at compound interest.[8]

Finding the paths of actual income and of moving equilibrium income is not so easy. Consider first actual income. To suggest the conclusion to be drawn, let us rewrite the first few elements in Table 62, as follows. Each investment block in one period is

[8] $$\frac{\Delta Y_t^s}{Y_t^s} = \frac{Y_{t+1}^s - Y_t^s}{Y_t^s} = \frac{KM^{t+1} - KM^t}{KM^t} = \frac{KM^t(M-1)}{KM^t} = M - 1 = 1 + r - 1 = r.$$

This indicates that the percentage increase in income is equal to r.

TABLE 62. Multiplier Table, Investment Increases at a Constant Percentage Rate

											.0101
										.0168	.0185
									.0280	.0308	.0339
								.0467	.0514	.0565	.0621
							.0778	.0856	.0941	.1035	.1139
						.1296	.1426	.1568	.1725	.1898	.2088
					.216	.2376	.2614	.2875	.3163	.3479	.3826
				.36	.396	.4356	.4792	.5271	.5798	.6377	.7016
			.6	.66	.726	.7986	.8785	.9663	1.0629	1.1693	1.2862
		1	1.1	1.21	1.331	1.4641	1.6105	1.7716	1.9488	2.1437	2.3581
I	1	1.1	1.21	1.331	1.4641	1.6105	1.7716	1.9488	2.1437	2.3581	2.5939
t	0	1	2	3	4	5	6	7	8	9	10
Y^a	0	1	1.7	2.23	2.669	3.0651	3.4500	3.8416	4.2538	4.6960	5.1758
Y^s	2.5	2.75	3.025	3.3275	3.6603	4.0263	4.4290	4.8720	5.3593	5.8953	6.4848
Y^m	—	2.2	2.42	2.662	2.9282	3.2210	3.5431	3.8975	4.2874	4.7161	5.1878

translated into an income block in the following period. Thus $(1.1)^2$ of investment in period 2 becomes the first block of income in period 3. Subsequently, each block of income becomes .6 of

TABLE 62A. A Regrouping of Elements of Table 62 for Clarity

					$(.6)^3 1$
				$(.6)^2 1$	$(.6)^2(1.1)$
			$(.6)1$	$(.6)(1.1)$	$(.6)(1.1)^2$
		1	1.1	$(1.1)^2$	$(1.1)^3$
Investment	1	1.1	$(1.1)^2$	$(1.1)^3$	$(1.1)^4$
Time	0	1	2	2	3

what it was, as this fraction is spent on consumption. Taking the sum at time period 4, we find: actual income at period 4 $= (1.1)^3 + .6(1.1)^2 + (.6)^2(1.1) + (.6)^3 1$. The pattern is obvious. We can see that actual income at time $t = (1.1)^{t-1} + (.6)(1.1)^{t-2} + (.6)^2 (1.1)^{t-3} + \cdots + (.6)^{t-2}(1.1) + (.6)^{t-1}$. In formal terms, this gives:

$$Y_t^a = M^{t-1} + cM^{t-2} + c^2M^{t-3} + \cdots + c^{t-2}M + c^{t-1}.$$

It is not hard to find the sum of this series. Let us first try to make M^{t-1} a factor of every term. We can do this by multiplying and dividing the second term by M, the third term by $M^2, \cdots,$ the last by M^{t-1}, giving:

$$\begin{aligned} Y_t^a &= M^{t-1} + \frac{c}{M}MM^{t-2} + \frac{c^2}{M^2}M^2M^{t-3} + \cdots + \frac{c^{t-2}}{M^{t-2}}M^{t-2}M \\ &\qquad + \frac{c^{t-1}}{M^{t-1}}M^{t-1} \\ &= M^{t-1} + \left(\frac{c}{M}\right)M^{t-1} + \left(\frac{c}{M}\right)^2 M^{t-1} + \cdots + \left(\frac{c}{M}\right)^{t-2} M^{t-1} \\ &\qquad + \left(\frac{c}{M}\right)^{t-1} M^{t-1} \\ &= M^{t-1}\left[1 + \frac{c}{M} + \left(\frac{c}{M}\right)^2 + \cdots + \left(\frac{c}{M}\right)^{t-2} + \left(\frac{c}{M}\right)^{t-1}\right] \\ &= M^{t-1}\frac{\left[1 - \left(\frac{c}{M}\right)^t\right]}{1 - \frac{c}{M}}, \end{aligned} \tag{13-9}$$

the last step being given by the formula for a geometric progression with t terms, with the common ratio $r = c/M$. See equation (12–12), Chapter 12. When t becomes very large, c/M being less than 1, $(c/M)^t$ tends to vanish. This gives for the moving equilibrium of income:

$$Y_t^m = M^{t-1}\frac{1}{\left(1 - \frac{c}{M}\right)}. \tag{13–10}$$

THE AVERAGE MULTIPLIER IN THE GEOMETRIC GROWTH PROCE

In virtue of the fact that $I_t = M^t$, and writing the definitio $k_t^a = Y_t^a/I_{t-1}$, we find from (13–9) that:

$$k_t^a = Y_t^a/I_{t-1} = \left[1 - \left(\frac{c}{M}\right)^t\right]\bigg/\left(1 - \frac{c}{M}\right). \tag{13–11}$$

We may think of this expression as the average multiplier during a geometric growth process. It represents the leverage effect of investment on income during the process. The dating of investment ahead of income is proper, since the investment of one period is translated into the income of the next.

In the average multiplier the term c/M plays the same role as $c = MPC$ in the ordinary dynamic multiplier of Chapter 9, equation (9–3). This suggests that it may be possible to identify c/M and to explain why it replaces c in the formula for a dynamic multiplier.

Basically, the cause lies in the fact that investment is increasing faster in each period in an absolute sense. As the economy proceeds through time, the original and induced increments of income of different ages which constitute the current level of income relate to different initial injections of investment. The last injections are the biggest. To reduce the earlier elements of the stream to the same basis as the latest we must "discount them" by the factor M. By discounting these earlier elements we account for the fact that they were derived from smaller injections of investment. By the same token this discounting process permits us to treat the current income level as the outcome of a steady investment rate of the same size as the current one.

To find the element of income provided by the preceding period we simply take the investment of that period. To find the

income element contributed by the investment of two periods ago we must make two adjustments: (1) we recognize the fact that it is income directly generated by consumption (though ultimately by investment) and therefore multiply it by c; (2) we recognize the fact that it must be discounted to allow for the fact that the investment of the earlier period was less and so multiply it by $1/M$. The processs described continues backward with the simultaneous operation of the consumption and discount effect. To sum up, we may refer to c/M as the *discounted marginal propensity to consume*. Its value is less than the marginal propensity to consume, since M is greater than 1.

Inspection of the average multiplier in equation (13–11) reveals that c/M plays the same role there as c = MPC in the dynamic truncated multiplier of Chapter 9, equation (9–3). This similarity suggests that c/M may be interpreted in terms of changes in C and Y. Since the C of one period helps to generate the Y of the next, we must date C earlier than Y in such a relationship. Following this line of thought leads us to calculate the "lagged marginal propensity to consume," a process which yields an equation of the following form:

$$\frac{\Delta C_{t-1}}{\Delta Y_t^a} = \frac{c}{M} + R_t, \tag{13–12}$$

where R_t is a positive amount, declining with t and ultimately tending to zero.[9] While we *cannot* say that c/M is the lagged MPC

[9]
$$R_t = \frac{(1-c)\left(1-\frac{c}{M}\right)}{\left(\frac{M}{c}\right)^t(M-1)-(c-1)}.$$

$$\frac{\Delta C_{t-1}}{\Delta Y_t^a} = \frac{c\Delta Y_{t-1}^a}{\Delta Y_t^a} = \frac{c(Y_t^a - Y_{t-1}^a)}{Y_{t+1}^a - Y_t^a} = \frac{cM^t - c^{t+1} - cM^{t-1} + c^t}{M^{t+1} - c^{t+1} - M^t + c^t}$$

$$= \frac{c}{M} + \frac{c^t(1-c)\left(1-\frac{c}{M}\right)}{M^t(M-1) - c^t(c-1)}, \text{ by division,}$$

$$= \frac{c}{M} + \frac{(1-c)\left(1-\frac{c}{M}\right)}{\left(\frac{M}{c}\right)^t(M-1)-(c-1)} = \frac{c}{M} + R_t.$$

As t grows without limit, $M - 1$ being positive and $M/c > 1$, the first term in the denominator grows without bound. Hence R_t approaches 0. We may also note that $R_t > 0$.

For this reason actual income ultimately tends to increase at the same rate as moving equilibrium income, namely r.[11]

With the passage of time actual income tends toward moving equilibrium income. By the same token it eventually increases at the rate r. Meanwhile, however, the growth rate for actual income exceeds r, a fact which may be observed in Figure 82.

To understand why actual income increases at a faster rate than r, let us note first that moving equilibrium income is proportional to lagged investment by (13–14a). In period 0 investment is 1 which implies that moving equilibrium income is k in period 1. However, the investment of 1 generates an actual income of only 1 in this period, implying that moving equilibrium income exceeds actual income by an amount $k - 1$ in period 1. In our example, with a c of .6, an M of 1.1, and a k of 2.2, this excess amounts to $2.2 - 1 = 1.2$. As time passes actual income tends toward moving equilibrium income. In the meantime actual income must rise at a faster rate than r to erase the difference of 1.2 or $k - 1$. Our conclusion is that investment, static equilibrium income, and moving equilibrium income all increase at the same proportional rate r. Actual income increases faster than r at first, but gradually increases more slowly until finally its growth rate approaches r.[12]

[11] From (13–10) we find,

$$\Delta Y_t^m = Y_{t+1}^m - Y_t^m = M^t\Big/\left(1 - \frac{c}{M}\right) - M^{t-1}\Big/\left(1 - \frac{c}{M}\right)$$

$$= M^{t-1}(M - 1)\Big/\left(1 - \frac{c}{M}\right);$$

$$\frac{\Delta Y_t^m}{Y_t^m} = \frac{M^{t-1}(M-1)}{1 - \dfrac{c}{M}} \div \frac{M^{t-1}}{1 - \dfrac{c}{M}} = M - 1 = r.$$

Consequently, the proportional rate of increase for moving equilibrium income is constant at rate r throughout.

[12]

$$Y_t^a = \frac{M^t\left[1 - \left(\dfrac{c}{M}\right)^t\right]}{M\left(1 - \dfrac{c}{M}\right)} = \frac{k}{M} M^t\left[1 - \left(\frac{c}{M}\right)^t\right] = \frac{k}{M}(M^t - c^t)$$

$$\frac{\Delta Y_t^a}{Y_t^a} = \frac{Y_{t+1}^a - Y_t^a}{Y_t^a} = \frac{\dfrac{k}{M}(M^{t+1} - c^{t+1}) - \dfrac{k}{M}(M^t - c^t)}{\dfrac{k}{M}(M^t - c^t)}$$

ABSOLUTE RATES OF CHANGE

As we have noted, the variables under consideration tend to increase at the same percentage rate. Let us now compare the changes in absolute amounts, concentrating on the comparison of static equilibrium income and moving equilibrium income. Since both increase at a rate r times their current values, $\Delta Y_t^m = rY_t^m$ and $\Delta Y_t^s = rY_t^s$. Taking the ratio of these changes, we find that $\Delta Y_t^s/\Delta Y_t^m = rY_t^s/rY_t^m = Y_t^s/Y_t^m$. Substituting from (13–8) and (13–14a),[13] we arrive at the conclusion that:

$$\Delta Y_t^s/\Delta Y_t^m = M\left[1 + K\left(c - \frac{c}{M}\right)\right]. \tag{13–16}$$

This equation implies that static equilibrium income increases at a faster absolute rate than moving equilibrium income. In part, this is the result of the time lag involved in translating investment demand into income, and is represented by the term M. This is a rather trivial difference, involving only the factor of increase, $M = 1 + r$, between one period and the next. However, the second element, $K(c - c/M)$ is not trivial in character. Note that $c - c/M$ is the excess of MPC over discounted MPC (to which lagged MPC tends with the passage of time). The excess consumption in static equilibrium is multiplied by K, the static multiplier, giving rise to an excess of static equilibrium income compared to moving equilibrium income. The statements apply

---

$$= \frac{(M^{t+1} - M^t) + (c^t - c^{t+1})}{M^t - c^t} = \frac{M^t(M - 1) + c^t(1 - c)}{M^t - c^t}$$

$$= \frac{M^t r + c^t(1 - c)}{M^t - c^t} = r + \frac{c^t(1 - c) + c^t r}{M^t - c^t},$$

remembering that $M - 1 = 1 + r - 1 = r$. Since $0 < c < 1 < M$, the fraction in the last line is positive. Accordingly, the rate of growth of actual income exceeds r. For this reason its rate of growth exceeds that of investment, static equilibrium income, and moving equilibrium income. However, as t becomes large, c^t tends to vanish. This implies that the last fraction tends to zero. In short, the growth rate of actual income tends to r.

[13] Making use of (12–8) and (12–10), we find that:

$$\Delta Y_t^s/\Delta Y_t^m = Y_t^s/Y_t^m = M^t[1/(1 - c)] \div M^{t-1}\left[1\Big/\left(1 - \frac{c}{M}\right)\right] = M\left(1 - \frac{c}{M}\right)$$

$$\div (1 - c) = M\left[1 + \left(c - \frac{c}{M}\right)\Big/(1 - c)\right] = M\left[1 + K\left(c - \frac{c}{M}\right)\right].$$

to the ratio of increments of the two quantities, as well as to the ratio of the two quantities, the latter proportion being constant.

In the equations (13–10) through (13–14), as well as (13–8), the rates of change of the various income concepts relative to investment and consumption are considered. As a result we have a fairly complete picture of the way in which these quantities change relatively to each other.

MONETARY REQUIREMENTS

This case follows exactly the same lines as the last. The rise in the quantity of money under the assumptions used is the same as the rise in transactions money. In turn, this is exactly equal to the rise in the quantity of income as given by equation (13–9). Income on an annual basis is V times (13–9), where V is the income velocity of transactions money.

Irregular economic change or growth

One of the most puzzling problems which arises in economic dynamics is the very irregularity of growth processes. When the growth occurs at a regular rate, the laws pertaining to such growth can be developed rather readily. When growth occurs in a seesaw fashion, it may be necessary to decompose the pattern of growth into regular elements. In this decomposition the irregular or oscillating element must be distinguished from the element leading to steady growth.

To illustrate this problem, consider an economic system whose investment follows the following pattern.

Investment	0	3	2	5	4	7	$\cdots$
Time Period	0	1	2	3	4	5	$\cdots$

It is quite obvious that growth is taking place here, but growth according to an irregular pattern. If we are to carry out a multiplier analysis of this problem, it is desirable to break up investment into components which may be more easily analyzed. In this case investment breaks down into two elements.

Growth Element	1	2	3	4	5	6	$\cdots$	$t+1$
Oscillation Element	-1	1	-1	1	-1	1	$\cdots$	$(-1)^{t+1}$
Total Investment	0	3	2	5	4	7	$\cdots$	$(-1)^{t+1}+t+1$
Time Period	0	1	2	3	4	5	$\cdots$	t

In a previous section we discussed the course of income which resulted from a steady autonomous increase in investment. Therefore the problem posed by the growth element has already been solved. It remains merely to add the consequence of the oscillation element. Such an oscillation might be caused by alternate investment and disinvestment in inventory or by an alternation of investment in equipment, the wearing out of such equipment (disinvestment), the subsequent reinvestment, and so forth. To find out the effect of an alternating sequence of investment on income we must apply multiplier analysis.

At first glance it may seem that the positive and negative investments would produce exactly opposite and equal effects on income—effects which would cancel. To some extent this is indeed the case. However, the first element tends to outweigh subsequent elements. Consequently, the sum of the series of incomes generated by the investment tends to have the same sign as the investment or the first block of income. Consider Table 63.

For period 6 the sum of the several income blocks may be written:

$$\begin{aligned} K_6 &= 1 - .6 + (.6)^2 - (.6)^3 + (.6)^4 - (.6)^5 \\ &= 1 + (-.6) + (-.6)^2 + (-.6)^3 + (-.6)^4 + (-.6)^5 \\ &= \frac{1 - (-.6)^6}{1 - (-.6)} = \frac{1 - (-1)^6(.6)^6}{1 + .6}, \text{ by equation (9–3), Chapter 9} \end{aligned}$$

with $\Delta C/\Delta Y = -.6$.

By the same line of argument, applied to the terms in the brackets,

$$\begin{aligned} K_5 &= -1 + .6 - (.6)^2 + (.6)^3 - (.6)^4 \\ &= -[1 - .6 + (.6)^2 - (.6)^3 + (.6)^4] \\ &= -\frac{1 - (-1)^5(.6)^5}{1 + .6}. \end{aligned}$$

Comparing K_5 with K_6, we see that the sign in front of the expression alternates. Writing c for MPC $= .6$, and taking account of the alternation in sign mentioned, we get:

$$Y_t^a = K_t = (-1)^t \frac{1 - (-1)^t c^t}{1 + c}. \tag{13–17}$$

TABLE 63. The Multiplier Effects of Alternating Doses of Investment and Disinvestment

										$-(.6)^8$
									$-(.6)^7$	$(.6)^7$
								$-(.6)^6$	$(.6)^6$	$-(.6)^6$
							$-(.6)^5$	$(.6)^5$	$-(.6)^5$	$(.6)^5$
						$-(.6)^4$	$(.6)^4$	$-(.6)^4$	$(.6)^4$	$-(.6)^4$
					$-(.6)^3$	$(.6)^3$	$-(.6)^3$	$(.6)^3$	$-(.6)^3$	$(.6)^3$
				$-(.6)^2$	$(.6)^2$	$-(.6)^2$	$(.6)^2$	$-(.6)^2$	$(.6)^2$	$-(.6)^2$
			−.6	.6	−.6	.6	−.6	.6	−.6	.6
		−1	1	−1	1	−1	1	−1	1	−1
Income	0	−1	.4	−.76	.544	−.6736	.59584	−.6424	.6146	−.6312
Investment	−1	1	−1	1	−1	1	−1	1	−1	1
Time Period	0	1	2	3	4	5	6	7	8	9

Since c is less than 1 in numerical value, c^t is less than 1 in value (except when t is 0). Therefore the numerator must always be positive when t is 1 or more, 1 exceeding c^t in numerical value. Hence the value of the series is positive or negative according to the sign of $(-1)^t$. If t is even the sign is positive; if odd, negative. For $t = 2$, $(-1)^2 = 1$, and the series is positive. For $t = 3$, $(-1)^3 = -1$, and the series is negative in value.

Since c is less than 1 in numerical value, c^t diminishes as t increases: $(.6)^1 = .6$ is more than $(.6)^2 = .36$ which in turn is more than $(.6)^3 = .216$. As t becomes large, c^t tends toward the value zero. Considering a very large value of t to be reached, we find the moving equilibrium value of Y_t to be:

$$Y_t^m = K_t^m = (-1)^t \frac{1}{1 + c}. \tag{13–18}$$

If $c = .6$, $1/(1 + c) = 1/(1 + .6) = .625$. This indicates that income tends ultimately to attain positive and negative values equal to .625.

To find the incomes corresponding to the total investment in the pattern assumed it is necessary to add the effects of the growth element to that of the oscillation element. The formulas for actual and moving equilibrium incomes are obtained by adding (13–4) and (13–17), (13–5) and (13–18), respectively. This information may be combined in a table.

It is somewhat difficult to see the pattern which emerges either from the table or the formulas. However, a graph representing actual and moving equilibrium income clears up a number of points. In Figure 83 moving equilibrium income is confined within two straight lines, alternating between positions on one and the other. These have been labeled the upper trend of Y^m and the lower trend of Y^m. The actual income, Y^a, starts out above the Y^m line, and gradually approaches it, but alternating between upper and lower values all the while. The set of upper values of Y^a gradually approaches the upper trend of Y^m, while the set of lower values of Y^a gradually approaches the lower trend of Y^m. In both cases the approach is from above, so that there is a process by which the upper points of Y^a gradually settle down on the upper trend of Y^m as the lower points of Y^a settle down on the lower trend of Y^m.

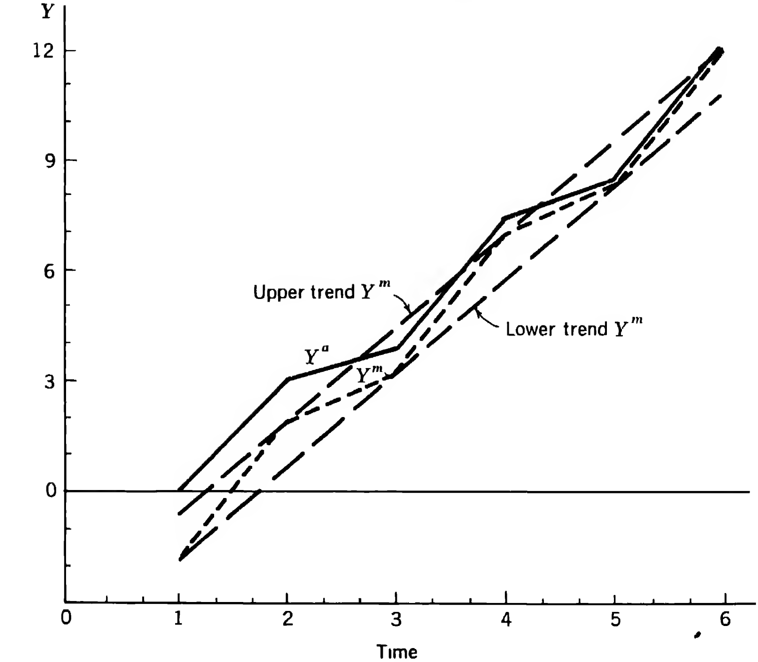


Figure 83. Irregular Investment Increase and Income Paths.

TABLE 64. Steady Growth Combined with Oscillation in Investment—the Effect on Income

							0	
						0	.39	
					0	.65	.43	
				0	1.08	.72	1.8	
			0	1.8	1.2	3	2.4	
		0	3	2	5	4	7	
I	0	3	2	5	4	7	6	$(-1)^{t+1} + t + 1$
Y^a	0	0	3	3.8	7.28	8.37	·· 12.03[a]	
Y^m	—	−1.875	1.875	3.125	6.875	8.125	·· 11.875[b]	
Y^s	0	7.5	5	12.5	10	17.5	··· 15	$2.5I$
Time	0	1	2	3	4	5	··· 6	t

[a] $Y_t^a = K(t+1) - K \quad K_{t+1} + (-1)^t \dfrac{1-(-1)^t c^t}{1+c} \qquad K = 2.5;\ c = .6$

[b] $Y_t^m = K(t+1) - K^2 + (-1)^t \dfrac{1}{1+c}. = K(t+1) - K^2 - 1/(1+c)$ for $t = 1.3.$ all odd numbers
$= K(t+1) - K^2 + 1/(1+c)$ for $t = 2,4,$ all even numbers

PROBLEMS

1. Construct a multiplier table under the following conditions: (a) investment is zero in the base period; (b) investment increases by 2 in each period, so that the pattern is: 2, 4, $\cdots$ in successive periods; (c) MPC is .8. Record investment, static equilibrium income, actual income in the table.
2. Graph investment, static equilibrium, and actual income. Using formula (13–5*a*) compute and draw in a moving equilibrium line, setting $A = 0$.
3. Repeat problem 1 with initial investment of 8, followed by 10, etc., so that the investment pattern is 8, 10, 12, etc.
4. Repeat problem 2 with the new data for investment.
5. Repeat problem 1 except that initial investment is 20, followed by 22, etc.
6. Find a pattern of investment in periods 0, 1, 2, $\cdots$ which will cause income to rise by exactly 1 in period 1 and remain at this constant level in all succeeding periods. (*Suggestion:* Assume an MPC $= .6$ and make $I = 1$ in period 0. Find the corresponding income in period 1 and record it in a table like Table 56. Then record the derived block for period 2. Find the one block necessary to fill out the total to 1. Working with these blocks fill in the necessary corresponding derived blocks in period 2. Then find the remaining block. The result will be apparent.) Prove the result by algebra.
7. Find a pattern of investment in periods 0, 1, $\cdots$ which will cause income to assume alternate values of 1, -1, 1, -1 etc. (*Suggestion:* Assume an MPC $= .6$ and make $I = 1$ in period 0. Allow negative values of investment. Follow the procedure suggested in the preceding problem.) Prove the result by algebra.
8. Find a pattern of investment in periods 0, 1, $\cdots$ which will cause income to rise according to the pattern: 1, M, M^2, M^3, $\cdots$, where M is a number greater than 1, in periods 1, 2, $\cdots$. (*Suggestion:* Assume an MPC $= .8$, make $I = 1$ in period 0, and let $M = 1.1$. Follow the procedure already outlined in problems 6 and 7. This result is perhaps harder to secure.) Prove the result algebraically.
9. Graph the investment and income pattern against time in problems 6, 7, and 8.
10. Use the results found in problem 8. With an MPC of .8 and income *falling* according to the pattern: 1, M, M^2, $\cdots$, where M is a number *less* than 1, say .9, find the pattern of investment associated.

Graph the result. Plot both income and investment against corresponding values of time.

11. From the viewpoint of the multiplier, income in period n equals:

$$Y_n = Y_0 \cdot c^n + I_0 \cdot c^{n-1} + I_1 \cdot c^{n-2} + \cdots + I_{n-1}$$

It may be desirable to aim at stabilization of income over time. With this objective in mind consider the following questions.

a. What change in I_1 will offset an increase in I_0 by 1 and leave Y_n constant? (*Suggestion:* Set $I_0 \cdot c^{n-1} + I_1 \cdot c^{n-2} = K$, a constant. Then let $(I_0 + 1)c^{n-1} + (I_1 + \Delta I_1)c^{n-2} = K$. Subtract and find the answer.)

b. What change in I_n will offset an increase of 1 in I_0 so as to leave Y_n constant?

12. In equation (13–9) it is normally assumed that $c < 1$ and $M > 1$, so that $c/M < 1$. However, some different cases are possible.

a. Suppose $c < M < 1$ so that investment, while above the base level, declines in each period. Note that k_t^a, given by (13–11), has a definite value, but that the element $M^{t-1} = I_{t-1}$ is declining steadily. Construct a table with $c = .6$ and $M = .75$. Also draw a graph of Y_t^a and Y_t^s.

b. Suppose that $c = M < 1$.

(1) What is the value of k_t^a given by (13–11)? *Hint*: Consider the value of the expression in parentheses immediately above (13–9). Or the student of calculus may differentiate the numerator and denominator of (13–11) and find the value of the resulting ratio when c/M approaches 1.)

(2) What is the value of Y_t^a given by (13–9).

(3) Does k_t^a approach the value given by (13–14) as t grows?

c. Suppose that $M < c < 1$ so that $c/M > 1$.

(1) Note that k_t^a can be written:

$$k_t^a = \frac{(c/M)^t - 1}{(c/M) - I}.$$

Let $c/M = 1 + i$. We can then write:

$$k_t^a = \frac{(1+i)^t - 1}{(1+i) - 1} = \frac{(1+i)^t - 1}{i}.$$

In compound interest theory this formula gives the amount of an annuity of unit value per period at a rate of interest i per period, and is designated by the symbol $S_{\overline{n}|i}$, where n corresponds to t in this problem.

(2) Note that $Y_t^a = M^{t-1} S_{\overline{t}|i}$, where $i = (c/M) - 1$. Let $M = .7, c = .721$. Find $c/M = 1 + i$, find i and compute a table of Y_t^a from a table of compound interest, found in any text on mathematics of finance, for ten periods. Graph the result.

(3) Show that $Y_t^a = \dfrac{M^t - c^t}{M}$, and from this show that Y_t^a tends to zero.

d. Suppose that $M < 1 < c$. Compare this result with the normal case. (*Hint:* Interchange c and M and refer to the formula used in part (c–3), above.)

13. In the case where investment increases by a constant percent per period it is possible to derive a formula for the cumulative increase in the level of income. Using the form given in $(12 - c - 3)$ show that:

(a) Cumulative $Y_t^a = \dfrac{c}{c - M}\left(\dfrac{1 - c^t}{1 - c}\right) - \dfrac{M}{c - M}\dfrac{1 - M^t}{1 - M}$

If $c < 1$, $M > 1$, show that:

(b) Cumulative $Y_t^a = \dfrac{c}{c - M}\left(\dfrac{1 - c^t}{1 - c}\right) - \dfrac{M}{c - M} S_{\overline{t}|r}$,

where $S_{\overline{t}|r} = \dfrac{(1 + r)^t - 1}{r}$, $r = M - 1$, t being used in place of n in the symbol $S_{\overline{n}|i}$, and r in place of i.

CHAPTER 14

Economic growth and cycles

In this chapter we will discuss the Harrod-Domar theory of *economic growth.*[1] *Then this theory will be related to the principal cycle models used: the Samuelson model and the Kaldor* theory with monetary factor. The cycle models share a characteristic with the growth model—the property of being "self-generating." In a self-generating growth or cycle theory forces within the structure of the model lead to the changes which the model describes. For this reason, perhaps, the two cycle theories bear a close relationship to the growth model.

The method of analyzing economic dynamics

In the dynamic analysis used so far the models employed (especially the multiplier and multiplier-accelerator sequences) have been "recursive." A recursive system is one in which the variables at one point of time directly determine the variables at the next point of time. This statement may be modified and generalized to extend over a number of time periods. In the model of the dynamic multiplier and the multiplier-accelerator analysis, the basic equation used was:

$$E_t = Y_{t+1} \quad \text{or} \quad E_t \rightarrow Y_{t+1}. \tag{14–1}$$

This equation means that the expenditure of one period generates corresponding income (production) in the following period. On the basis of the income of Y_{t+1}, expenditure of E_{t+1} is

[1] R. F. Harrod, *Towards a Dynamic Economics*, London, Macmillan, 1949, and E. D. Domar, *Essays in the Theory of Economic Growth*, New York, Oxford, 1957. Harrod first published his model in an article in the *Economic Journal* in 1939. Domar later rediscovered the same principle. The two models differ in the matter of technique. Here we follow the Harrod treatment as given by W. J. Baumol, *Economic Dynamics*, 2d ed. New York, Macmillan, 1959.

made. In turn, this generates corresponding income, Y_{t+2}, in the next period. Schematically, this appears as follows:

$$\begin{tikzcd}[row sep=small]
E & E_0 \arrow[dr] & E_1 \arrow[dr] & E_2 \arrow[dr] & \cdot \\
Y & Y_0 \arrow[u] & Y_1 \arrow[u] & Y_2 \arrow[u] & \cdots \\
\text{Time} & 0 & 1 & 2 & \cdots
\end{tikzcd}$$

Reflection yields the conclusion that in this scheme income and expenditure in each period are determined unambiguously from the conditions of the last period. There is no question as to whether the variable in question will reach the values determined by the sequence. For this reason the models can purport to describe the behavior of observable variables in the real world.

Now let us consider how the model we have already used to explain cycles may be modified to account for growth. Intuitively, it appears obvious that one relationship, such as the acceleration principle, cannot account for both fluctuations and growth. For this reason the modified model will account for growth but *not* fluctuations. In the business cycle model consumption was considered to depend on the income of the same period. Symbolically, this can be written:

$$C_t = aY_t,$$

where a is the marginal and average propensity to consume. In the particular formulation given here consumption is treated as proportional to income. In terms of our analysis of consumption this corresponds to a long-run consumption function with income at a level which exceeds the previous maximum income. If our consumption theory is accurate this will be the case only when income is advancing annually. As a numerical example, MPC $=$ APC might be .8, meaning that .8 of all income is consumed, and .8 of any additional income is consumed. Hence $C_t = .8Y_t$, or consumption is .8 of income.

As before, investment is assumed to be proportional to the difference between this period's consumption and last year's. This is written:

$$I_t = g(C_t - C_{t-1}) = gaY_t - gaY_{t-1} = ga(Y_t - Y_{t-1}),$$

where g is the acceleration coefficient, the ratio of increased investment to increased consumption. Suppose $g = 2$, $Y_t = 100$,

$Y_{t-1} = 90$. In this case, $I_t = 2[.8(100) - .8(90)] = 2(80 - 72) = 2(8) = 16$. Clearly, the difference in consumption is 8, and investment is twice this difference or 16.

We are now in a position to discuss income determination. By adding C and Y we get:

$$E_t = C_t + I_t = aY_t + ga(Y_t - Y_{t-1}).$$

In our discussion of cycles we assumed that the expenditure of period t generates the income of period $t + 1$. This is expressed: $Y_{t+1} = E_t$, or, in consideration of the above equation,

$$Y_{t+1} = aY_t + ga(Y_t - Y_{t-1}). \tag{14–2}$$

This equation yields values of Y which register ups and downs with the passage of time.

To secure an explanation of growth we assume $Y_t = E_t$. By virtue of the first equation of the preceding paragraph we find that:

$$Y_t = aY_t + ga(Y_t - Y_{t-1}). \tag{14–3}$$

With certain values of a and g this equation may be used to define a steadily increasing sequence of incomes.

The meaning of equation (14–3) is that income is adjusted so as to make E and Y equal in the same period. If this relation is to be meaningful, it must be interpreted in the same way as our equilibrium equations of static equilibrium. In that analysis it was argued that an equilibrium tends to be stable. If income takes on a value in stable equilibrium other than the required one, certain economic forces should act to return it to the equilibrium value. If the equilibrium is unstable, the value of income cannot be expected to occur spontaneously in the real world. Furthermore, any displacement of income from the equilibrium value would lead to continued change away from the desired position. Since the model we are about to study is not recursive, some assurance should be offered that the values of income determined from the equation will actually be reached. It seems evident that the only such assurance lies in the stability of the situation in the static sense.

Let us note that the simple dynamic systems used thus far have not been characterized by equilibrium. (One exception is found

in the nonlinear cycle theory presented earlier.) In fact, we find that they are characterized by the relation:

$$Y_{t+1} = E_t,$$

which is much different from the present assumption,

$$Y_t = E_t.$$

The former leads to a well-defined sequence of events at no point of which "equilibrium" exists in the sense that $E_t = Y_t$. For example, Figure 78 illustrates the presence of a difference between income and expenditure. In this problem an equilibrium is never found, but income is well defined or determined through time. In the recursive case, equilibrium is *not* required for a determinate level of income. In the present case equilibrium is apparently required.

WARRANTED INCOME

In Figure 84 the equilibrium appears as follows. The consumption function is represented as a straight line through the origin—

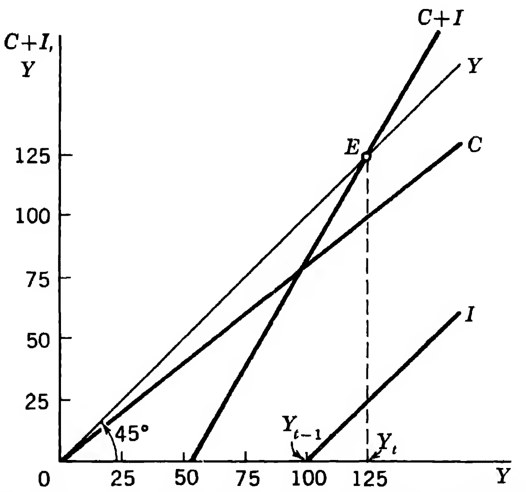


Figure 84. Equilibrium in Harrod Growth Model.

a typical long-run function. Last period's income is marked out on the horizontal axis at Y_{t-1}. When Y exceeds Y_{t-1}, investment becomes positive. Whether we should extend the investment

curve below zero is a moot point, since it is easier to expand than to contract the capital stock. In any event, $C_t + I_t = E_t$ reaches equilibrium with Y_t at point E. If the level of income, Y_t, is attained, the market will be cleared. In the literature income Y_t is known as "warranted income." It is that level of income which, if attained, will cause expenditure to equal income (production). Also, saving will be equated with investment. The increase in capital, which is the investment of the current period, is that amount of capital growth warranted by the necessity of clearing the market.

Returning to equation (14–3) we may solve for Y_t as follows:

$$Y_t - aY_t - gaY_t = -gaY_{t-1};$$
$$Y_t(1 - a - ga) = -gaY_{t-1};$$
$$\begin{aligned} Y_t &= -\frac{ga}{1 - a - ga} Y_{t-1} \\ &= \frac{ga}{ga + a - 1} Y_{t-1} \\ &= \left(1 + \frac{1 - a}{ga + a - 1}\right) Y_{t-1} \\ &= (1 + r)Y_{t-1}, \end{aligned} \tag{14–4}$$

where $r = (1 - a)/(ga + a - 1)$. If r exceeds zero, income in period t exceeds income in period $t - 1$ by r percent.[2]

If $a < 1$, that is, if MPC < 1, the numerator is positive. If, in addition, $ga + a - 1 > 0$, or $a > 1/(g + 1)$, the denominator will be positive. To illustrate, let $a =$ MPC $= .8$ and $g = 1.25$. In this case $r = (1 - .8)/(.8 \times 1.25 + .8 - 1) = .2/.8 = .25 =$ 25 percent. In order for equilibrium to prevail in successive periods income must grow at a rate equal to 25 percent per year.

From (14–4) we can secure a more informative solution. Setting t successively equal to $t, t - 1, t - 2, \ldots, 2, 1$, we get a chain of equations from (14–4) which appear as follows:

$$Y_t = (1 + r)Y_{t-1},\ Y_{t-1} = (1 + r)Y_{t-2}, \cdots, Y_1 = (1 + r)Y_0.$$

If we substitue the value of Y_{t-1} given by the second equation into the first equation, we get, successively:

$$Y_t = (1 + r)Y_{t-1} = (1 + r)(1 + r)Y_{t-2} = (1 + r)^2 Y_{t-2}.$$

[2] Cf. Baumol, *op. cit.*, pp. 165–66.

Continuing these substitutions until we reach the last equation, we get:

$$Y_t = (1 + r)^t Y_0. \tag{14–5}$$

In verbal terms, income grows at a compound interest rate, r, from the value Y_0 in the initial period. The path of growth of income is precisely like that of a sum of money which accumulates at compound interest. Thus the form is precisely like that of Figure 75(A) when r as defined in (14–4) equals 8 percent.

Let us return now to one of the main issues pertaining to this theory. What does "warranted income" represent? Is it equilibrium income? If this is to be the interpretation, the income level should be the outcome of a stable equilibrium. Consider the situation, as pictured in Figure 84 (p. 389). Examining the relation between income and expenditure, we see that the $C + I$ line cuts the Y line from below. It seems clear that, if the $C + I$ line is to cut the Y line at all, the $C + I$ line must have a steeper slope, and cut the Y line from below. This implies the existence of an *unstable* equilibrium.

We can state our conclusion to this phase of the matter. If an equilibrium income exists, the equilibrium which produced it is unstable. With this observation the picture of the warranted income path as a sequence of equilibrium incomes becomes distinctly blurred. The question now arises whether the sequence of incomes under consideration has a strong justification in the theory which underlies it. Our previous justification of static equilibrium breaks down in this situation. In fact, we cannot point to the sequence of warranted incomes and say, "This is a sequence of incomes which economic forces will tend to bring about at each point of time." The most we can say is, "This is a sequence of incomes which will clear the market in each successive period, but which static forces operating at any time will tend to upset if the least disparity from the required value occurs." Evidently, this is not a sequence which economic forces of the type shown will spontaneously maintain and assure from digressions.

In this model the investment is induced by changes in income. If the model is changed to make it stable in a static sense by adding autonomous investment, the simplicity of the model tends to

disappear. It does not seem worth while to pursue this investigation, since no clear-cut analysis emerges as a consequence.

In view of the desirability of finding a simplified explanation of economic growth this outcome is disappointing. One compensation of this outcome is as follows: The growth theory developed above depends on the operation of the acceleration principle. Simply by introducing a one-period time lag of income behind expenditure we get a model for economic cycles. Evidently, we must choose between a lag and no lag. If the lag is chosen, we are led to a simplified model of cycles. We will comment later on this matter.

Models explaining growth and cycles

In the following discussion the Harrod model plays an important role in explaining the element of growth. However, this model is very closely related to that of Samuelson, differing mainly in the time lag. Accordingly, in part of the discussion we take the existence of the Samuelson model for granted and see what can be done with it alone to get the desired answers, i.e., cycles *and* growth.

NOTE ON THE HICKS THEORY

Let us remark on one development of the subject effected by Hicks.[3] Starting with a Samuelson-type model, somewhat generalized, Hicks adds a growth factor for investment. If investment grows at a constant percentage rate, National Income may grow at the same rate. Concurrently, it is possible for a cyclical fluctuation to occur around this line of growth. The cyclical fluctuation requires that a and g assume values within related ranges which are not changed by having the cyclical process associated with growth and have been stated earlier in the text. However, Hicks does not set out those additional restrictions required by growth.

[3] J. R. Hicks, *A Contribution to the Theory of the Trade Cycle*, Oxford, The Clarendon Press, Ch. 8. See also a theory based on the use of ratchet effects: Arthur Smithies, "Economic Fluctuations and Growth," *Econometrica*, Vol. 25, No. 1, January 1957, pp. 1–52.

Hicks's theory has been criticized on the ground that a distinction between autonomous investment and investment induced by added income or consumption is not feasible in practice. If the distinction cannot be put to the test in practice, it is useless to make it in the first place. Since much of Hicks's theory rests on this distinction, the allegation is a significant one. The only way to determine the validity of the assertion is to seek a test of the Hicks model in practice.[4]

In the Hicks model, growth is made to depend upon the factor of autonomous changes in the level of investment. As most economists see the problem, the source of growth should lie within the system.[5] Evidently, in imputing growth mainly to an unexplained outside force Hicks has fallen somewhat short of a complete explanation. However, neither of the foregoing criticisms can detract from the neatness and virtuosity of Hicks's work in this field.

Successive periods of cyclical fluctuation and growth

With the aid of certain rather plausible assumptions we can build a model in which alternating periods of cyclical fluctuation and growth occur. There are two sets of assumptions which will occasion the result in question: (1) appropriate changes in the MPC and the acceleration coefficient as full employment is attained, and (2) elimination of the assumed time lag of income behind expenditure. Consider now the first of these.

VARIABILITY OF MPC (a) AND g DURING THE CYCLE

When the economy passes through a period of depression and approaches full employment, the MPC tends to increase. We have outlined the reasons for this in the chapter on consumption. It also seems quite possible that the acceleration coefficient will exhibit an increase at full employment. As long as considerable unemployment exists there will be excess capacity in most industries. At this stage an increase in consumption will be met largely

[4] See James S. Dusenberry, "Hicks on the Trade Cycle," *Quarterly Journal of Economics*, Vol. 64, August 1950, p. 473.
[5] Smithies, *op. cit.*, p. 5

by an increase in production from fixed plant and equipment. Hence the acceleration coefficient is rather low at this stage.

By the time the economy approaches full employment (of labor) additional production can scarcely be squeezed from existing plant and equipment. If additional output is to be produced, further investment must be undertaken. For this reason the acceleration coefficient will tend to increase, expressing the need for further capital as consumption expands. With an increase both in the MPC(a) and acceleration coefficient (g) the economy may move from fluctuation into steady growth.[6] This result will come off only with sufficiently large increases in these coefficients.

Suppose the economy then proceeds in a state of steady growth. If the various factors underlying a and g remain unchanged, the economy will continue to expand in a steady manner. However, the situation will probably develop in such a way as to terminate the growth phase after a time. First, it is necessary to maintain a proper balance between capital and other factor supplies. The assumed relationship between output and capital expressed in the acceleration coefficient actually rests on the relations of labor and land supplies to capital. If some of these factors, notably land, fail to expand during the growth process, additional capital may prove less productive than earlier quantities and the acceleration coefficient will fall. In a growth process new inventions may permit the growth of output to be maintained, even if land does not expand in supply to the required extent. Owing to the jerky nature of invention, this tendency will grow weaker at some point, making it probable that g will fall.

In the second place, statistical evidence tends to show that people with high incomes save a larger and consume a smaller

[6] Samuelson has provided a figure in which a (α in his notation) and g (β in his notation) are represented along the vertical and horizontal axes, respectively. He then maps out the a-g plane into four areas: A, marked by steady convergence to a fixed value; B, marked by damped oscillations; C, marked by explosive oscillations around a fixed value; D, marked by steady expansion. As a and g (α and β) both increase from an initial position in area C, the point which they define moves toward and into area D. When the boundary or interior of D is reached, steady expansion occurs. It is the thesis advanced here that a movement from area C to D may well occur as full employment is reached.

proportion of their incomes. This suggests that a growth in an individual's income may lead to a decline in his APC $= C/Y$. By the same token a general rise in per capita incomes during a growth process might be expected to lead to a decline in society's APC $= C/Y$. Only if new products, a greater variety of products, and, above all, products more closely adapted to the needs of society appear will this adverse effect be forestalled. In short, innovations must appear steadily to effect the maintenance of APC. Since innovation is jerky by its nature, we cannot really expect a steady value of APC. Sooner or later a lapse in the flow of innovation will occur, and APC will fall. By the same token $a =$ MPC will tend to fall.

When the growth process falters, as evidenced in a decline in g or a, the system will lapse into a state of oscillation. In some cases it will lapse into steady decline to a fixed value or strongly damped oscillations around a fixed value.[7] Generally, the system could be expected to exhibit oscillations which grow in amplitude. When income again rises to the full employment level, forces making for growth would again become strong.

AN ALTERNATIVE HYPOTHESIS FOR ALTERNATIONS OF CYCLES AND GROWTH

We have seen that variations in a and g in the neighborhood of full employment can cause oscillation to give way to growth and vice versa. Evidently, the attainment of full employment is an

[7] The point defined by the combination (a, g) would move from region D in the Samuelson figure to region C. In more extreme cases the point might end up in region A or B with the results mentioned.

Note that some ratchet effects may take effect. The growth process generates a new peak income. In turn, this leads to an upward shift in the short-run consumption function. The essential result of the upward shift is the presence of a higher break-even point at which $C = Y$. Since investment cannot fall below a figure equal to maximum disinvestment, this break-even point helps establish a floor for income. In virtue of the higher income at which this floor is reached, the trough of the depression is reached and overcome at higher income levels than in cycles occurring before the latest growth phase.

Another ratchet effect resulting from the growth process arises from the accumulation of capital. This process increases productive capacity over former levels and leads to a pressure for sales at low prices. More technically, the short-run marginal cost curve of a firm moves to the right and tends to intersect with marginal revenue at a larger output. This tends to maintain a higher level of output than would be the case without the larger capacity.

important event for cyclical and growth processes. Another element which may undergo an important change at this point is the timing of events. Such a change in timing at full employment, reflecting a more optimistic attitude on the part of employers, may lead to a shift from cycles to growth. We will proceed to examine the nature of this possibility. As a preliminary let us recall this important fact. *The only formal difference between the Samuelson and Harrod models, as stated in the text, is that the former exhibits a unit time lag of income behind expenditure, whereas the latter does not.*

INCOME LAG AND SYNCHRONIZATION

When expenditure generates corresponding income in the following period, an income lag exists. On the other hand, when expenditure and income reach equality in every period "synchronization" may be said to occur. Consider the underlying economic situation as it affects the question of timing during the course of a cycle.

Suppose that National Income is below the full employment level in a depression. Factories are operating below capacity in many instances, and some labor is unemployed; total demand for all products is insufficient to cause production to increase to capacity. Under such conditions uncertainty about the course of events is likely to prevail. In this atmosphere businessmen are likely to play it safe. Rather than trying to produce in anticipation of demand, they allow demand to reveal itself. On the basis of revealed demand producers hire factors and pay out income. Since it takes time to adjust production to the needed level, incomes paid lag behind revealed demand. Clearly, this argument expresses the notion that an income lag exists.

Let us consider this point a bit more closely. Businessmen have two main problems with which to cope: (1) the desire for profit or capital gain; (2) the desire to hold down risks or uncertainties involved in the attainment of these returns. When demand is deficient, returns tend to decline and uncertainties multiply. To avoid disaster in a tight situation the business man must take precautions; that is, he must take steps to reduce risks that he could afford when profits were higher. Among such precautions is a strict limitation of production to demand actually in sight.

In consequence, production will increase only following some concrete evidence in the form of additional buying.

It is obvious that sales must be synchronized with demand in a modern industrial society with division of labor and exchange. When the customer comes in to buy, he will expect to find the goods on hand. However, this does *not* mean that an increase in demand will be met by a simultaneous increase in production. If the producer needs to make a sure return on each outlay, he will be inclined to wait for *changes* in demand before adjusting production. All this is summarized in the equations:

$$E_t = Y_{t+1},$$

$$E_t - Y_t = Y_{t+1} - Y_t = \Delta Y_t.$$

In the second equation an excess of demand over supply in one period generates a rise in production.

When full employment is reached, the situation takes on a changed complexion. In the first place, the profit situation is such that most firms are both able and willing to bear increased risks on *extra* business. Thus an additional outlay to produce and sell products in anticipation of demand can no longer endanger the firm. In short, when the firm approaches a profitable position with an assured basic demand, it also is in a position to take something of a gamble on new business.

In the second place, one condition on which an advance beyond full employment depends is that employers are constructing new plant and equipment. In turn, the new plant embodies technological advancements and often turns out a new product. Clearly, it is the deliberate intent of businessmen who are innovators to create conditions in which these products will be bought. Thus the businessman seeks simultaneously to increase plant capacity and to generate corresponding demand. If income has exceeded previous levels for several successive periods, confidence in a further advance is engendered. When this happens, businessmen are encouraged to increase output in the expectation that the advance will continue. A symptom of the expectation of such growth is found in the construction of plants for the production of new goods. In summary, the spirit of confidence that animates

businessmen at this point prompts them to seek a more immediate relationship between production and demand.

COORDINATION OF SAMUELSON AND HARROD MODELS

In our discussion of the Samuelson model we found that the case of explosive oscillation led to the most satisfactory explanation of cycles. Starting off with cyclical fluctuation in this model, marked by the time lag noted, income will tend ultimately to reach either a well-defined floor or the full employment level. Assume the latter is reached. By the argument just reviewed the attainment of full employment will be marked by a "time shift" from income lag to income–expenditure synchronization. Conceptually, we represent this by a shift from a Samuelson to a Harrod type of model.[8] Consequently, a synchronized growth process will occur until the growth forces are interrupted.

Eventually, the flow of new resources or innovation required to sustain the growth will falter. As this happens, a reverse "time shift" will occur, with a resultant tendency to take up cyclical fluctuation again. In the course of the ensuing fluctuations income will sooner or later attain the full employment level. At this juncture the growth process could pick up again. If income first strikes its lower limit in the course of its cyclical fluctuation, the fluctuation will be altered but will continue. Eventually, the fluctuation will bring income to the full employment level. At this point the system can move from fluctuation into steady growth.

As we have restated the theories in this text the Samuelson cycle model and the Harrod growth model are identical but for a time factor. By a "time shift" from income lag to synchronization in time we move from one model to the other. If we are to make use of both models, it is necessary to provide a theory of this time shift. An attempt to do this was made above.

[8] The condition for a positive rate of growth in the Harrod model is: $a > 1/(1 + g)$, assuming that $0 < 1 - a < 1$. This is derived from a consideration of equation (14–4). We may plot the relation $a = 1/(1 + g)$ on the Samuelson map mentioned in an earlier footnote in this chapter. The area above this line indicates combinations of a and g which yield steady growth. Any point (a, g) lying in Samuelson's area C will lie within the area of steady growth in the Harrod model. If a time shift occurs, a combination (a_1, g_1) leading to explosive fluctuations in a Samuelson model will now lead to steady growth in a Harrod model.

Another model for the coordination of cycles and growth

We have advanced two hypotheses to account for a possible shift from fluctuation to growth around the full employment point. One is based on a change in the coefficients, the other on a change in the time relations between income and expenditure. We could also combine the hypotheses. There remains the limitation that these hypotheses do not explain very adequately cycles with growth arising from autonomous causes. It is perfectly true that we can work this in via ratchets, and this was done in a footnote. Nevertheless, the theory as it stands is not complete.

We now wish to advance a somewhat different approach to the coordination of cycles and growth. As a prelude, we must take up once again the Harrod model.

HARROD'S THEORY AND THE MONETARY FACTOR

One issue left unresolved in the Harrod theory is the instability of income or of the situation which defines "warranted income." Consider the effect of the monetary and interest factor on the growth problem. Here we employ the same framework as was used to discuss the generalized Kaldor model and the over-all Keynesian equilibrium. Consider now the revisions in the construction of the basic diagrams caused by the assumptions of the Harrod theory.

We use here the saving–investment diagram rather than the income–expenditure approach. In so doing we represent the saving line as passing through the origin, since full employment has been attained, and the *C* and *S* lines are assumed to have the shapes assigned by the Modigliani-Dusenberry theory. In addition, the *I* line is represented as cutting the income axis at a positive income, following the Harrod theory. Let us assume also that lowering the rate of interest causes the *I* curve to shift upward, whereas a rise causes it to shift downward. At a given level of income and a lower rate of interest the MEC exceeds the rate of interest, causing an increase in the rate of investment. At a given level of income and a higher rate of interest the MEC falls short

of the rate of interest, causing a decline in the rate of investment. In either case an appropriate change in the investment rate causes the MEC to move to equality with the rate of interest.

Let us derive in Figure 85 the I,S curve which helps to portray general equilibrium. Assume that the S curve is independent of the rate of interest so that it lies in one fixed position. Draw in three I curves at 3 percent, 4 percent, and 5 percent rates, respectively. Note that the I curve starts from last period income, Y_0, if the rate of interest remains unchanged. However, a drop in the rate of interest will make it possible to commence investment at a lower income. On the other hand, a higher rate of interest will make it necessary for income to be even higher than last period's income before investment commences.

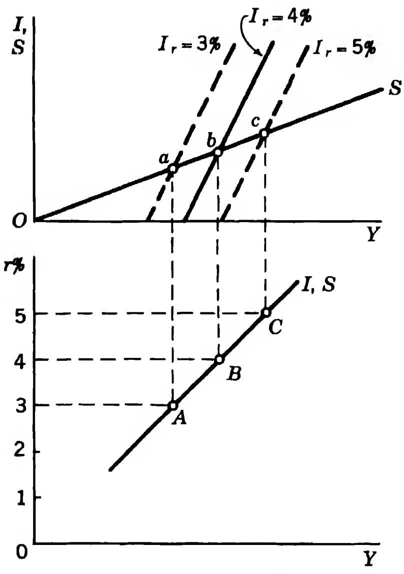


Figure 85. I,S Curve for the Harrod Model.

We now draw in three investment curves at 3 percent, 4 percent, and 5 percent interest rates, respectively, in the investment–income plane. We also draw in the saving curve. The $I_{r=3\%}$ curve, generated by a 3 percent interest rate intersects with the S curve at point a. We then drop a line vertically downward into the interest–income plane. This line intersects a line drawn horizontally to the right at an interest rate of 3 percent, the point of intersection being A. Point A defines the income level consistent with a 3 percent interest rate.—By a similar (and familiar) process we define points B and C which give the equilibrium income levels at interest rates of 4 percent and 5 percent, respectively. Connecting the points with a smooth line, we get the I,S curve.

Now we put the I,S and L,M curves together as in Figure 86.

Note that the I,S curve resulting from the assumptions used has a positive inclination throughout its length. By applying a familiar argument we can derive the conclusion that income OA and interest rate AE define a stable equilibrium. Our assumption is that the interest rate is determined in the money market, that this market is always in equilibrium, and that all temporary settlements lie on the L,M curve. At income OU the interest rate is UV. But this is lower than the natural rate UW which equates I and S. Consequently, at V, I exceeds S, and income tends to expand from OU toward OA. At income OU' the interest rate is $U'V'$ with a higher rate than equates I and S. Consequently, at V', S exceeds I, and income contracts from OU' toward OA.

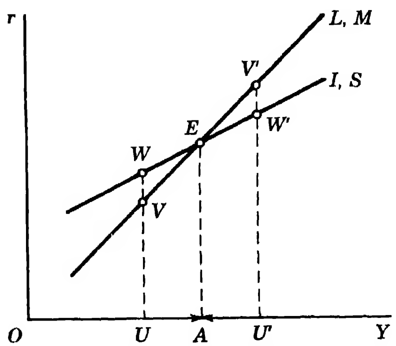


Figure 86. Equilibrium in Harrod Model with Monetary Factor.

As an alternative, we can use the more classical argument, cast in terms of the difference between the money rate and the natural rate. At income OU the natural rate exceeds the money rate. It then pays to borrow in the money market in order to invest in the capital market. In turn, this injects a stream of new money into circulation which increases income, pushing it toward OA. If the income level is OU', the money rate exceeds the natural rate, and it pays to divert saving from the capital market into the repayment of bank debt. This results in a decline of investment, a reduced supply of money, and falling income.

We now see that under the assumption given the rate of interest and the level of income attain a stable equilibrium at E. The next question is whether the sequence of incomes in each period is an expanding one under the new conditions. The solid lines portraying the S curve and the $I^1_{r=4\%}$ curve cross to determine an equilibrium income at Y_1 in the upper part of Figure 87. In turn, this income represents a value higher than that of the preceding period in the amount $Y_1 - Y_0$. The new I curve in period 2 cuts

the Y axis at Y_1, giving the $I^2_{r=4\%}$ curve shown by the second line. Evidently, the I,S curve tends to shift to the right in each successive period.

What effect does the shift of the I,S curve have on the situation? First, suppose the rate of interest were to be held constant by monetary action. In this case the L,M curve would have to

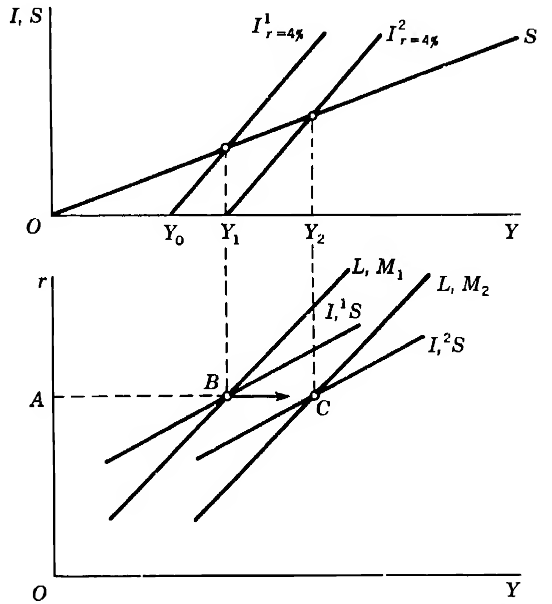


Figure. 87. Growth in Harrod Model with Monetary Factor.

move to the right in each successive period, the shift just sufficing to hold the rate of interest constant. This situation is pictured in Figure 87, income increasing from AB to AC, at interest rate OA.

Such a policy on the part of the banking system brings about the exact change in the income level indicated by the Harrod model. In the Harrod model the I curve for each successive period is drawn up on the assumption that the rate of interest is constant. Assuming that the supply of money increases sufficiently

to meet the additional demand at a higher income, this implied requirement of the Harrod model is satisfied. Consequently, the situation pictured in Figure 87 is essentially that of the Harrod model, but with one significant difference. Monetary restraint in each period makes the equilibrium stable.

If the money supply increases more slowly, a situation less favorable to an advance will occur. The L,M_2 curve will intersect the I^2,S curve at an income smaller than AC. If the money supply increases more rapidly, the L,M_2 curve will intersect the I^2,S curve at an income larger than AC. In the first case, monetary factors impede the expansion; in the second, they add impetus to it.

Clearly, the additional money supply needed arises from the income expansion. If the additional money demanded as a result of the income expansion is proportional to the income change, the amount needed could be expressed by the equation, $\Delta M = K \cdot \Delta Y$. In this special case the monetary requirement for the Harrod warranted income path is that the money supply increase in some fixed proportion to income.

KALDOR CYCLE THEORY AND THE CONCEPT OF ECONOMIC GROWTH

From the discussion of the Harrod model just given it is clear that the I,S curve shifts to the right in each time period, provided the assumptions of the former analysis are accepted. Granted a corresponding increase in the money supply, income increases in each successive period. How does this concept of growth relate to the modified Kaldor cycle theory discussed earlier? Before answering this question let us note one or two points about that analysis.

In the chapter on business cycles we ignored the effect of capital accumulation in causing an increase in output. Consequently, income either increased or decreased, but with no marked tendency to growth. The Harrod theory affords an opportunity to introduce this element into the account of cycles. As the preceding graphical analysis shows, the effect of economic growth of the Harrod type is to cause a movement to the right of the I,S curve. If we assume the I and S curves to be straight lines, as illustrated

previously, the movement is a constant percentage at a given interest rate.[9]

Suppose that even when the I curve is a curved line conditions are such as to cause a constant percentage change in I and Y. This would be occasioned by a uniform proportional shift in the I curve to the right. In turn, this will occasion a uniform proportional shift in the I,S curve to the right. By scaling off the values of

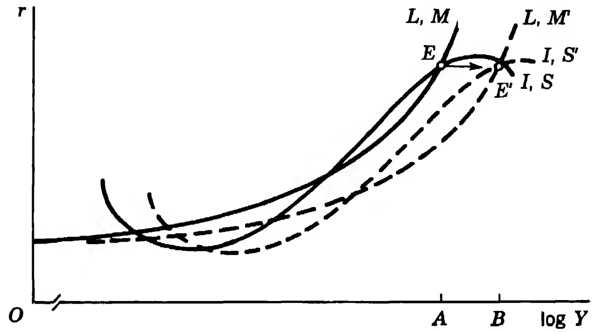
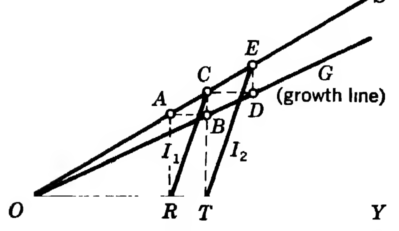


Figure 88. Growth in Kaldor Model with Monetary Factor.

income in logarithms we can register this movement by a constant horizontal distance. Such a setup is illustrated in Figure 88. In

[9] The constant percentage advance in (real) income may be taken as a postulate or it may be derived as a consequence from the shapes of the I and S curves. Take the former approach. First, lay off a growth line by the condition that $OT/OR = 1 + RT/OR$, where RT/OR is the required growth rate. If income was OR in the initial period, it must become OT to sustain the growth rate. For OT to be the equilibrium income I must equal S. Hence I, commencing at the old output OR, must grow to equal S at C (and income OT). Note that income OT represents capacity at the time it is attained. The next increase in income is CD. The new investment line is TE; beyond E the investment line is horizontal. It can be shown by similar triangles that income, saving, and investment advance at a common percentage rate. Note that points R, T etc. represent points of capacty for periods 1,2, etc.



Alternative Illustration of Harrod Growth Process.

order to sustain the advance in the level of income the movement of the L,M curve must exactly match the shift of the I,S curve.

The case of an equiproportional change in I and in the L,M curve is illustrated in the preceding diagram. Each successive shift is equal to AB, since the horizontal scale is now a ratio or log scale. Note that the solution requires that the L,M curve intersect the I,S curve from below, so that stability results.

THE FACTOR OF TECHNOLOGY

It appears that the theory sketched will account for periods of steady growth, as well as cycles. If the I,S and L,M curves move to the right in the same proportion, income will be higher in that proportion. As in the Harrod theory, this event requires a delicate balance of assumptions. In order that income and capital grow at a constant percentage rate certain things must be happening to the factors. Suppose that constant returns to scale prevail. If the labor supply, land supply, and capital all grow at a common rate, their product can grow at the same rate. No change in technology is needed. Since land (or natural resources) is subject to limitations, technology must generally improve to permit a constant rate of growth. Once technology appears in the argument, the cat is out of the bag. There is a variety of possible assumptions which may be made.

Assume that technology progresses sufficiently fast for a time to permit the growth process to continue. At some later date technology may fail to keep up the needed pace. Such an event causes the I,S curve (or the natural rate of interest) to decline. For a while this tendency may be offset by a rightward and downward movement of the L,M curve. However, the L,M curve is restricted in its downward movement by institutional considerations. In short, a lower limit to the fall of the money rate is set by the institutional minimum. Eventually, the downward shift in the I,S curve brings a critical point of contact between the L,M and I,S curves. Then a large downswing is inevitable from the shapes and relations of the curves. In Figure 89 this is shown by the shifting of the equilibrium point from E to e where the equilibrium is unstable in a downward direction, making necessary a substantial decline in the level of income.

Let us note several points about this development from a growth phase to the downswing of a cycle. First, it is in accord to a considerable extent with the views of Keynes who asserted that the main cause of the trade cycle is the variability of the MEC.[10] It is precisely a decline in the MEC curve which underlies the decline in the *I,S* curve in our model. Beyond the initial decline in the

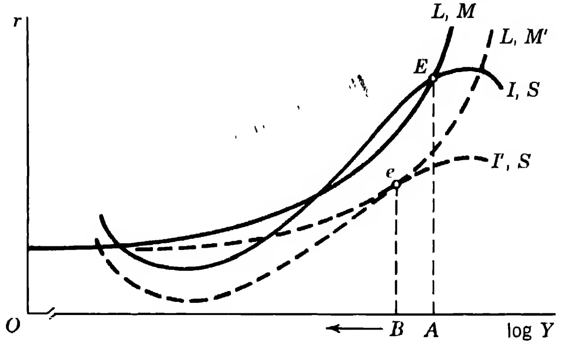


Figure 89. Downturn After Growth in Revised Kaldor Model.

MEC is the further decline in natural rate (to which the MEC is equal) as a result of the unstable relations between the *I,S* (natural rate) curve and the *L,M* (money rate) curve. As the natural rate declines, the MEC declines, since MEC is always equal to it.

Second, the situation is roughly in accord with the ideas of Schumpeter. During the boom or growth phase of the cycle potentialities of some basic innovation are being exploited and the MEC as well as the *I,S* curve are shifting to the right. Eventually, the possibilities of the innovation are worked out, and the MEC and *I,S* curves fall, leading ultimately to a large contraction. One of the critical assumptions underlying the cycle and growth analysis defines the changes in the supplies of factors other than capital. If the supply of land fails to grow with sufficient rapidity, it is the factor of technology or innovation which may make for either continued growth or cyclical decline.

[10] John Maynard Keynes, *The General Theory of Employment, Interest and Money*, New York, Harcourt Brace, 1936, p. 313.

CHAPTER 15

Dynamic aspects of economic policy

Elementary problems of inflation

At this juncture we need to discuss in broad terms the elements of inflation. Since this subject presents such an important practical problem, it lends itself to a unified treatment of theory and policy. To keep the analysis manageable we restrict the analytical methods to simple diagrams.

Let us consider briefly the various assumptions to be used. First, we use a $C + I$ line of the usual form up to the full employment income level. Beyond this point, or as money income expands without an increase in output, buyers will increase demand in proportion to income. Only in this manner will they be able to purchase the same collection of goods. If incomes and the prices of goods have increased in proportion, buyers will *want* to purchase the same collection. In fact, real income is the same, and buyers will wish to purchase the same real quantity of goods. In Figure 90 this is represented by a line through the origin passing through the full employment expenditure point. At f the $C + I$ line coincides with a line passing through the origin. Beyond f buyers spend increased sums on output in proportion to increased prices and income.

If income rises from OF to OF_1, a movement on the $C + I$ line from f to g occurs. However, once prices and factor costs have risen, they never go back to the same levels. Consequently, a price ratchet is in effect which permits prices to rise between f and g. Now the full employment money income is OF_1. If money income recedes, the price ratchet holds and expenditure does not move back down the solid curve from g to f. Instead, at permanently higher prices a movement down a higher $C' + I'$ line occurs. Note that point S on the new $C' + I'$ line corresponds to R on the old; the difference between these points involves just the price level which is higher in the proportion $OS/OR = Og/Of = OF_1/OF$. In

short, it is assumed that there is a point on the new demand line, corresponding to the old, as S corresponds to R, representing the same real situation, but with a proportionally higher price.

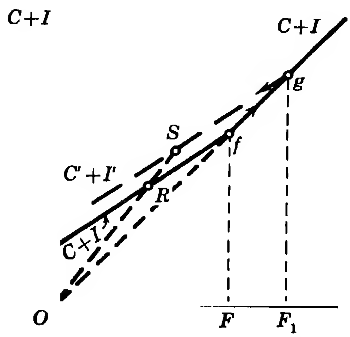


Figure 90. $C + I$ Lines at Varying Income Levels.

Next, we introduce a one-period time lag of expenditure behind income. The expenditure, $C + I$, of one period determines the income at factor cost, Y, of the next. Such an assumption simplifies the explanation of the mechanism of change.

Note that as money income moves beyond point F and real income remains the same, the full employment income increases. When money income reaches the level OF_1, this value now represents full employment income. Prices having risen to higher levels and having downward rigidity, a decline in money income will immediately lead to some unemployment.

It is customary at the present time to classify inflationary cases into two types: (1) demand–pull; (2) cost–push. In the first case the occasion for the inflation is "too much money chasing too few goods." Whenever this happens, the demand for goods exceeds the supply at full employment, and an inflationary gap appears.

On the other hand, a cost–push inflation occurs when businessmen seek to collect a higher level of receipts as a condition of furnishing a given quantity of output. If wages are increased by union action, the firm seeks to recover the added cost by additional receipts. Also an attempt to exploit a monopoly position more fully may lead businessmen to ask for a greater level of receipts. The source is not vital in this connection. The criterion is that firms require a larger level of receipts as a condition of furnishing the same output.

DEMAND–PULL INFLATION

In this case the $C + I$ line lies above the Y line at full employment income. Consider Figure 91 in this connection. The

expenditure registered at a in period 1, which exceeds income at A, generates income at B in period 2. In turn, with income at B, expenditure at C is generated. Pursuing this sequence, income and expenditure follow an increasing stairstep pattern, $AaBCDE\ldots$. Since full employment income was initially found at A, the subsequent increases denote a rise in factor prices. In virtue of the rise in factor payments (with constant quantities of the factors) factor prices must be increasing. By the same token, since expenditure is exhibiting a pattern of increase, and output is constant, this output is being sold at ever higher prices. Clearly, factor and product prices are undergoing inflation.

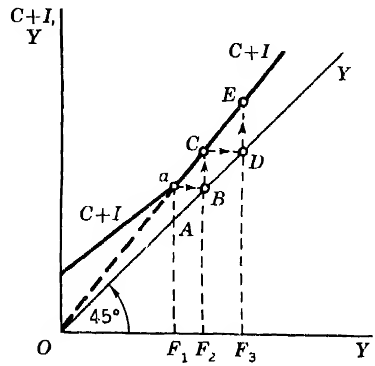


Figure 91. Demand–Pull Inflation Process.

Let us entitle the ratio of this year's to last year's price "the price index." We can define this quantity by the equation, $p_t = P_t/P_{t-1}$, where $t-1$ and t represent successive periods of time. Suppose P_t is greater than P_{t-1}, the values being, say, 220 and 200, respectively. Then the price index is $220/200 = 1.1$. As is the case with indexes, when we get the answer in the form of a ratio, we generally multiply by 100, an operation which gives the answer in the form of the number 110. Such an index number implies that this year's price is 110 percent of last year's price. In turn, this implies that the price has risen by 10 percent over the last period value.[1]

If the sequence of prices in three years, 0, 1, and 2, is 200, 220, and 242, the price indexes are: $p_1 = 220/200 = 1.1$, or 110; $p_2 = 242/220 = 1.1$, or 110. In short, $p_1 = p_2 = 110$, and the

[1] Let us define the proportional increase in price by the equation, $r_t = (P_t - P_{t-1})/P_{t-1}$. In the numerical illustration this is $r_t = (220 - 200)/200 = 20/200 = .1$ or 10 percent. Note that $p_t = P_t/P_{t-1} = \{P_{t-1} + (P_t - P_{t-1})\}/P_{t-1} = 1 + r_t$; in the case cited above p_t is $1 + .1$, where .1 or 10 percent is the price increase.

price index is 110 in every year. This implies that the price is rising 10 percent a year. In this event the price is increasing at compound interest and follows the growth pattern indicated by the compound interest law.[2]

Under what conditions will the price level rise and how is this increase measured? Note that once full employment is attained output can increase no further. If expenditure on output exactly equals full employment output, $C + I = Y_f$, then the price level remains the same. If $(C + I) > Y_f$, then demand for goods exceeds the flow of output. In this event there is no avoiding a price rise. When the flow of output, measured by money income, meets a larger flow of expenditure, prices must be higher than before in the ratio of expenditure to income.[3]

By the argument stated previously, each increase in the level of income beyond the full employment level generates a proportional increase in expenditure. From period to period the ratio of E to Y remains the same. Since the constant proportional disparity generates a similar price rise, the proportional increase in the price remains the same. As we interpret this condition in Figure 91, the index of price increase is shown by the common ratio of $C + I$ to Y in each period, or:

$$\begin{aligned} p &= P_1/P_0 = aF_1/AF_1 \\ &= P_2/P_1 = CF_2/BF_2 \\ &= P_3/P_2 = EF_3/DF_3.^4 \end{aligned}$$

Just as the index of price increase remains the same, so also does the percentage increase in price exhibit the same value in each

[2] In the example given in the text, $p = p_1 = p_2$. We then find that $P_2/P_0 = (P_2/P_1)(P_1/P_0) = p_2 \cdot p_1 = p^2 = (1 + r)^2$. In the particular case in question, $P_2/P_0 = 242/200 = 1.21 = (1 + .1)^2$. In short, starting with a price of 200, increasing at 10 percent per period, compounded, we arrive at a price of 242 at the end of period 2. Cross-multiplying, we get: $P_2 = P_0(1 + r)^2 = 200\ (1.21)$.

[3] Let $E_0 = Y_1$, since this period's expenditure generates next period's factor income. Also, by definition, $E_t = P_t \cdot Q_t$, since expenditure is the value of output. At full employment income and beyond Q_t assumes a constant value Q. Assume that full employment is reached at time O. Since $E_0 = P_0 \cdot Q$ and $E_1 = P_1 \cdot Q$, $E_1/Y_1 = E_1/E_0 = P_1Q/P_0Q = P_1/P_0$. This proves the textual statement.

[4] In the figure, $E_1/Y_1 = aF_1/AF_1 = aF_1/OF_1$; $E_2/Y_2 = CF_2/BF_2 = CF_2/OF_2$. Since triangles OaF_1 and OCF_2 are similar, the sides are proportional. Consequently $aF_1/OF_1 = CF_2/OF_2$; this amounts to saying that $E_1/Y_1 = E_2/Y_2$ which implies by the preceding note, $P_1/P_0 = P_2/P_1$.

successive year. The percentage increase in price is measured by the excess demand expressed as a fraction of the income level. Thus we can show that:

$$r = aA/AF_1 = CB/BF_2 = ED/DF_3,$$

these common values expressing the constant percentage increase in prices from year to year.[5] The numerical example given earlier illustrates how prices develop. Evidently, prices of products are increasing at a constant percentage rate through time, as indicated by the unchanging value in question.

In this analysis the $C + I$ line is drawn up on the assumption that the banking system is holding constant the rate of interest. No matter how great the demand for money, enough is supplied to keep the rate of interest constant. This implies complete passivity with respect to inflation on the part of the banking system. Let us see what measures can be taken to check inflation, given this general attitude of the banking system.

MEASURES TO CHECK DEMAND–PULL INFLATION

Perhaps the most obvious solution is to reduce the demand for goods. By means of fiscal policy the $C + I$ line can be lowered. Here we are classifying government spending under I, so that the contemplated action would reduce I. If taxes on personal income are raised, consumption will decline and the $C + I$ line will be shifted downward. If government spending is reduced, the line will also decline. Another alternative is to invoke monetary policy and raise the rate of interest, thus reducing investment and lowering the $C + I$ line. In the last case we may assume that the higher interest rate is maintained, regardless of circumstances.

Suppose one or a combination of these policies is followed to the extent that a point of equality between demand and supply is found at full employment income. In Figure 92 the $C + I$ line drops to the position indicated by the dashed line for incomes up to full employment and to a position coinciding with the Y line

[5] Remembering from footnote 4 that $P_1/P_0 = E_1/Y_1$, we find $P_1/P_0 = E_1/Y_1 = aF_1/AF_1 = (aA + AF_1)/AF_1 = 1 + aA/AF_1 = 1 + r_1$; $P_2/P_1 = E_2/Y_2 = CF_2/BF_2 = (CB + BF_2)/BF_2 = 1 + CB/BF_2 = 1 + r_2$. Since $P_1/P_0 = P_2/P_1$, as shown in the previous two notes, $1 + r_1 = 1 + r_2$, implying that $r = r_1 = r_2 = (aA/AF_1) = (CB/BF_2)$. It is also possible to prove this by similar triangles.

for equal and higher incomes. The interest rate is assumed to be held constant in the new situation. In this case the income level cannot fall below the full employment level, OF.[6] However, any level of income above OF is possible, the system being in neutral equilibrium. This indeterminancy in an upward direction is caused by the assumed passivity of the banking system, evidenced by its attitude in holding the interest rate constant. In any event the policies mentioned will remove the necessity, though not the possibility, of continued inflation from the side of demand.

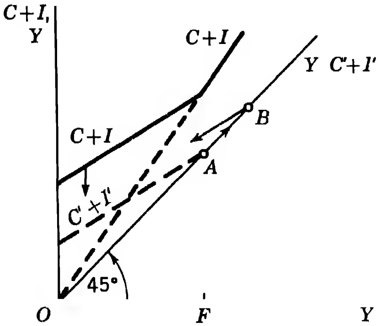


Figure 92. Demand-Pull Inflation Remedied by Lowering $C + I$ line.

A second policy is to increase the supply of goods. This might be accomplished by private initiative through the discovery of new techniques of production which would increase full employment output. On the other hand government might do things which would stimulate productive effort such as greater support of research, or policies designed to increase the benefits accruing from new inventions. From another direction government might seek to improve the allocative efficiency of the economic system. This would involve encouragement of purely competitive enterprise at the expense of monopoly and the encouragement of a more competitive situation in the labor market. From all these sources one might provoke an increase in capacity output.

In Figure 93 this increase in output is represented by a movement of full employment income from OF to OF'. Since real income increases, the $C + I$ line is extended by the dashed segment BC. Beyond C the $C' + I'$ line coincides with the Y line. In this situation income is stable in a downward direction. At incomes

[6] If the system moves from A to B, a ratchet engages at B. If income should decline, expenditure will follow the direction indicated by the arrow pointing backward from B. This would cause $C' + I'$ to exceed Y, and precipitate a return to B. Thus income is stable in a downward direction.

above OF' income and expenditure are equal at any income level. Accordingly, any income level above OF' is possible, but there is no positive force pushing income to ever higher figures.

In both this and the preceding case income is stable in a downward direction once full employment equilibrium has been reached. In the last case suppose money income increased from OF' to OF'' without a change in real income. Then a movement from C to D occurs, marked by rising prices. If income should now recede, expenditure on output will not decline from the level at D to that at C. Instead a price ratchet takes hold, and the level of output and employment falls along with price, as income declines. Thus a movement from D in the direction of the arrow takes place, implying that a permanently higher expenditure in money resulting from a higher price level is now in effect. Such a movement would be corrected, since it causes expenditure to exceed income. Hence once an income level such as OF'' has been attained it is stable in a downward direction.

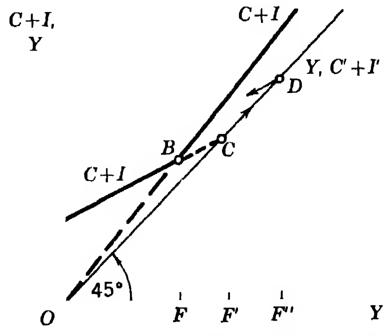


Figure 93. Demand-Pull Inflation Remedied by Increasing Supply.

INFLATIONARY SITUATION IN GOODS MARKET WITH INELASTIC MONEY SUPPLY

Suppose the situation is the same as before, except for conditions in the money market. In the preceding case the money supply was perfectly elastic, and any quantity called for by the state of the goods market was provided by the money market without a rise in the interest rate. Normally, the greater the quantity of money supplied, the higher is the interest rate asked by the banks. When the level of income increases, the demand for money increases, and in order to elicit an increased supply from the banks it is necessary to offer a higher rate of interest. All this follows from the usual assumptions made in the book and a supply curve of money, offered by the banks, which is less than perfectly elastic.

Referring to the upper part of Figure 94 we find the $C + I$ and Y lines in an inflationary relation to one another. Recall our assumption that equilibrium is always attained in the money market, but may be reached only after a time lag in the capital market. As seen in the lower figure, the interest rate OV and income level OF prevail at point a on the L,M curve. Obviously, the goods market is not in equilibrium, as shown in the upper figure. At an interest rate OW, $C + I$ would equal Y, as seen at a' on the I,S curve. Since the natural rate exceeds the bank rate by $aa' = VW$, income will tend to expand until they become equal at income OF_1. At interest rate OW the new $C + I$ line follows the dashed segment SB to the Y line, coinciding at higher income levels. Note that the price ratchet has taken effect, so that the $C + I$ line has shifted upward and to the right. In turn this moves part of the I,S curve to the right.

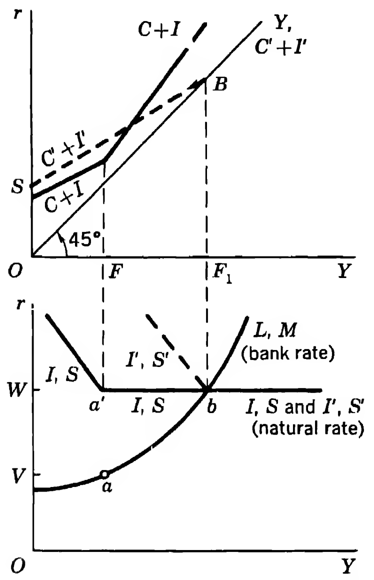


Figure 94. Demand-Pull Inflation Remedied by Tight Money.

Even if the interest rate were to be held constant, income would be stable in a downward direction. This is seen by the relation of the new $C' + I'$ curve to the Y line at lower incomes. If the interest rate were to be held constant, the situation exhibits neutral stability at higher incomes, as seen in the upper figure. However, the bank rate rises as income expands, because the money supply is assumed not to be perfectly elastic. Hence the bank rate rises above the natural rate at higher incomes, causing a contraction to the new full employment income OF_1. In short, income OF_1 is stable in either direction. The stability at higher income levels is traceable to tightness of money.

CONCLUSIONS ON DEMAND–PULL INFLATION

In the case we have analyzed the presence of an inelastic money supply is sufficient to bring a definite halt to inflation. Other measures will lead in the same direction, especially a reduction in demand. However, the willingness to create larger and larger quantities of money at constant interest rates appears to be one of the most essential conditions for this type of inflation. Likewise, the most effective remedy or, at any rate, one which is an essential part of the remedy is to have an inelastic money supply when output is constant and money income increases.

An even more stubborn and alarming type of inflation can arise from the unstable relations of the $C + I$ and Y curves suggested by Harrod. This could cause a positively inclined I,S curve, implying that the natural rate rises with income. Such a condition could cause runaway inflation, if the natural rate rose faster with income than did the bank rate.[7] We will not go into this problem in detail here, since it involves numerous complications.

A second phenomenon may develop during inflation which greatly adds to its force. This is a "flight from money." Suppose prices have been increasing for a number of time periods at a rather rapid rate. The holding of money then becomes quite costly. If the holders of money realize what is happening, they may wish to hold various forms of securities or stocks of goods in preference to money. In turn, this may lead to a reduction in the demand for money as income rises. As a result the rate of interest will fall with the rise in income. If the natural rate is constant, as represented in the last analysis, or falls more slowly than the bank rate, the expansion of money income and prices will continue.

COST–PUSH INFLATION

Let us assume that the economic system is in a state of full employment. In Figure 95 the economy is shown in a position of neutral equilibrium at A. At lower incomes than OF_1 income tends to expand, since $C + I$ exceeds Y. At incomes of OF_1 and higher the $C + I$ line and the Y line coincide for reasons we have already discussed. This implies that the system is in neutral

[7] See Knut Wicksell, *Interest and Prices*, London, Macmillan, 1936.

equilibrium at incomes of OF_1 and higher. In the absence of a definite push from some source the system will remain at A. If pushed, it will move to a new point and remain. Only a succession of pushes will lead to a sustained movement in this situation. As explained before, income is stable in a downward direction.

Suppose that with a given income in a certain period the factors want an increase in a given proportion each year. To represent this desired result in the figure we construct a new line, labeled Y', which is shifted to the right of Y in a given proportion. Points such as B and D compare with like points A and C in that the corresponding incomes are larger in the proportions, OF_2/OF_1 and OF_3/OF_2. By construction $OF_2/OF_1 = OF_3/OF_2$, indicating a desire by owners of factors for an increase in factor incomes by a given proportion at any given income level.[8] If the increase is not granted, corresponding factor services are withheld.

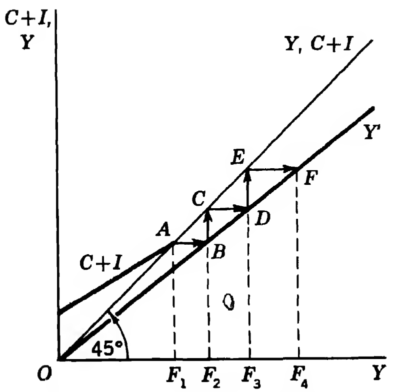


Figure 95. Cost-Push Inflation Process.

Suppose owners of factors cause factor incomes to rise in the proportion, OF_2/OF_1, leading to an increase $AB = F_1F_2$ in the level of income. Since full employment exists, output is constant, and buyers wish to expand outlay $(C + I)$ in the same proportion as income. Since expenditure on output was initially equal to income, expenditure must increase by the same amount. Clearly, then, the rise in expenditure, BC, equals the rise in income, AB. Tracing out this process, we get the path $ABCDE$ with a constant proportional rise in income. As far as mechanics go, this is much the same case as demand pull inflation. However, the economy

[8] By similar triangles we have:

$$OF_2/F_2B = OF_3/F_3D.$$

Since $F_3D = F_2C = OF_2$, and $F_2B = F_1A = OF_1$, we find by substitution:

$$OF_2/OF_1 = OF_3/OF_2.$$

is in equilibrium at points like A, C, and E where income and expenditure are determined for succeeding periods. In terms of interest rate terminology, the natural and money rate curves are horizontal and equal at income levels equal to or greater than full employment. If the income axis is a log scale (ratios being represented), equal steps are taken along the income axis in each period. The natural and money rates remain equal after each step.

COST–PUSH INFLATION AT LESS-THAN-FULL-EMPLOYMENT

One of the important developments of the past decade has been the emergence of a new variety of inflation. This is the phenomenon of inflation during periods of underemployment. In the past it was believed that a necessary condition for inflation was an excess of demand for goods over supply. With the emergence of this new and more stubborn inflationary tendency has come a

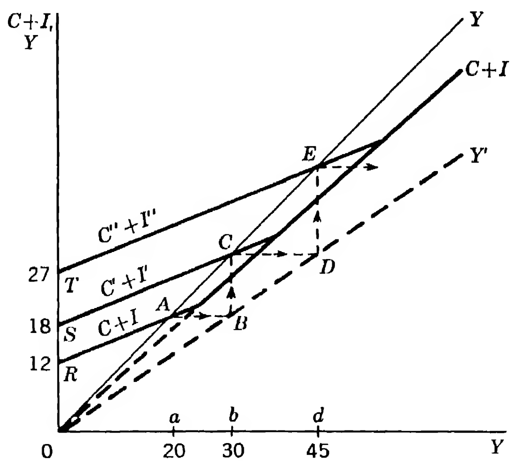


Figure 96. Cost-Push at Less-than-Full Employment.

realization that the excess demand condition is not necessary. Let us illustrate this possibility with a simple model closely resembling the previous ones.

Assume an underemployment equilibrium exists at point A with income of Oa as shown in Figure 96. In this situation the owners

of factors decide to press for higher incomes at a certain percentage rate per year. This is represented in Figure 96 by a shifting of the Y curve to the position Y'. As in the previous models, this implies that a given factor income in one period must be succeeded by an income which is higher by a certain percentage in the next period. If such an increase takes place it will be represented by an increase of $AB = ab$ in income (at factor cost). The expected proportional increase in each year is ab/Oa *by construction.* By the usual argument the proportional increase in each year is the same: $ab/Oa = bd/Ob$.

In an inflationary situation with underemployment the employers will want to raise price of goods in proportion to factor price. As income increases from Oa to Ob, prices of goods change in the same proportion, and the purchasing power of the new income is the same as before. Consequently, demand in real terms should remain the same. The money demand at the new income should increase in proportion to the increase in money income. Since income and money demand were equal to begin with, they will be equal after a common percentage increase. Evidently, the new demand at the increased income is found at C, where $AB = BC$ is the rise in income and total spending.

In a situation like this income and expenditure will rise in the stairstep pattern, $ABCDE \ldots$; income and prices (both factor and product) will rise in the common proportion, $ab/Oa = bd/Ob = \ldots$; income and expenditure in dollars of constant purchasing power will remain *constant.* Underemployment equilibrium will exist at all times. The short-run consumption line shifts upward in the same proportion as the rise in factor incomes. This construction was explained in an earlier diagram (Figure 90). Only when an increase in real income (along with the normal price increase) occurs does a movement along a short-run line occur.

Clearly then, it is possible to have an inflationary situation with less than full employment. The condition necessary for the emergence of this state of affairs is the willingness of factors to press for higher returns. In turn, either laborers or employers may be responsible for this tendency. Regardless, the higher factor returns are required as a condition for furnishing the same real income (or output).

REMEDIES FOR COST–PUSH INFLATION: OUTPUT EXPANSION

What can be done to prevent cost–push inflation. First, consider the possibility of an increase in output. If the output of the economy can be made to grow in the same ratio as factor income, prices will not increase. Before going into complications note that output would have to grow in the proportion, $OF_2/OF_1 = OF_3/OF_2$, etc., from year to year. Recall Figure 95.

Note that expenditure is equal to income in each period, the points of equality being found at A, C, E and similar points on the Y, $C + I$ line. Consequently, expenditure is rising in the proportion, OF_2/OF_1, etc. If output also increases in the proportion, OF_2/OF_1, etc., output will maintain the same proportion to expenditure as before. Obviously, this implies that commodity prices remain unchanged.[9] This does not necessarily imply, however, that the prices of the factors are constant.

As a matter of fact, it is not critical whether factor prices rise or not so long as this change is not translated into higher product prices. If the increase in production were provided by a corresponding increase in the factors there would have been no change in the level of factor prices. If the quantities of all factors increased 10 percent, and output increased 10 percent, so that the social production function is homogeneous and linear, the condition in question would be satisfied.[10] In this event the economy is expanding in scale with a flow of additional money income which just suffices to keep factor and product prices constant.

If the result discussed took place, the factors would not be succeeding in obtaining a higher return per unit of the factor. Consequently, the cost–push discussed merely arises from the increased quantities of the factors seeking employment. In a sense, then, the desire on the part of the factors for increased incomes is not strictly inflationary. In fact, an increased flow of money

[9] Write $E = C + I = P \cdot Q$, where Q is the level of output. By assumption $E_2/E_1 = OF_2/OF_1 = Q_2/Q_1$. Hence $E_2/E_1 = P_2Q_2/P_1Q_1 = P_2/P_1 \cdot Q_2/Q_1 = P_2/P_1 \cdot E_2/E_1$. Crossing out E_2/E_1 on either side of the equation, we get: $P_2/P_1 = 1$ or $P_2 = P_1$.

[10] Let Q be output and A, B, C be quantities of factors, all connected by a social production function, $Q = f(A,B,C)$. This function is a homogeneous linear function, if, when the factors are increased in a common proportion k to become kA, kB, kC, the output increases in the proportion kQ. In short, we have $f(kA,kB,kC) = kf(A,B,C) = kQ$.

incomes is necessary in the circumstances to keep product prices constant. Without the increased flow of income and consequent expenditure prices of products would fall with detrimental effects on production. Our first "remedy," then, really turns out to be a description of the balanced growth of the economy. In such a case the expansion of incomes is strictly justifiable.

Consider a somewhat different situation in which the increased production offsetting the increased flow of factor incomes results purely from technological change. Assume that the quantities of the factors are constant. In this case the pressure for increased factor incomes implies a like pressure for increased factor prices. As the increased incomes are generated, they are passed back to the recipients in the form of higher factor prices for the same quantities. As in the previous case, the increased flow of output causes the corresponding price level to remain the same. Obviously, the increased flow of income must be imputed to the factors. Since factor quantities are constant, the imputation process forces up the prices of the factors. Without this imputation incomes would not rise, and without increased money income the prices of products would not be maintained. Again, the remedy has led to a description of an economic system which is expanding in a rational manner. In this case factor prices are rising, but only in a fashion justified by changes in technology.

From the foregoing discussion it is evident that a cost-push to incomes associated with increased quantities of the factors or improved technology, leading to proportional output increases, does not involve inflation. In fact, it is the pressure for increased factor incomes out of proportion to increased production which constitutes the main problem under consideration. Only by dealing with the source of such excessive desires for increased incomes can the inflation be checked.

REMEDIES AIMED AT THE CAUSES OF UNWARRANTED INCREASES IN FACTOR PRICES

A. Wage—Cost Push

In recent years it seems probable that labor has sought wage increases exceeding the growth in labor productivity. To eliminate inflation from this source it would be necessary to cause labor

to abate its demands for increased wages. In particular, the unions are a logical focus of efforts to dampen the increase in wages. Yet the logic of the union movement seems to involve pushing for large annual increases, irrespective of the progress of the economy. Consequently, efforts to reduce the rate of wage increase appear to cut across the basic design of labor institutions.

In actuality, the unions themselves represent something desired by the membership. To produce a basic change in the situation it is necessary to induce both the leaders and the rank and file to face the consequences of heavy wage pressure. Only by seeing clearly the damage which inflation wreaks on the structure of the economy can they be induced to modify this course of action. This is a very difficult step to accomplish, because people tend to see what they want to in cases where their self-interest is greatly involved.

If persuasion is difficult, actual coercion in any form is hard to imagine. At present, the only obvious method would be to press an attack through the medium of the antitrust laws on unions asking for excessive wage increases. Steadily advancing wages which exceeded productivity growth could be taken as a symptom of monopoly power and treated as such. Just what action could be taken against such unions is not apparent—the main idea would be to establish the notion that excessive wage pressure would lead to unpleasant consequences.

B. Profit Push

In some cases manufacturers may plan to impose a rising price trend on the public.[11] Clearly, this is impossible without some degree of monopoly power. Moreover, it is possible that such a tendency could be presented in the guise of necessity. Under the cloak of meeting wage cost increases, of providing an improved product, or other plausible pretexts the price could be raised annually. Clearly, the purpose of the rising price trend is to provide increasing profits, but it is quite difficult to separate this from the push of rising factor prices. In this case the remedy of antitrust law prosecution seems more realistic and reasonable than in the preceding one.

[11] Gardiner C. Means. *Administrative Inflation and Public Policy,* Anderson Kramer Associates, Washington, D.C., 1959.

MONETARY REMEDIES FOR COST–PUSH INFLATION

Underlying this analysis of cost–push inflation is the assumption that the economy is in neutral equilibrium in an upward direction at any income level. When the income level is given a shove by higher factor cost, it moves to a new level without any further tendency to change. For this to be possible it is necessary that the money supply be perfectly elastic so that the interest rate remains the same even when the demand for money increases. As in the initial analysis of demand–pull inflation, the monetary and banking system acquiesces in any desired change in the money supply without changing the terms on which loans are made.

If this assumption is dropped, and it is assumed that the supply curve of money is less than perfectly elastic, an increase in factor costs (income) will lead to an increased demand for money and a rise in the interest rate. In turn, this will lower demand for capital goods, and cause aggregate demand to fall below supply (valued at factor cost). In alternative terminology, the bank rate will be forced above the natural rate by the increased demand and inelastic supply of money.

Evidently, the inelastic supply of money is a condition which stabilizes the level of income. To put it another way, beyond full employment the goods market tends to move into a neutral equilibrium of the type described in Say's Law. In this condition the level of income is subject to change from a variety of forces among which the cost–push is prominent. If the monetary system puts up no resistance to inflationary forces originating in the goods market, prices will rise. However, if the supply of money is only increased to the accompaniment of higher interest rates, a definite resistance to the inflation is present. From this we see that the burden of resistance to inflation tends to pass to the monetary system once a condition of full employment is reached, the goods market tending to lose its self-stabilizing characteristic.

Problems of cycle policy

A good many of the basic problems in this area have already been considered. What remains to be covered is the timing of

monetary and fiscal policy and the indirect effects of policy changes on private business.

In the timing of monetary changes we have a minimum of difficulty. It is always possible to plan and execute monetary changes with a minimum of time lag. If the economic situation changes for the worse, discount rates can be reduced, bonds purchased on the open market, and reserve requirements reduced in a limited period of time. Now it is not evident that the effects will be immediately apparent, but, at least, the initial policy changes can be effected rapidly.

On the other hand, fiscal changes require more time to execute. Suppose an excessive expansion in economic activity is in prospect. Proper fiscal actions consist in a reduction of government spending and an increase in taxes. If the activities of the relevant government bureaus have been at all carefully planned, it will not be easy to interrupt them on short notice with a view to cutting outlay. Contracts may have been let, personnel hired for a year, and other obligations undertaken. If cost is cut, a considerable loss or downright waste will be incurred as a result of a budget cut. Even more important is the loss in efficiency owing to a slump in morale. Obviously, short-term cutting of expenses is going to be both wasteful and difficult in that the loss in results achieved is likely to outweigh the reduction in money cost.

Raising taxes on short notice may prove to be even more difficult. Whereas expenses may be cut by executive action, taxes must be levied by Congress. There are obvious limitations on the speed with which this can be accomplished. Not to be overlooked also is the fact that tax increases are extremely controversial. Such changes must be carefully explained and justified to the public.

Fiscal changes designed to offset depressions face many of the same difficulties. A short-term increase in expenditure designed to offset a recession is difficult to manage. Budgets for existing departments are set up well in advance, and are not easy to expand on short notice. More important, it is not easy to make a sudden expansion in outlay so as to stimulate the economy in those directions where the need is greatest. For example, it would be easy simply to raise everyone's pay, but hard to plan a program involving effective use of more personnel. This problem is also

complicated by the fact that expansion of existing activities tends to be hindered by diminishing returns.

Planning and initiating new enterprises on short notice is also quite difficult. For one thing the lead time required is substantial. By the same token crash programs tend to develop "bugs" which are damaging to their continuance. If new projects are kept on file for emergency use, they lose currency and interest. By keeping such projects continuously up to date the authorities might avoid this difficulty, but the cost of developing unused programs would be high.

Fortunately, one part of a fiscal program to increase incomes is easy to execute. This is a cut in individual income taxes. Such an action would combine the possibility of reasonably rapid action and effective results. Obviously, the political climate during a period of depressed economic conditions would also tend to be favorable.

Consider a more fundamental kind of difficulty—the problem of calculating effects. In the realm of monetary policy this has long been known. Under certain circumstances cheapening credit may cause a rapid expansion of the economy and increased income. Under other circumstances, such as deep depression, credit may not expand in the desired way. To some extent the causes of the varying effects of monetary policies are known and corresponding allowances may be made, but an exact calculation is rarely possible.

FISCAL POLICY IN AN UNSTABLE ECONOMY

As our study of cycles has shown, the economy is capable of passing rather suddenly from a stable to an unstable situation. When the unstable phase is reached, the situation has passed out of the control of the fiscal or monetary authorities. Such a possibility must not be forgotten in determining fiscal and monetary policy.

Suppose that the economy is in stable equilibrium in the lower phase of the cycle. The natural rate is equal to the money rate at a rather low figure, but a rather slight rise in the natural rate will bring the economy to the critical point. If the government then expands spending and cuts taxes, investment plus government

spending will rise while saving plus taxes will fall.[12] To bring about equality between $I + G$ and $S + T$ it is necessary for the rate of interest to rise. Such a rise in the interest rate will reduce I (and possibly increase S) until $I + G$ is reduced to equality with $S + T$. In short, the natural rate of interest which equates $I + G$ with $S + T$ (in this slightly generalized analysis) has risen.

If the economy is near the critical point, this jump in the natural rate may cause it to rise above the money rate. Owing to the unstable character of investment as income expands, the natural rate continues above the money rate, and the level of income will advance to or beyond the point of full employment. If the government entered into its spending program seriously with firm intent to lift the income level, it will find cancellation of its program somewhat difficult. This will involve the government in a possible inflation.

What may happen, in brief, is that the Treasury may overdo its plans, not being able to anticipate a recovery of such magnitude. Once involved in large-scale enterprises the government may find withdrawal well-nigh impossible. Exactly the same problem arises in halting a boom verging on inflation. A cut in spending and a rise in taxes may so lower the natural rate (I,S) curve to bring it to the critical point of contact with the money rate (L,M) curve. At this juncture the fat is in the fire and a severe contraction sets in. Nor will countercyclical action by the government have a big effect at this time. Consequently, an anti-inflation program may have the effect of precipitating or encouraging a recession.

THE TIMING OF ANTICYCLICAL MEASURES

If fiscal and monetary actions are to have their due effect, they must be carefully timed. When a recovery movement is relatively weak, monetary and fiscal measures for expansion should be pushed with all vigor well into the recovery stage. As the time comes to let off pressure, it should be done gradually. In 1937, with Treasury and Federal Reserve overly afraid of a repetition of 1929, the brakes were applied too rapidly with the result that a sharp recession set in thereafter. Nowadays these authorities

[12] If taxes fall by 1 with an MPC of .8 and an MPS of .2, saving will increase by .2 compared to a decrease of 1 in taxes. This implies that saving and taxes decrease by .8.

seem to have more respect for the possibly fragile quality of a recovery.

When a recession is felt to be rather mild, and the ensuing recovery is strong, expansionary measures may be abondoned before full employment is reached. In fact, this was the policy actually followed by the Federal Reserve in 1958. Failure to reverse actions taken shortly before to ease credit might have accelerated the recovery unduly and led straight into a period of inflation.

Roughly speaking, action in the cycle by the fiscal–monetary authorities should follow a pattern which is the reverse of private activity. As National Income begins to fall, actions designed to stimulate the economy should be initiated. As National Income falls, these actions should be strengthened in proportion to the fall, etc. However, plans must precede actions, if they are to be implemented in a timely fashion.

The basic obstacle to implementation of this approach is that we cannot foresee the future. In consequence, errors in estimating the need for government action will be made, implying that the actions taken may be ineffective or serve to increase the magnitude of fluctuations. For example, expansionary action extended well into the recovery phase may push the economy into an inflationary process.

If public action is to be effective, there must be ways of adjusting policies fairly frequently. Failing this, a mistaken policy may be continued in such a way as to encourage and increase economic instability. This is the real bug-a-boo of monetary-fiscal policy. The ideal safeguard is the capability of altering policy actions at reasonably short intervals. An overshoot of public action in one direction may be corrected by an opposite policy, provided the error is recognized and the policy altered.

PRIVATE REACTION TO PUBLIC ACTION

It is suggested by some that actions such as additional public spending will lead to compensating changes in private investment. In this way the effect of the public policy is nullified. Consider briefly the bases of such an event. The causes may be classified under two headings: (1) the balanced budget shibboleth; (2) the fear of socialism.

From their business experience leaders of industry know that a firm which fails to cover all its costs will ultimately fail. In short, a firm's private budget must be balanced or the firm will go out of business. Similar considerations apply to the household in its use of money. Reinforcing this attitude is an extensive historical record of mismanagement of fiscal affairs by government. Consequently, a firmly balanced budget is regarded by most businessmen as a prime condition of sound fiscal policy. A policy of spending more than income to relieve unemployment is likely to be regarded with great disquiet. A situation of this kind might portend such things as a future disorganization of the financial markets as government debt begins to pile up, an increase in the future level of business and private taxation, the possibility that the government will resort to the printing press or other inflationary means to cover expenses or repay debt. These fears are not wholly imaginary, but bear little relation to the short-run situation and the need for its solution.

In addition, businessmen tend to regard any expansion of government spending as the first step on the road to socialism. More precisely, they are inclined to feel that once the government has initiated a program in a certain field it will never get out. Indeed, they fear that the initial step in such a field will lead to gradually expanding government activity. Pushed far enough, this will tend to eliminate private initiative, leaving the field to the government. Under the influence of the budget shibboleth and fears of socialism, businessmen may suffer a loss of confidence as a result of countercyclical fiscal action. Such an adverse reaction may lead to a decline in the MEC schedule and a corresponding fall in investment.

At the other end of the cycle the reaction may be equally perverse. As income and receipts rise, with constant government spending, a nice government surplus may emerge. If government improves on this by actually cutting outlays and increasing taxes, the surplus will be even larger. In itself, such a pattern of events tends to dampen any inflationary tendency in private business. On the other hand, the prospect of operating in an economy with such a "soundly managed" treasury may prove very stimulating to business. There is no prospect of printing press inflation or

future tax increases and the reduced government activity may betoken a greater scope for private enterprise. Such considerations may stimulate the confidence of businessmen and cause the MEC curve to rise. In turn, this rise will tend to raise investment in a perverse and untimely manner.

If all this happens, and it seems a bit fanciful, we will have to reckon with less leverage effect from government spending. In fact, the revised multiplier is:

$$\frac{\Delta Y}{\Delta G} = \frac{1 + \text{MPD}}{1 - \text{MPC}}, \tag{15-1}$$

where MPD $= \Delta I/\Delta G$ is the marginal propensity to disinvest with additional government spending. We assume that MPD is negative so that the numerator of the fraction is less than 1. If MPD is negative and of the same magnitude as MPC, the government multiplier is only 1. If negative and smaller than MPC, the multiplier is greater than 1, but less than the ordinary multiplier. If negative and larger than MPC, the multiplier is less than 1.

All this simply suggests that a perverse reaction on the part of businessmen necessitates more vigorous action by the Treasury. With much of the leverage stripped from the multiplier it would be far more difficult to pursue a policy of "compensatory spending."

This sort of thing can easily be exaggerated. Businessmen are nothing if not practical. If a profit is to be made, they will probably try to make it, even if they break a shibboleth in so doing. So long as public policy is clearly defined (say, as an anticyclical one), businessmen will adjust to the situation. The worst policy is an ill-defined, pussy-footing approach to the situation. This increases the range of uncertainty and lowers the MEC. Businessmen are hardy creatures and tend to react in a healthy manner in a well-defined situation.

Economic growth

The topic of economic growth is much too large to receive thorough treatment here, and only one or two points will be covered. We have already considered the relationship between

the growth of money income and that of output. If output increases in the same proportion as demand, prices of goods will remain the same. From the viewpoint of growth the role of economic policy is to bring about conditions which make possible the steady expansion of output. Roughly, these conditions may be divided into two parts: (1) those conditions of factor supply which make possible a steady expansion of output; (2) those conditions of demand which make possible the purchase of these goods at prices which cover cost.

Consider the availability of factors during a growth process. If the economy is an expanding one, the supplies of the factors will probably be expanding also. Suppose that constant returns to scale prevail. If all the factors are expanding at the same proportionate rate, output increases in the same proportion. This being the case, a steady expansion of output can take place. If the factor supplies increase at an unequal rate, a different sort of situation arises.

Suppose all factors but one increase at the desired rate. The factors increasing at the same rate can be treated as one factor and the slowly increasing factor as another. Suppose one rate is 3 percent, while the other is 5 percent, the desired growth rate for output. Clearly, a uniform 3 percent increase in all the factors under constant returns to scale leads to a 3 percent increase in output. However, the desired rate is 5 percent. The extra 2 percent increase in one factor with no increase in the other factor tends to lead to a smaller proportional increase in output, because one of the factors does not increase to the extent necessary. Hence output grows at a rate less than the desired 5 percent.

These considerations lead to the obvious conclusion that factor supplies which increase at unequal rates constitute a condition which limits growth rates. In certain economies capital increases more slowly than other factors. In turn, this leads to capital scarcity and a straitjacketing of certain types of expansion. In industrial economies like that of the United States the factor tending to greatest scarcity is land. The inability to expand the supply of land to the same extent as labor and capital leads to diminishing returns on land. This is the case analyzed by the

classical writers; it applied then to the English economy and applies prospectively to that of the United States.

In order to keep the supply of land up to the desired level it is necessary to engage in vigorous exploration for new resources. If this is insufficient, new techniques must be developed which make available existing materials for use in new ways. Although the flow of resources may change in composition, new techniques can sustain its quantity and usefulness.

During an extended period of expansion the flow of capital may grow at a constant rate, while land eventually increases more slowly. Only if techniques improve can the growth rate be maintained. Yet it is too much to expect that the technical growth can continue indefinitely at the desired rate. If and when this improvement slackens, the growth rate for output will decline. In our combined cycle-growth model this led to a decline in the MEC curve and in the natural rate which equates *I* and *S*. If the effects stopped here, the situation would not be much to worry about. However, this lapse may carry the system into a relationship in which a large contraction of economic activity is inevitable. In turn, this contraction may be succeeded by a period of economic fluctuation, rather than straight growth.

If the monetary-fiscal apparatus is employed to sustain economic growth, the situation may not be improved. In fact, this apparatus is desiged to encourage investment or government spending. Since government spending of the public works variety amounts to government investment, the MEC and private investment opportunities tend to be depressed even further. Working out opportunities for the use of capital at this time may lead to a deferment of the recession at the cost of increasing its amplitude and duration when it comes.

MAINTENANCE OF DEMAND NECESSARY FOR GROWTH

Next consider the need to maintain demand to the end that producers may sell their output at a remunerative price. Assume that inputs and outputs increase in proportion. Now a proportional increase in the several types of output is not necessarily what is wanted by consumers. Empirical evidence tends to indicate that as individual incomes rise, there is a tendency to spend a

larger share on such items as housing, automobiles, and consumer durables and a smaller share on food and clothing. In addition to this, some of the evidence tends to indicate that consumption may rise less rapidly than saving as individual incomes expand. Recall the evidence of Table 11. In short, the ratio, C/Y, may drop if the right conditions are not fulfilled.

In order to maintain the ratio of C to Y attractive new products should make their appearance. With a stimulus of this kind households will be inclined to maintain their consumption-income ratio. As a case in point, the automobile industry has maintained its position by offering a changing product, a wider variety of products, and products adapted to consumer tastes. If the growth of income is to be long sustained, entirely new products or innovations may be necessary. What seems to present the greatest problem is that the maintenance of a steady flow of new products of types wanted by consumers may not be possible. By its nature the process is jerky and irregular. If a lag occurs at some point in the process, the ratio C/Y may fall. Saving increases, the natural rate falls relative to the money rate, and a contraction may set in forthwith.

SUMMARY: BARRIERS TO CONTINUED ECONOMIC GROWTH

Summarizing the discussion briefly, there appear to be two main barriers to continued economic growth. First, the supply of land does not grow except by exploitation and the discovery of new techniques. If the flow of new discoveries and techniques is insufficient to offset the tendency for resource supply to lag, diminishing returns will tend to be experienced. In turn, this implies a slackening in the growth of output, declining MEC, setting the stage for possible contraction. Second, the maintenance of the consumption–income ratio with growing individual incomes requires a stream of new products. In all probability the introduction of such products is a jerky process. When a lag in the process occurs, the ratio, C/Y, may fall with adverse repercussions on the growth process.

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